1. (Practical capacitive level sensor) Hambley Problem 3.32 (10 points)

2. (Use of a Wheatstone bridge) Hambley 2.86 (10 points)

3. (Limitations of the Wheatstone bridge) Hambley 2.88 (10 points)

4. (Thevenin equivalent, energy storage density) (35 points)
   Measurements made on a AA battery gave the following results: When a voltmeter (a Fluke digital multimeter – a DMM -- that can be considered an ideal voltmeter for this problem) was connected across the ends of the battery, the display read 1.586 V. When the meter was switched to read current (consider it an ideal ammeter) the reading was 6.82 A.
   a. Sketch the equivalent circuit for this battery and indicate the values of the components.
   Next a precision 10.0-ohm resistor was connected as a load across the terminals of the battery.
   b. Find the current through the load resistor.
   Battery life is usually indicated in terms of a current times the battery life. For this battery assume the life is 60mA-hrs (it could produce a current of 60 mA for 1 hour, or 30 mA for 2 hrs, etc.).
   c. If you assume that the battery voltage remains constant until the instant the battery dies (when the chemicals inside have been exhausted), how long would the current flow in the circuit described above?
   d. How much chemical energy in joules is converted to electrical energy (actually, heat) during the battery’s life?
   e. Physically an AA battery measures about 4.8 cm in height and 1 cm in diameter. What is the density of energy storage (joules per cubic meter) for this form of chemical energy storage?
   f. One can store electrical energy in a capacitor also. Consider a 30,000 μF electrolytic capacitor that is 8 inches tall and 3.5 inches in diameter and that is charged to 60 volts. How much energy is stored and what is the energy storage density in J/m^3? How does this energy density compare with the value for chemical storage in the AA battery?
   g. Determine the approximate density of storage in the form of magnetic energy in the region inside and around an inductor carrying 5A current, having an inductance of 1 mH, and dimensions equal to those of a AA battery.

5. (Condenser microphone) (15 points)
   A condenser microphone biased by a DC voltage source was shown in the Week3b lecture notes. Suppose the area of the flexible diaphragm of the microphone is 1 cm^2, the
height of the condenser is 2 mm, the bias voltage is 200 V, and when a sinusoidal pressure wave impinges on the diaphragm the displacement of the diaphragm is \( x(t) = 2000 + \sin(300t) \) microns.

a. Find the alternating current \( i(t) \) through the condenser. You can use the approximation \( I/(1+x) \sim 1-x \) when \( x<<1 \).

b. Since the value of the GΩ resistor isn’t specified precisely, find the output voltage between the flexible membrane and the ground using the approximate expression given beneath the sketch in the notes: \( V_{out} \sim 8x \ E_{const} \). (Assume that the series capacitor and the unity gain buffer are equivalent to a wire connection to the flexible diaphragm.)

6. (Capacitors and inductors at low and high frequencies) (10 points)
A 1 μF capacitor \( C \) is connected across a function generator that produces a triangular voltage waveform. The frequency is adjusted from 1 cycle every second (1 Hz, low frequency) to \( 10^6 \) cycles every second (1 MHz, high frequency). The peak-to-peak voltage is 2 V.
Determine the capacitor current, \( i_C(t) \), for successive time intervals when the voltage is rising and when it is falling, for low and high frequencies.
   a. At low frequency does a capacitor behave more like an open circuit or a short circuit? (Consider whether this seems intuitively correct, given the structure of a capacitor.)
   b. At high frequency does a capacitor behave more like an open or a short circuit?
   c. Suppose now that a 1mH inductor is put in place of the capacitor.
   d. With the same low and high frequency triangular waves calculate the voltages across the inductor.
   e. Does an inductor behave at low frequencies more like an open circuit or a short circuit? (Consider whether this seems intuitively correct, given the structure of an inductor.)

7. (Are ideal capacitors and inductors linear elements?) (10 points)
You’ve seen that the ideal resistor has a linear I-V relationship – we call that Ohm’s Law. It means that when we double the voltage across a resistor, the current through the resistor doubles. Show that the ideal capacitor is also linear in that sense – even though it isn’t obvious at first because the I-V relationship involves a time derivative. Start with the \( i(t)-v(t) \) relationship for a capacitor \( C \) and apply two different voltages: First apply voltage \( v_1(t) = V_1\sin(\omega t) \) and find \( i_1(t) \). Next apply a three times bigger voltage at the same frequency and find \( i_2(t) \). Is this ideal capacitor linear? (You could show the same is true of ideal inductors.)

Comment: Why should we care about linearity? One reason is that we can apply the principle of superposition to circuits composed entirely of linear elements, and use that approach to solve some circuit problems. Also, if we drive a linear circuit with a source at some single frequency (say, 150 kHz) we can be sure that all the voltages and currents that arise in that circuit are also at 150 kHz. If nonlinear circuit elements are in a circuit, additional frequency components may arise.