Lecture 10 - Chapter 6, 7 - RC/RL Circuits

Administrivia: None 😊

Today - Chapter 6: I will cover sections 6.1, 6.2. READ 6.3

That's all you need to know from chapter 6. Skip 7.1, 7.7.

Chapter 7: [Natural/Step response of RL/RC Circuits]

Section 7.4 today, do examples all next week from 7.3, 7.5

Note: We will do chapter 5 in two weeks.
Chapter 6 - Inductance & Capacitance

Inductors (also "choke" aka "coil") & Capacitors are elements that store energy in a magnetic field & electric field respectively.

Inductance, Unit: Henry Typical: 1 mH

Capacitance, Unit: Farad Typical: 1 uF

Inductor Symbol: \[ L \]
Capacitor Symbol: \[ C \]
I-V RELATIONSHIP

\[ V = iR \]

\[ P = vi \]
\[ P = \frac{v^2}{R} = i^2R \]

\[ \frac{\partial i}{\partial t} = \frac{C}{\partial V} \]

\[ i = C \frac{dV}{dt} + \frac{1}{R} \frac{dv}{dt} \]

Note: Passive sign convention

\[ E = \int P = \frac{1}{2} Li^2 \]
more important (Joule)
Continuity Property for Inductors & Capacitors

\[ \text{Inductors: } V = L \frac{dI}{dt} \]

(i) What happens to \( V \) if \( I \) changes instantaneously?

i.e., \( \frac{dI}{dt} \rightarrow \text{finite value} \) \( \Rightarrow \quad V \rightarrow \infty \)

\( \frac{dI}{dt} \rightarrow \text{zero} \)

Current through an inductor does not change instantaneously.
Similarly, Voltage across a capacitor cannot change instantaneously (i.e. \( \frac{dv}{dt} \)).

Mathematical details — I look up Dirac delta functions.

You are not responsible for this.

Mini - HW: Read 6.3; \( \frac{1}{2} \) \( \left\lfloor \frac{1}{2} \right\rfloor \) = 0.25 \( \text{leg 4 the} \)

Now, Chapter 7!
Chapter 7 - RC / RL Circuits

RC Circuit:

\[ \text{find } v_c(t) \text{ and } i(t) \]

\[ \text{Sol: } v_{DC} = U_R + V_c \]

First, let's find \( v_c(t) \)
Note: For capacitance, we had \( v_c \) first, instead of \( i \). Because it is
\[ \frac{dv_c}{dt} \]

\[\Rightarrow \quad V_{dc} = iR + v_c \]

\[\Rightarrow \quad v_c = \left[ C \frac{dv_c}{dt} \right] R + v_c \]

\[\Rightarrow \quad V_{dc} = \left( RC \right) \frac{dv_c}{dt} + v_c \quad \cdots (1)\]
0 is an example of a first-order ordinary linear differential equation with no partial derivatives. Nothing like \( \frac{d^2 y}{dt^2} \) unless \( \frac{dy}{dt} \) is constant coefficient because \( R, L, V_0 \) are constant.

Q: What makes differential equations hard to solve? That is, why is \( R + 2 + 5 \) easier?
sol. (1) Solutions are not usually unique.
\[ \text{e.g. } x(t) = 5 \text{, unique, } a > 2 \]

(2) Solutions should satisfy a differential equation for all time — hard to do!
best way to solve diff. eqn. == guess at a solution.

\[ v_{dc} = K \frac{dc}{dt} + V_c \to \text{looks like } K = \frac{A + \Delta x}{\Delta t} \]
Based on our intuition above, let us guess $v_c(t) = A + Be^{-t}$.

Note: $\beta \frac{dx}{dt} + x = 0 \Rightarrow \frac{dx}{dt} = -\frac{x}{\beta} \Rightarrow x(t) = e^{-\frac{t}{\beta}}$.
Therefore, our "first" guess is: 

$$V_{OC} = A + BC$$

Goal: Find A, B, C

Equation: 
$$V_{OC} = RC \frac{dv_c}{dt} + V_c$$

(1) Find A: 
$$\lim_{t \to \infty} v_c(t) = \lim_{t \to \infty} (A + Be^{-t/\alpha})$$

$$= A \quad \Rightarrow \quad A = V_c \ (t \to \infty)$$
After a very long time, capacitors are modelled as open circuit because they are fully charged.

\[ V_c(t) = V_{oc} = A \]

So, \( i = c \frac{dv_c}{dt} = 0 \) A

\( \Rightarrow \) as the circuit has been in operation for a very long time, \( v_c = A \) is a constant.

\[ V_{oc} = v_c(t+\omega) \]
\[ V_{\text{out}} = V_{\text{dc}} + B e^{-\frac{t}{\tau}} \]

(2) Find \( B \): Suppose \( V_{C}(t=0) = V_0 \)

Initial voltage across the capacitor.

\[ B = V_{C}(t=0) = V_{\text{dc}} + B \Rightarrow V_{\text{dc}} + B = V_0 \]

\[ B = V_0 - V_{\text{dc}} \]

\[ V_C(t) = V_{\text{dc}} + (V_0 - V_{\text{dc}}) e^{-\frac{t}{\tau}} \]
(8) Find $C$ : Notice $2$ satisfies $2t_3 = 7$

$$V_{oc} = RC \frac{dV_C}{dt} + V_0$$

$$= \frac{V_{oc}}{RC} \frac{d}{dt} \left[ V_{oc} + \left( V_0 - V_{oc} \right) e^{-\frac{t}{RC}} \right]$$

$$= V_{oc} = RC \left[ 0 + \left( V_0 - V_{oc} \right) e^{-\frac{t}{RC}} \right] + V_{oc} \left( V_0 - V_{oc} \right) e^{-\frac{t}{RC}}$$
\[ RC \left( V_o - V_{in} \right) e^{-\frac{t}{RC}} = V_o - V_{in} \]

\[ T = RC \]

\[ V_c(t) = V_{in} + (V_o - V_{in}) e^{-\frac{t}{RC}} \]

Observation 1:

(1) \( V_c(0^-) = V_o \)

(2) \( V_c(\infty) = V_o \)

(3) Units: check RC
\[ \frac{dV_c}{dt} = \frac{V_s}{\tau} \left( \frac{Q}{C} \right) = \left( \frac{G}{\tau} \right) = \left( \frac{E}{\tau} \right) = \text{Second} \]

Observations II:

1. Significance of \( \tau = RC \)

Now, \( V_c (t = \frac{5}{2} \tau) \) = \( V_{DC} + (V_0 - V_{DC}) e^{-5\frac{\tau}{\tau}} \)

\[ = V_{DC} + (V_0 - V_{DC}) e^{-5} \approx V_{DC} \]

That is, problem is "over" in 5 time constant.
(2) Easier to understand from a plot of $V_c(T)$ (Assume $V_o = 0 \, V$).

$$V_c = V_{dc} - V_{oc} e^{-\frac{t}{\tau}}$$

Note: This $V_c(T)$ is called the step response. It is a forced response, as opposed to...
Natural response to initial conditions only

Step response

\[ t = 0 \] switch closes at \( t = 0 \)

\[ V_{c(0)} = V_0 \]

\[ V_c(t) = V_{dc} + (V_0 - V_{dc})e^{-\frac{t}{R_C}} \]

\[ V_{st} = V_0 \]
Example: Example 2.8, p. 289

Set up only, solve on Tuesday

(Q2) Switch has been open for a very long time, close at \( t = 0 \). Find \( i(t) \), \( v(t) \), \( t > 0 \).
Find i by finding v_c \[ i = C \frac{dv_c}{dt} \]
4 can be solved if

\[ V_c(t) = V_{oc} + (V_o - V_{oc}) e^{-t/\tau_{mc}} \]

Notice! To find \( V_o \), you have to use circuit analysis.

That is, problem stated the switch had been open for a very long time.
\[ V_0 = 0 \]

The instant after switch close.
Next time
I finish example above
More examples!!!