Elapsed Time Formula

\[ t_k - t_j = \tau \ln \frac{x(t_j) - x(t_\infty)}{x(t_k) - x(t_\infty)} \]

any point lying on an exponential waveform with positive or negative \( \tau \)}
1st-order Linear Time-Invariant Circuits Driven by DC Sources

State Equation

\[
\dot{x} = -\frac{x}{\tau} + \frac{x(t_{\infty})}{\tau}
\]

\(\tau = \) time constant

\(x(t_{\infty}) = \) Equilibrium Point

\(x(t_0) = \) Initial State at \(t = t_0\)

Solution:

\[
x(t) = x(t_{\infty}) + \left[ x(t_0) - x(t_{\infty}) \right] e^{-\frac{(t-t_0)}{\tau}}
\]

Stable Case: \(\tau > 0\)

(a) \(x(t_0) < x(t_{\infty})\)
1st-order Linear Time-Invariant Circuits Driven by DC Sources

State Equation

\[ \dot{x} = -\frac{x}{\tau} + \frac{x(t_\infty)}{\tau} \]

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Solution:

\[ x(t) = x(t_\infty) + \left[ x(t_0) - x(t_\infty) \right] e^{-\frac{(t-t_0)}{\tau}} \]

Stable Case: \( \tau > 0 \)

(b) \( x(t_0) > x(t_\infty) \)

\[ 0.63 \left[ x(t_0) - x(t_\infty) \right] \]

\( \tau > 0 \)
Untable Case: $\tau < 0$

(a) $x(t_0) > x(t_\infty)$
Untable Case: $\tau < 0$

(b) $x(t_0) < x(t_{\infty})$
\[
x(t_j) - x(t_\infty) = \left[ x(t_0) - x(t_\infty) \right] e^{-\frac{(t_j-t_0)}{\tau}}
\]
\[
x(t_k) - x(t_\infty) = \left[ x(t_0) - x(t_\infty) \right] e^{-\frac{(t_k-t_0)}{\tau}}
\]
\[
\frac{x(t_j) - x(t_\infty)}{x(t_k) - x(t_\infty)} = e^{-\frac{(t_j-t_0)}{\tau}} = e^{-\frac{(t_k-t_0)}{\tau}} = e^{-\frac{(t_k-t_j)}{\tau}}
\]
\[
\ln \left[ \frac{x(t_j) - x(t_\infty)}{x(t_k) - x(t_\infty)} \right] = \frac{(t_k - t_j)}{\tau}
\]
\[
\Rightarrow \quad t_k - t_j = \tau \ln \left[ \frac{x(t_j) - x(t_\infty)}{x(t_k) - x(t_\infty)} \right]
\]
Driving-Point Characteristic

The 2 nodes \{①, ②\} where the voltage source is connected are called driving-point terminals.

The \(i\)-vs.-\(v\) driving-point characteristic is the set of all \((i,v)\) which simultaneously satisfy:

1. KCL
2. KVL
3. Constitutive Relation of all elements inside \(\mathbf{N}\)
The $v_o$-vs.-$v_i$ transfer characteristic is the set of all $(v_i, v_o)$ which simultaneously satisfy:

1. KCL
2. KVL
3. $v$ - $i$ characteristics of all elements inside $N$
4. $i_o = 0$ (no-loading condition)
DP (Driving-Point) Plot

\[ \beta = \frac{R_B}{R_A + R_B} \]

\[ \text{slope} = -\frac{R_A}{R_B} \left( \frac{1}{R_f} \right) \]

\[ \frac{1}{R_f} \quad \frac{1}{R_f} \quad \frac{1}{R_f} \]

\[ \beta E_{\text{sat}} \]

\[ -\beta E_{\text{sat}} \]