

3.1 The Concave and Convex Resistors

The usual convention for plotting characteristics of nonlinear resistors is to use the v - i plane instead of the i - v plane. For the circuit in Fig. 3.1a, the characteristic plotted in the v - i plane is shown in Fig. 3.1b. (See Fig. 2.6 for comparison.) The shape of the curve suggests a name: concave resistor. Its symbol is shown in Fig. 3.1c. Thus, the *concave resistor* in Fig. 3.1 is a piecewise-linear voltage-controlled resistor which is uniquely specified by two parameters: G , the slope of the linear segment, and E , the breakpoint voltage. In terms of a function representation, a concave resistor can be specified by

$$i = \frac{1}{2}G[|v - E| + (v - E)] \quad (3.1)$$

Let us demonstrate that Eq. (3.1) indeed leads to the v - i characteristic of Fig. 3.1b. We use two approaches. First, consider Fig. 3.2a where $i = (G/2)(v - E)$ is plotted. In Fig. 3.2b, we plot its absolute value, i.e., $(G/2)|v - E|$. Adding the two, we obtain the characteristic of the concave resistor shown in Fig. 3.1b.

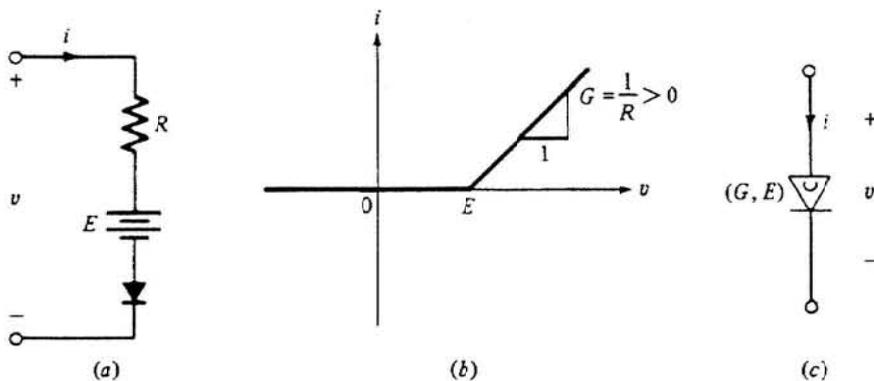


Figure 3.1 (a) Equivalent circuit, (b) characteristic, and (c) symbol for a concave resistor.

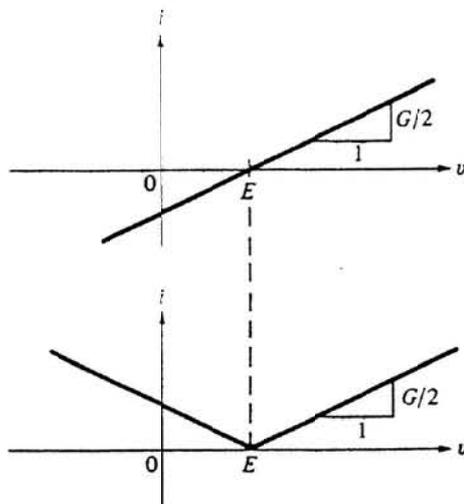


Figure 3.2 Graphic interpretation of Eq. (3.1): (a) $i = (G/2)(v - E)$ and (b) $i = (G/2)|v - E|$.

Second, we consider directly Eq. (3.1) which states

$$i = G(v - E) \quad \text{for } v - E \geq 0 \quad (3.2a)$$

This is precisely the characteristic of the one-port in Fig. 3.1a when the diode is conducting. Now Eq. (3.1) also states

$$i = 0 \quad \text{for } v - E < 0 \quad (3.2b)$$

This is the characteristic of the same one-port when the diode is reversed biased.

The circuit shown in Fig. 3.3a has the v - i characteristic shown in Fig. 3.3b. This characteristic defines a convex resistor whose symbol is shown in Fig. 3.3c. The *convex resistor* in Fig. 3.3 is a piecewise-linear *current-controlled resistor* which is uniquely specified by two parameters: $G = 1/R$, the slope of the linear segment, and I , the breakpoint current. Being current-controlled, it can be represented by

$$v = \frac{1}{2}R[|i - I| + (i - I)] \quad (3.3)$$

Clearly,

$$v = R(i - I) \quad \text{for } i \geq I \quad (3.4a)$$

and

$$v = 0 \quad \text{for } i < I \quad (3.4b)$$

Note that concave and convex resistors are dual elements provided $G = R$ and $E = I$.

Exercise Demonstrate that if the circuit in Fig. 3.3a has $G < 0$, the v - i characteristic of the one-port is given by that of Fig. 3.4b and not by that of Fig. 3.4a. Note that the characteristic in Fig. 3.4b is not current-controlled.

REMARK Concave and convex resistors realized by the circuits in Figs. 3.1a and 3.3a require that $G > 0$. However, as will be demonstrated in the next section, it is convenient to use concave and convex resistors in device modeling even with G negative. For example, if $G < 0$, the convex resistor

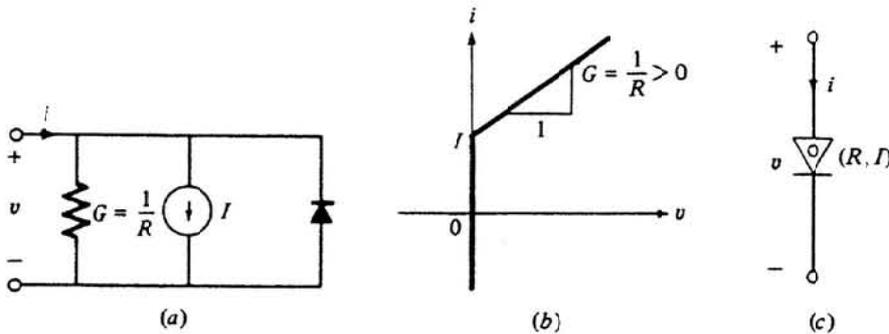


Figure 3.3 (a) Equivalent circuit, (b) characteristic, and (c) symbol for a convex resistor.

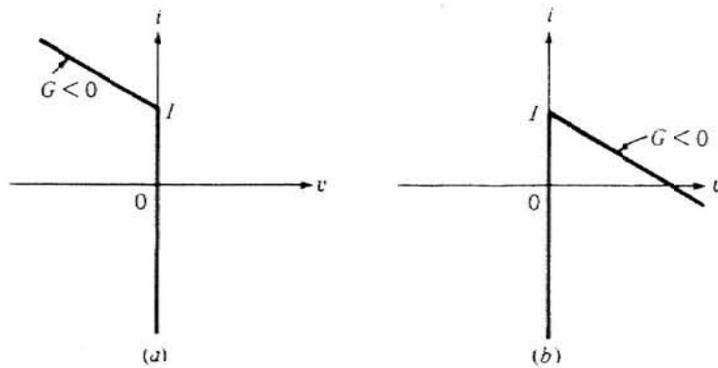


Figure 3.4 (a) The characteristic of a convex resistor with G negative. (b) The v - i characteristic of the one-port in Fig. 3.3a when G is negative.

is defined by the current-controlled characteristic shown in Fig. 3.4a. In this case, Eq. (3.3) is still valid but the circuit shown in Fig. 3.3a no longer gives the correct v - i characteristic. However, other circuits (using the operational amplifier) exist for realizing a concave or convex resistor having a negative G . For device-modeling purposes, concave and convex resistors are simple two-terminal resistors defined by Eqs. (3.1) and (3.3), respectively, where G may assume either positive or negative values. In such applications, since we do not intend to build the circuit model—models are used mainly for analysis purposes—the question of a physical circuit realization is irrelevant.

3.2 Approximation and Synthesis

We have used graphic methods to study the properties of nonlinear resistors. Graphic methods are useful not only for analyzing simple circuits but also for providing insight and understanding of nonlinear behavior of circuits. We will have further opportunities to illustrate various graphic techniques in this book.

When we deal with complex circuits, graphic methods are either too cumbersome or not applicable. In such cases, we need to rely on computers for calculation. Therefore it is important to develop analytic methods to formulate problems precisely and to approximate nonlinear characteristics in mathematical form. Sometimes, a mathematical characterization can be obtained from the physics of the device, e.g., the pn -junction law [see Eq. (1.6)]. However, often we have to rely on measurements or curves provided by device manufacturers. Therefore we need to introduce methods of approximation. For example, the tunnel-diode characteristic can be approximated by a polynomial

$$i \approx \sum_{k=0}^n a_k v^k$$