Tricks for non-linear circuits

(1) v_∞, i_∞

Capacitors: $\frac{i}{c} = \frac{dv}{dt}$ $\quad$ i_∞ always 0

Inductors: $\frac{-v}{L} = \frac{di}{dt}$ $\quad$ v_∞ always 0

(2) Always use $\frac{-i}{c} = \frac{dv}{dt}$ for Cap. $\frac{-v}{L} = \frac{di}{dt}$ for Ind.

& so general soln: $x(t) = x_\infty + (x_0 - x_\infty)\exp[-(t-t_0)/\tau]$

(3) (i) If you're not at eq or you're not heading towards stable equilibrium, determine a switching time.

(ii) If headed to stable eq. → don't find a switching time

(4) (i) In a negative slope region & x is increasing? Draw

(ii) In a " " " " " decreasing? Draw

(iii) In a positive " " " " increasing? Draw

(iv) In " " " " " decreasing? Draw

(5) Easy way to find shape of curves? → See Problem 3

Tricks for Op-amps

(1) Ideal? → Does not saturate → Ignore railing

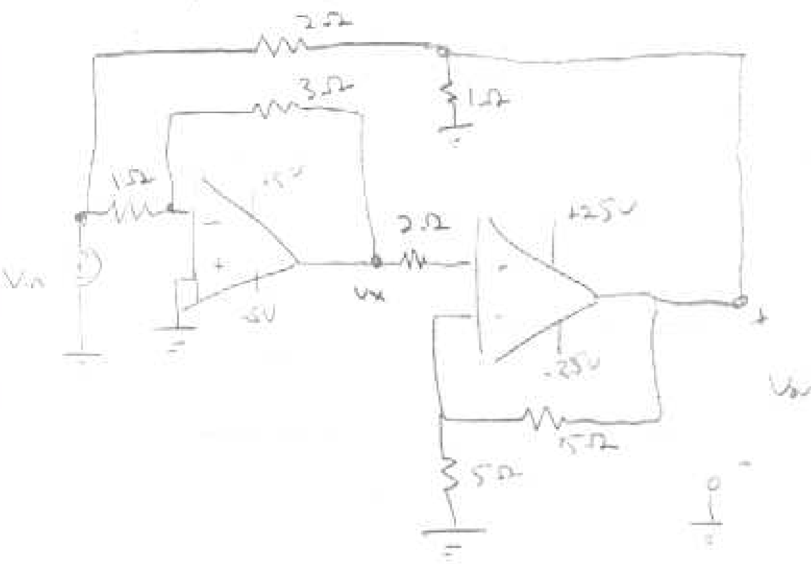
(2) Non-ideal → Can saturate → Account for railing → $v_n \neq v_p$

(3) Do nodal analysis at all nodes you see EXCEPT v_{out}.

Current can flow into output of op-amp

Pg 2

From the Summer:



a) Find type of amplifier

b) Find v_x/v_{in} (Ideal)

c) Find v_{out}/v_{in} (Ideal)

d) $v_{in} = 3V$, $v_{out} = ?$

e) $V_{in} = -4V$, $v_{out} = ?$

f) $v_{in} = 1V$, $v_{out} = ?$

Soln

a) Inverting + Non-inverting = Inverting

b) Inverting op-amp $\frac{v_{in} - 0}{1} = \frac{0 - v_x}{3} \Rightarrow \boxed{\frac{v_x}{v_{in}} = -3}$

c) Non-inverting op-amp $\frac{v_{out} - v_x}{15} = \frac{v_x - 0}{5} \Rightarrow \frac{v_{out}}{v_x} = 4$

$$\left(\frac{v_{out}}{v_x}\right)\left(\frac{v_x}{v_{in}}\right) = \boxed{\frac{v_{out}}{v_{in}} = -12}$$

d) $v_{in} = 3$. 1st op-amp $v_x = -9 \rightarrow$ Saturates $\Rightarrow v_x = -5$

2nd op-amp $\boxed{v_{out} = -20V}$

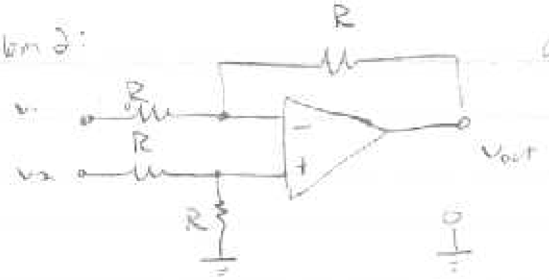
e) $v_{in} = 4$. 1st op-amp $v_x = 12 \rightarrow$ Saturates $\Rightarrow v_x = +5$

2nd op-amp $\boxed{v_{out} = +20V}$

f) $v_{in} = 1$ 1st op-amp: $v_x = -3V$

2nd op-amp: $\boxed{v_{out} = -12V}$

Problem 2:



a) Find $v_{out} = f(v_1, v_2)$
Weighting Factors of v_1 & v_2?

Soln: $V_n = V_p = V_x$

a) $\frac{v_2 - v_x}{R} = \frac{v_x}{R} \Rightarrow v_2 = R\left[\frac{2v_x}{R}\right] \Rightarrow v_x = \frac{v_2}{2}$

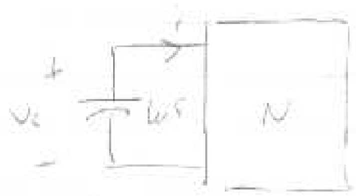
$\frac{v_1 - v_2/2}{R} = \frac{v_2/2 - v_{out}}{R} \Rightarrow v_1 = v_2 - v_{out}$

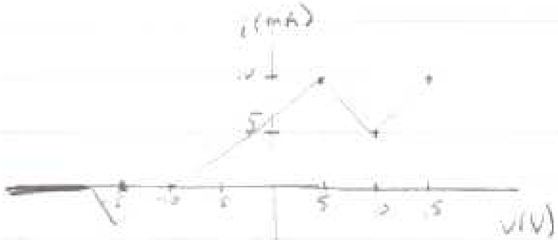
$\boxed{v_{out} = v_2 - v_1}$

b) Weighting Factors $\quad v_{out} = \alpha v_1 + \beta v_2$

$\alpha = -1 \qquad \beta = 1$

Problem 3

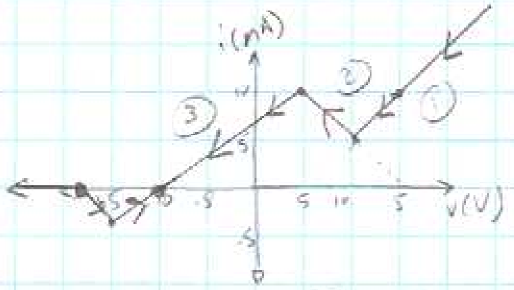




(i) Indicate Dynamic Route, Eq Pts & Stability
(ii) $v_C(0) = 15V$ Find $v_C(t)$, $i_C(t)$ for $t \geq 0$

See Soln on Pg 3

Pg4



$i = -C\frac{dv}{dt} \Rightarrow \frac{-i}{C} = \frac{dv}{dt}$ Eq: $\frac{dv}{dt} = 0 \Rightarrow i = 0$ 2 eq. pts.

$i > 0 \rightarrow \frac{dv}{dt} < 0$ $i < 0 \rightarrow \frac{dv}{dt} > 0$ $v = -10 \rightarrow$ Stable
$v = -10 \rightarrow$ Unstable

Start ③ $v_c(0) = 15V$

Region 1: Need 9 pieces of info:

1st 8: $v_0 = 15V$ $v_\infty = 5V$ $R = 1000\,\Omega$ $\tau = .001s$
$i_0 = 10mA$ $i_\infty = 0mA$ $C = 1\times10^{-6}C$ $t_0 = 0$

$v(t) = 5 + 10e^{-1000t}$ (V)
for $0 < t < t_1$
$i(t) = 10e^{-1000t}$ (mA)

9th piece of info: Switching time: $t_1 \rightarrow$ time to get to (10V, 5mA)

Use i Because Simpler $5 = 10e^{-1000t_1} \rightarrow$ $\boxed{t_1 = \ln 2/1000}$ = .69 mS

Region 2:
$v_0 = 10V$ $v_\infty = 15$ $R = -1000\,\Omega$ $\tau = -1\times10^{-3}$
$i_0 = 5mA$ $i_\infty = 0$ $C = 1\times10^{-6}C$ $t_1 = \ln 2/1000$

$v(t) = 15 - 5\exp[+1000(t-t_1)]$ for $t_1 < t < t_2$
$i(t) = 5\exp[+1000(t-t_1)]$

t_2 is @ (5V, 10mA) $\rightarrow t_2 = t_1 + \ln 2/1000 = \ln 4/1000 = 1.38mS$

Region 3: To Stable Eq
$v_0 = 5V$ $v_\infty = -10$ $R = 1500\,\Omega$ $\tau = .00015s$
$i_0 = 10mA$ $i_\infty = 0$ $C = 1\times10^{-6}C$ $t_2 = \ln 4/1000$
$v(t) = -10 + 15\exp[-667(t-t_2)]$ for $t > t_2$
$i(t) = 10\exp[-667(t-t_2)]$

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Now to graph.

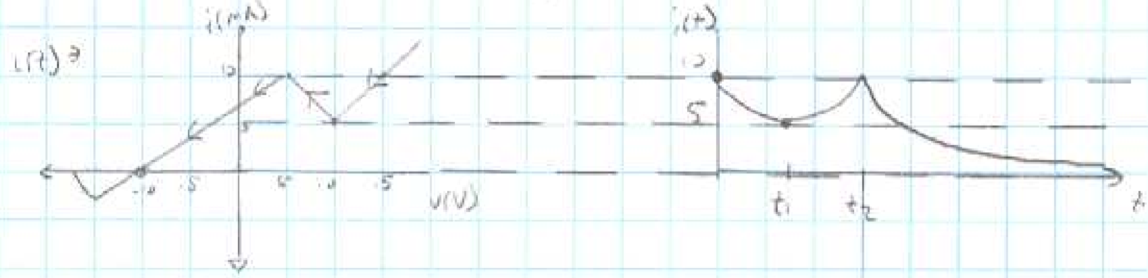
We <u>COULD</u> just graph what we got, but here's an easy way to eyeball the plots.

Good if running out of time & don't want to write out equations.

For $i(t)$, use "Drag Approach"

For $v(t)$, use "Invert & Drag Approach"

$i(t) \Rightarrow$

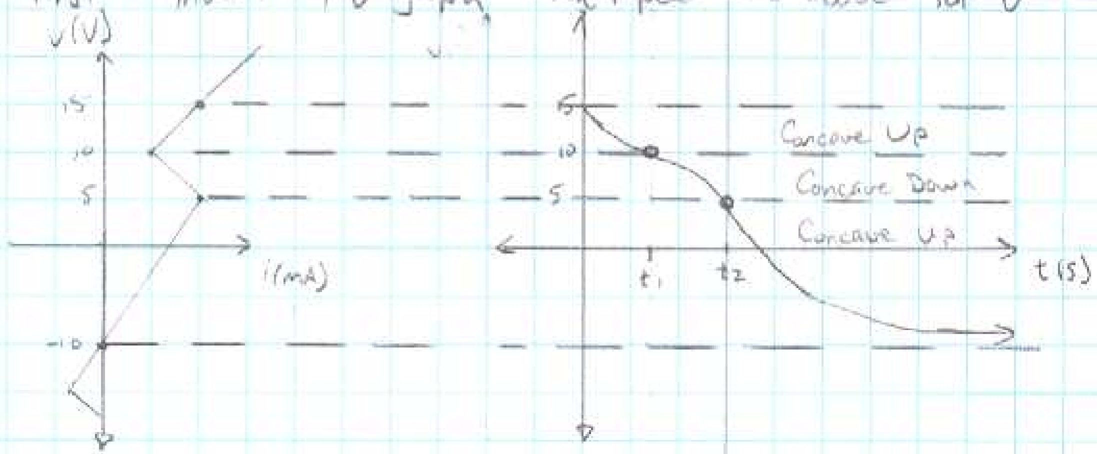


Extrapolate all essential values ($i = 5, 10$ mA)

Start at 10 → decrease to 5 @ t_1

→ increase back to 10 @ t_2 → decrease to 0 asymptotically

$v(t)$ First: Invert i-v graph. Then repeat the above for v



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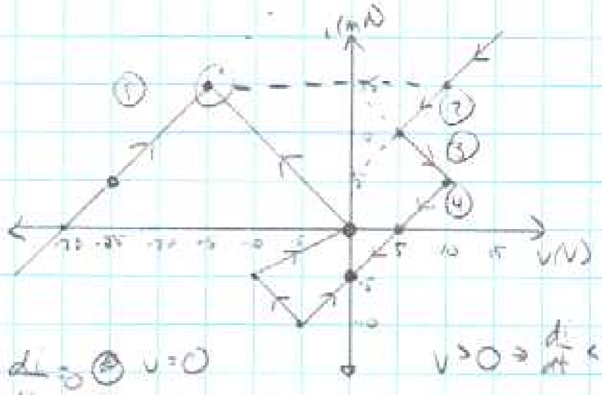
Problem 4 The "Justin is a Crazy Bastard" (JCB) Problem



Find $v(T)$ & $i(T)$ for $t \geq 0$

Start @ $V_L(0) = -25V$

$-\frac{v}{L} = \frac{di}{dt}$ Eq: $\frac{di}{dt} = 0$ @ $v = 0$ $v > 0 \Rightarrow \frac{di}{dt} < 0$ $v < 0$ $\frac{di}{dt} > 0$

Impasse Pt ② $(-5V, 15mA) \rightarrow$ jumps to $(10V, 15mA)$

$\tau = L/R$

4 Regions

Region I: $v_0 = -25V$ $v_\infty = 0$ $R = 1000\Omega$ $\tau = .001s$
$i_0 = 5mA$ $i_\infty = 30mA$ $L = 1H$ $t_0 = 0$
$v(t) = -25\exp[-1000t]$
$i(t) = 30 - 25\exp[-1000t]$ for $0 \leq t \leq t_1$
t_1 @ $v = -5 \Rightarrow$ $t_1 = \ln 6/1000$

Region II: $v_0 = 10$ $v_\infty = 0$ $R = 1000\Omega$ $\tau = 0.001s$
$i_0 = 5$ $i_\infty = 5$ $L = 1H$ $t_1 = \ln 6/1000$
$v(t) = 10\exp[-1000(t - t_1)]$
$i(t) = 5 + 10\exp[-1000(t - t_1)]$ for $t_1 \leq t \leq t_2$
t_2 @ $v = 5$ $t_2 = t_1 + \ln 2/1000 = \ln 12/1000$

Region III: $v_0 = 5$ $v_\infty = 0$ $R = -1000\Omega$ $\tau = -0.001s$
$i_0 = 10$ $i_\infty = 15$ $L = 1H$ $t_2 = \ln 12/1000$
$v(t) = 5\exp[1000(t - t_2)]$
$i(t) = 15 - 5\exp[1000(t - t_2)]$ for $t_2 \leq t \leq t_3$
t_3 @ $V = 10 \Rightarrow t_3 = t_2 + \ln 2/1000 = \ln 24/1000$

Region IV: $v_0 = 10$ $v_\infty = 0$ $R = 1000\Omega$ $\tau = .001s$
$i_0 = 5$ $i_\infty = -5$ $L = 1H$ $t_3 = \ln 24/1000$
$v(t) = 10\exp[-1000(t - t_3)]$
$i(t) = -5 + 10\exp[-1000(t - t_3)]$ for $t \geq t_3$

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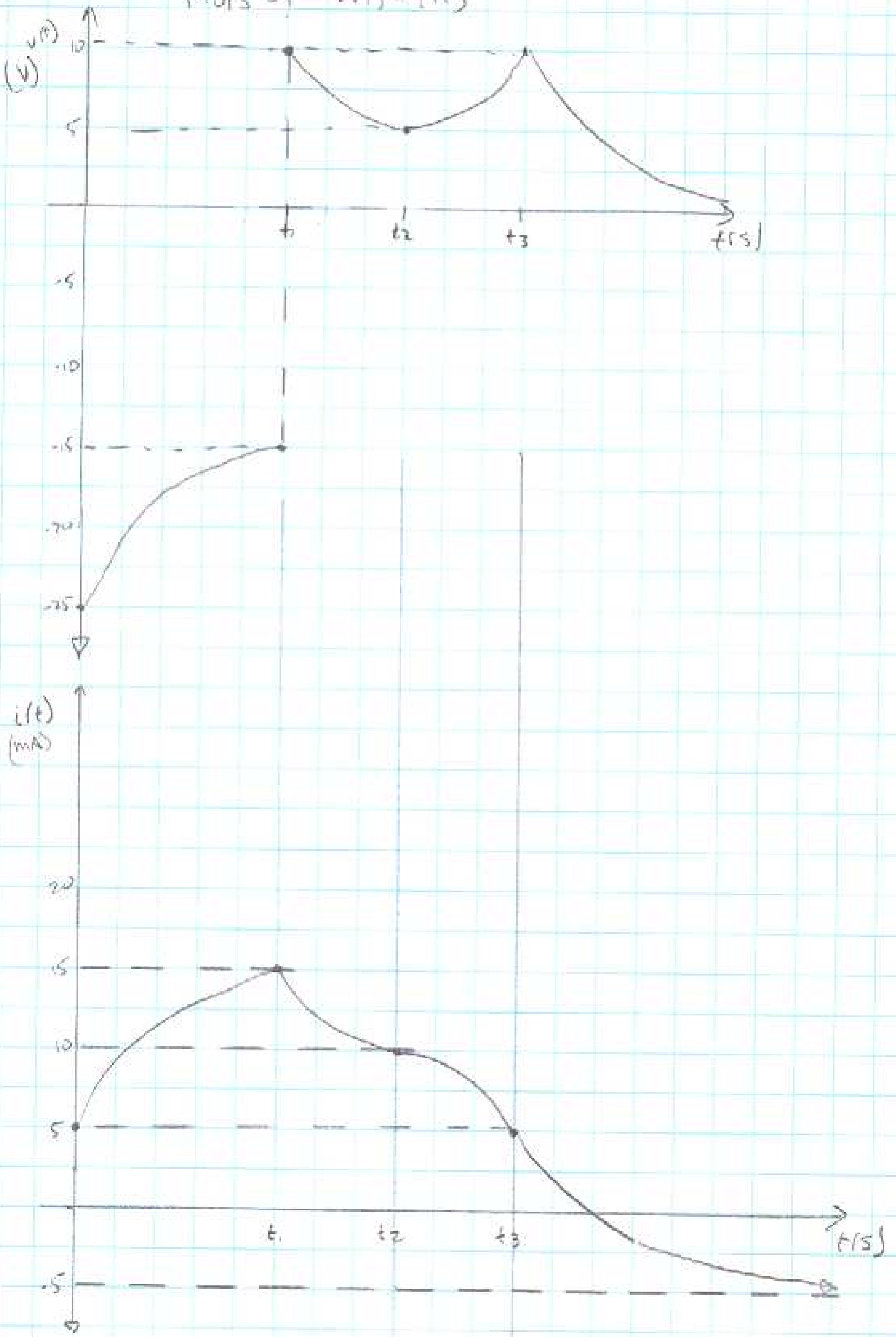
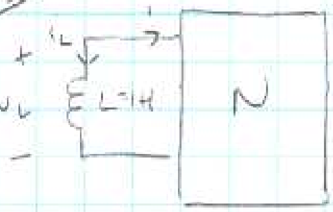
Plots of v(t) & i(t)

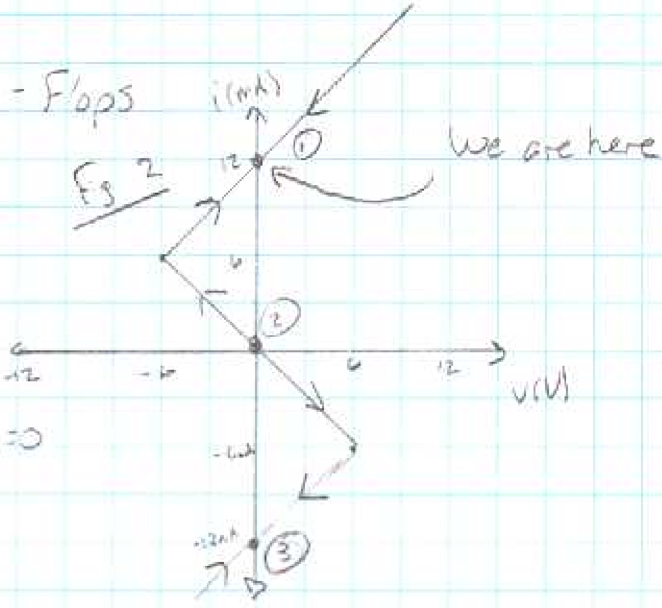


Problem 5: Flip - Flops

Fig 1



Fg 2



$-\frac{v}{L} = \frac{di}{dt}$ $\frac{d_i}{dt}$ ⓪ $v = 0$

$v > 0 \Rightarrow di/dt < 0$

$v < 0 \Rightarrow di/dt > 0$

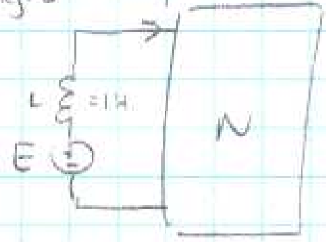
2 Stable Eq & 1 unstable

2 Possible Questions:

a) Min voltage you must add in Fig 3 to flip from 1 eq pt to another ① → ③

b) Time to flip from one to another. ① → ③

Fig. 3



a.) If symmetric: always difference between V(unstable eq) & V(cusp)

→ $E = 0 - (-6) = \boxed{+6V}$

b) If E = 12V, what is min time for E to be "on" to get from ① to ③?

Hint: Use dynamic route.

Trick: If we can get to pt ② & turn off E, dynamic route will take us to ③

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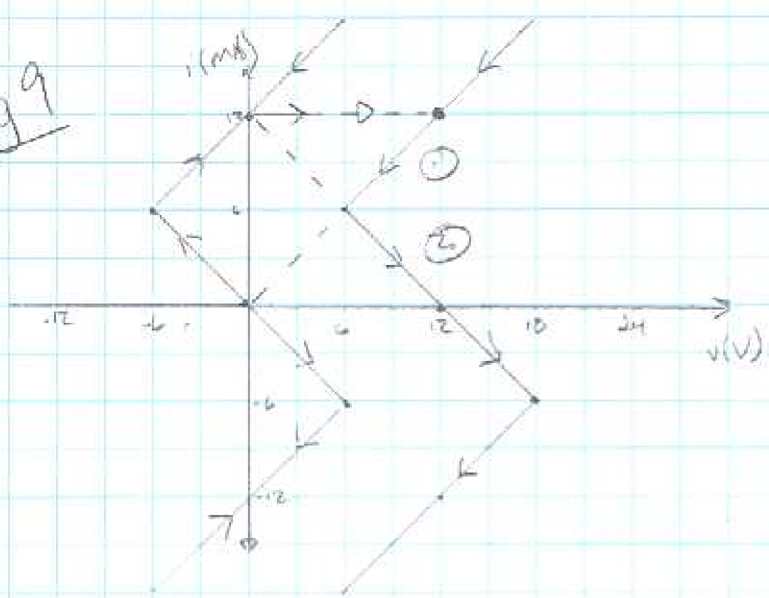

① Shift from (0, 12mA) → (12V, 12mA)

② Draw new dynamic route

③ Find t_1 & t_2 $\quad t_2 = t_{min}$

Region ①

$v_0 = 12 \quad v_\infty = 0 \quad R = 1000 \quad \tau = .001$

$i_0 = 12 \quad i_\infty = 0 \quad L = 1H \quad t_0 = 0$

Only need 1 → $v(t) = 12\exp(-1000t)$

↘ $i(t) = 12\exp(-1000t)$

t_1 @ $v = 6 \rightarrow t_1 = \ln 2/1000$

Region ②

$v_0 = 6 \quad v_\infty = 0 \quad R = -1000 \quad \tau = -.001$

$i_0 = 6 \quad i_\infty = 12 \quad L = 1 \quad t_1 = \ln 2/1000$

Only need 1 → $v(t) = 6\exp[1000(t - t_1)]$

→ $i(t) = 12 - 6\exp[1000(t - t_1)]$

t_2 @ $v = 12 \quad t_2 = \ln 2/1000 + t_1 \quad t_2 = \ln 4/1000$

$$\boxed{t_{min} = t_2 = \ln 4/1000}$$

THAT'S ALL THERE IS TO FLIP-FLOPS