

## Review Problem Solutions:

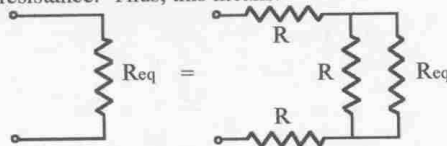
If anything is incorrect: blame Justin

1.) We must find the equivalent resistance of the following:



But first, an analogy, say you are finding the sum:  $\sum \frac{1}{n^2}$ . If you add up the first million terms, you will get what is essentially the exact solution. Adding any more terms will change your sum insignificantly.

Thus, getting back to our semi-infinite resistive ladder, we can find that adding another rung to the ladder will not change the equivalent resistance. Thus, this means:



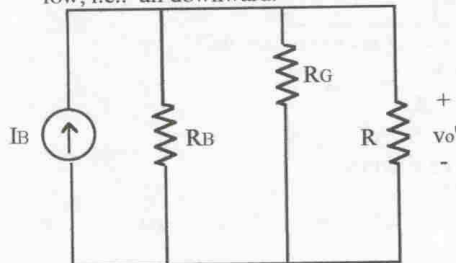
So, solve for the equivalent resistances of each and set them equal and solve for  $R_{eq}$ .

$$R_{eq} = 2R + \frac{R \cdot R_{eq}}{R + R_{eq}}$$

$$R_{eq}^2 + R \cdot R_{eq} = 2R^2 + 2R \cdot R_{eq} + R \cdot R_{eq} \rightarrow R_{eq}^2 - 2R^2 - 2R \cdot R_{eq} = 0$$

$$R_{eq} = (1 + \sqrt{3})R$$

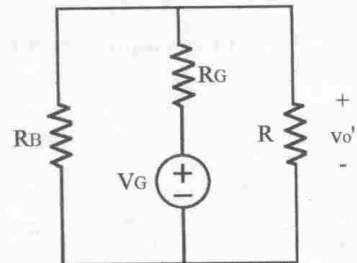
2.) Before we begin, make sure that whatever polarity you define your resistors for one part of the problem, you stick with those polarities for the rest. We need to solve 2 circuits for the total voltage across resistor R; let's call that  $v_o$ . Let's have the bottom node be grounded. This means that the top node is all at the same voltage,  $v_o$ , and thus all currents through the resistors will flow from high to low, i.e.: all downward.



KCL at the top node yields:

$$I_B = \frac{v_o'}{R_B} + \frac{v_o'}{R_G} + \frac{v_o'}{R}$$

Thus,  $v_o' = 1.382 \text{ V}$



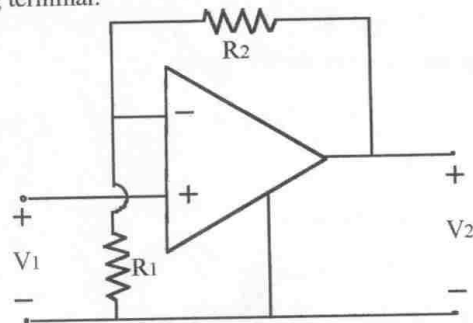
KCL at the top node yields:

$$\frac{v_o''}{R_B} + \frac{v_o'' - V_G}{R_G} + \frac{v_o''}{R} = 0$$

$$v_o'' = 4.608 \text{ V}$$

So,  $v_o = v_o' + v_o'' = 5.99 \text{ V}$

3.) Before we begin, note that the curvy part of the wiring means that the wire going to R1 is not in direct contact with the non-inverting terminal.



$$V_p = V_n = V_1$$

KCL at  $V_n$ :

$$\frac{V_2 - V_1}{R_2} = \frac{V_1}{R_1}$$

$$\frac{V_2}{V_1} = \mu = \frac{R_1 + R_2}{R_1}$$

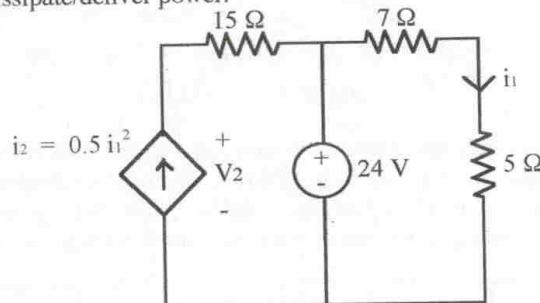
(A.) Thus,

$$-E_{sat} \leq V_2 \leq E_{sat}$$

$$-E_{sat} \leq \frac{R_1 + R_2}{R_1} V_1 \leq E_{sat}$$

$$(B.) \frac{-E_{sat} R_1}{R_1 + R_2} \leq V_1 \leq \frac{E_{sat} R_1}{R_1 + R_2}$$

4.) (a.) Sources always dissipate/deliver power.



(b.) Let's ground the bottom node. The center node at the top is thus at 24V.

$$i_1 = \frac{24 \text{ V}}{7 \Omega + 5 \Omega} = 2 \text{ A}$$

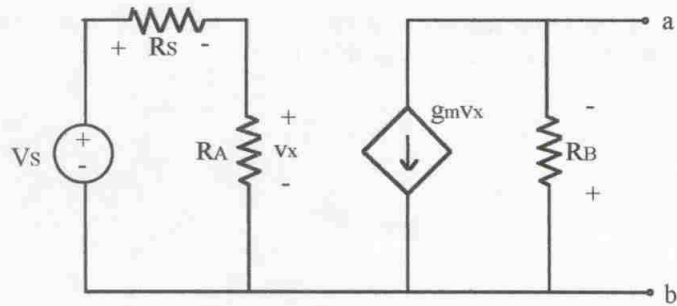
$$i_2 = 0.5 i_1^2 = 0.5 (2)^2 = 2 \text{ A}$$

KVL around the left loop:

$$v_2 - (2 \text{ A})(15 \Omega) - 24 = 0 \rightarrow v_2 = 54 \text{ V}$$

$$P = -i_2 v_2 = -(2 \text{ A})(54 \text{ V}) = -108 \text{ W} = 108 \text{ W delivered}$$

5.)



$$v_x = \frac{V_S R_A}{R_A + R_S}$$

(a.) First off: By voltage divider:

$$i = g_m v_x = \frac{g_m V_S R_A}{R_A + R_S}$$

Current through controlled source:

Immediately, if terminal ab were short circuited,  $i_{sc}$  would be the same as above, only negative:

$$i_{sc} = \frac{-g_m V_S R_A}{R_A + R_S}$$

To find the equivalent resistance, we set  $V_S$  to 0. Thus  $v_x$  and the dependent source is zeroed. Thus, we are left with a single resistor:

$$R_{eq} = R_B$$

By ohm's law:

$$v_{oc} = \frac{-g_m V_S R_A R_B}{R_A + R_S}$$

(b.)

$$P = .012 = IV = \frac{V^2}{R} = \frac{24^2}{10k + R_S}$$

$$i.) \quad R_S = 38 \text{ k}\Omega$$

$$v_x = \frac{V_S R_A}{R_A + R_S} = \frac{240}{48} = 5 \text{ V}$$

$$P = .17 = -IV = -g_m v_x (-5V) = g_m (25)$$

$$ii.) \quad g_m = 0.0068 \text{ }\Omega^{-1}$$

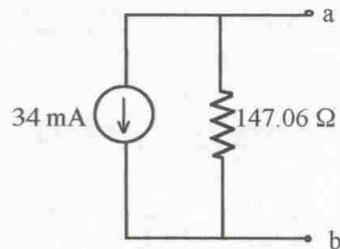
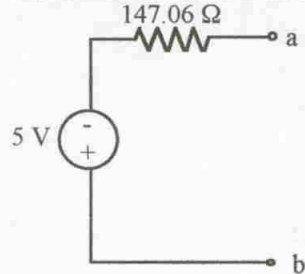
$$P = .17 = I^2 R = (g_m v_x)^2 (R_B)$$

$$iii.) \quad R_B = 147.06 \text{ }\Omega$$

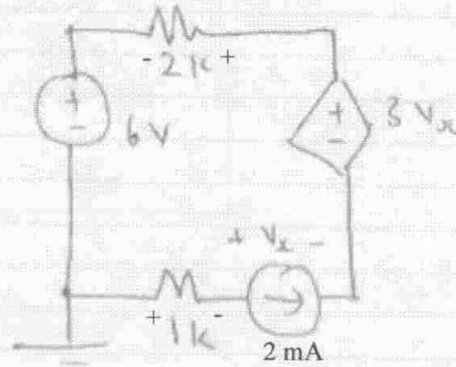
(c.) From the given information on the website,  $V_{ab} = V_{oc} = -5 \text{ V}$

Substituting all our values gives us:  $i_{sc} = -34 \text{ mA}$

So, the Thevenin and Norton equivalent circuits are, respectively:

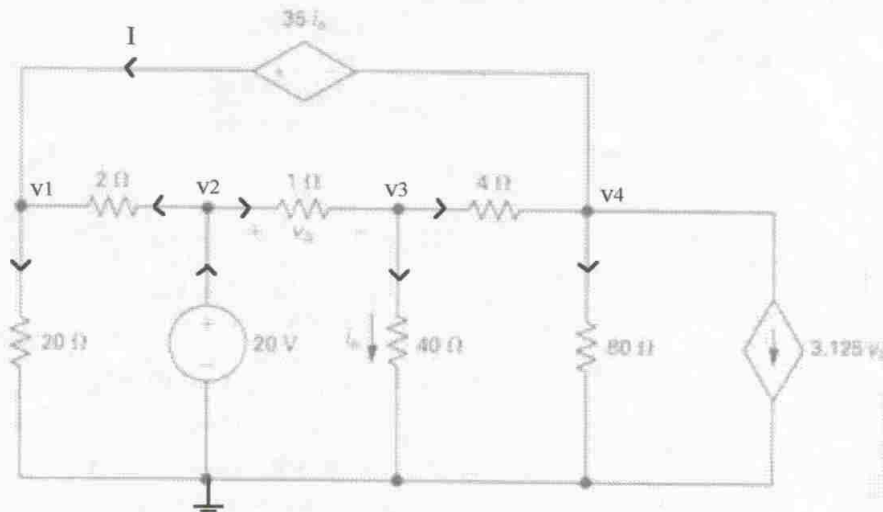


6.) The current source forces the current through the entire circuit to be 2 mA, meaning the resistors must have the polarities as shown.



Doing KVL clockwise, starting at the ground:  
 $6 + 2000(.002) - 3v_x + v_x + 1000(.002) = 0$   
 $12 - 2v_x = 0 \rightarrow v_x = 6 \text{ V}$

7.)



First let's ground the bottom node, leaving us with 4 essential nodes.

Immediately we see:  $v_2 = 20\text{V}$ ;  $v_\Delta = v_2 - v_3$ ;  $i_\phi = v_3/40$

$$\frac{20 - v_3}{2} = \frac{v_3}{40} = \frac{v_3 - v_4}{4}$$

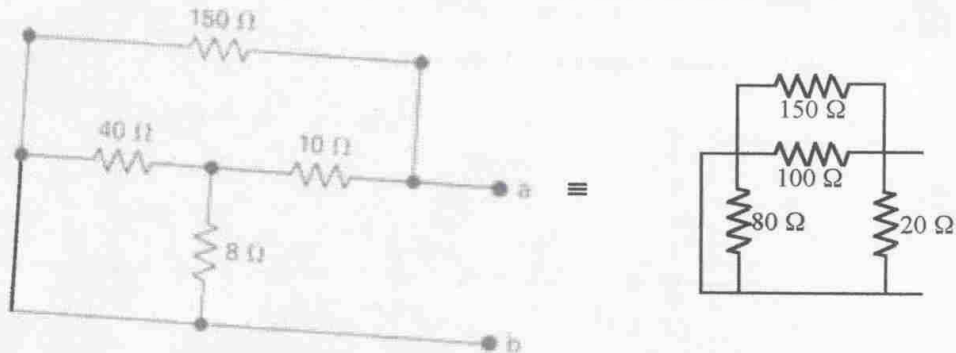
KCL at v3:  $\frac{20 - v_3}{2} = \frac{v_3}{40} = \frac{v_3 - v_4}{4}$

Dependent voltage source:  $v_1 - v_4 = 35 i_\phi = 35 (v_3/40)$

KCL at v1:  $I + \frac{20 - v_1}{2} = \frac{v_1}{20} \rightarrow I = \frac{v_1}{20} + \frac{v_1 - 20}{2}$

KCL at v4:  $\frac{v_3 - v_4}{4} = I + 3.125 v_\Delta + \frac{v_4}{80} \rightarrow$   
 $\frac{v_3 - v_4}{4} = \frac{v_1}{20} + \frac{v_1 - 20}{2} + 3.125 (20 - v_3) + \frac{v_4}{80}$

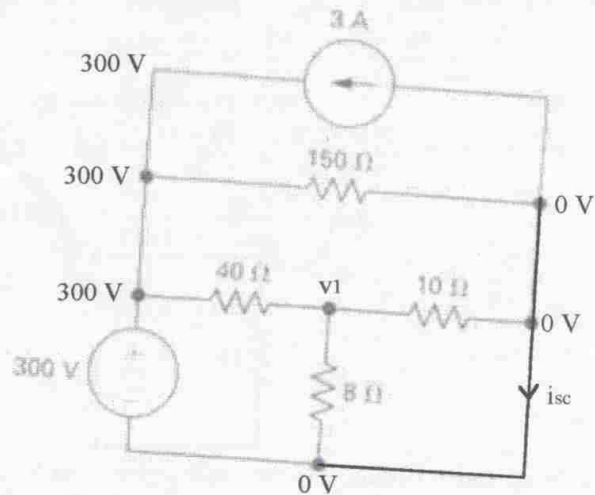
- 8.) Let us first solve for the equivalent resistance. Replace the voltage source with a short circuit. Replace the current source with an open circuit. Apply a tee-to- $\pi$  transformation. Solve.



Because of the short circuit, the  $80\ \Omega$  resistor is not seen. The equivalent resistance is:

$$R_{eq} = 20 \parallel 100 \parallel 150 = 15\ \Omega$$

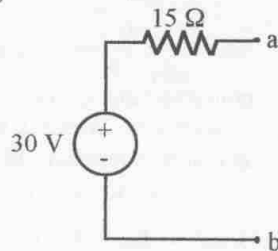
Now, instead of finding the open circuit voltage, let's do the simpler calculation: solving for the short circuit current. So, connect terminals a and b with a wire and ground b.



$$\text{KCL at } v_1: \frac{300 - v_1}{40} = \frac{v_1}{8} + \frac{v_1}{10} \rightarrow v_1 = 30\ \text{V}$$

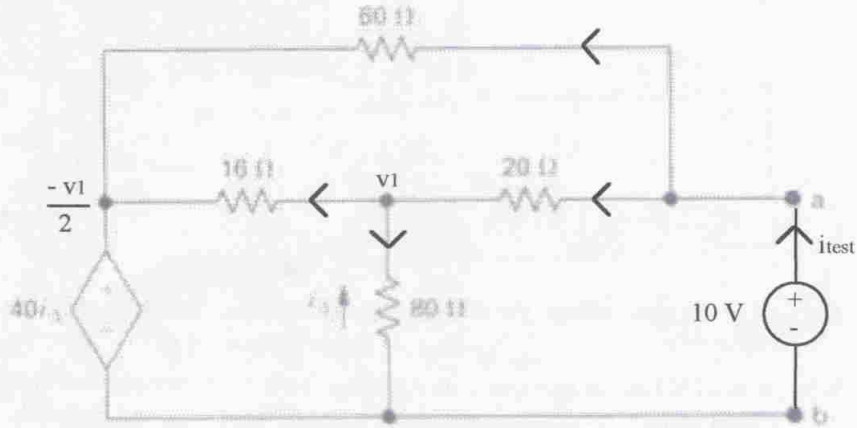
$$\text{KCL at terminal a: } \frac{300}{150} + \frac{30}{10} = 3 + i_{sc} \rightarrow i_{sc} = 2\ \text{A}$$

$$\text{Thus, } V_{oc} = R_{eq} i_{sc} \rightarrow V_{oc} = 30\ \text{V}$$



- 9.) Before we begin, notice: there is NO independent source, thus this means that there will be no open circuit voltage or short circuit current. In other words, the thevenin equivalent is ONLY A RESISTOR.

So to solve for it, let's apply a 10 V test voltage. Note:  $i_{\Delta} = \frac{0 - v_1}{80} = \frac{-v_1}{80}$ ;  $40i_{\Delta} = \frac{-v_1}{2}$



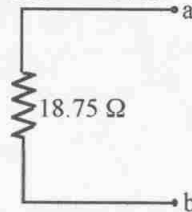
KCL at  $v_1$ :

$$\frac{10 - v_1}{20} = \frac{v_1}{80} + \frac{v_1 - v_1/2}{16} \rightarrow v_1 = 3.2 \text{ V}$$

KCL to solve for  $i_{\text{test}}$ :

$$\frac{10 + v_1/2}{60} + \frac{10 - v_1}{20} = i_{\text{test}} = 8/15 \text{ A}$$

$$R_{\text{eq}} = \frac{V_{\text{test}}}{i_{\text{test}}} = \frac{10}{8/15} = 18.75 \Omega$$



- 10.) When the op-amp is operating in the linear region, then  $V_n = 3 \text{ V}$ . As long as  $v_o$  stays above  $9 \text{ V}$ , this remains true.  $i_L$  remains constant for this period and is equal to the current flowing through the  $1.5 \text{ k}\Omega$  resistor.

$$(a.) \text{ Hence, } i_L = \frac{3\text{V}}{1500\Omega} = 2 \text{ mA}$$

For part b, as described above, the first moment the output is  $9 \text{ V}$  is when the max value of  $R_L$  is obtained:

$$(b.) \frac{9 - 3}{R_L} = 2 \text{ mA} \rightarrow R_L = 3 \text{ k}\Omega$$

- (c.) The op-amp saturates and outputs only  $9 \text{ V}$ . This means that  $V_n$  no longer is  $3 \text{ V}$  and the current changes from  $2 \text{ mA}$ . We find  $V_n = 1.6875 \text{ V}$ , and  $i_L = 1.125 \text{ mA}$ .

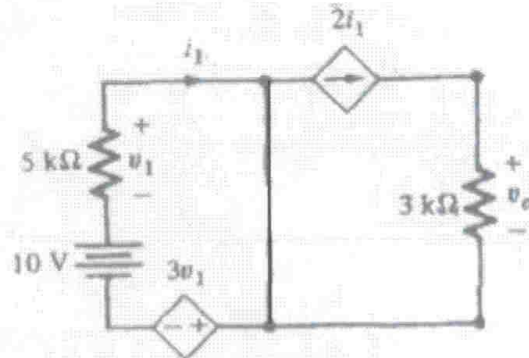
- (d.) From a and b, the graph should have the following qualities:

$$0 \leq R_L \leq 3 \text{ k}\Omega \rightarrow i_L = 2 \text{ mA}$$

$$3 \text{ k}\Omega \leq R_L \leq 6.5 \text{ k}\Omega \rightarrow i_L = \frac{9}{1500 + R_L}$$

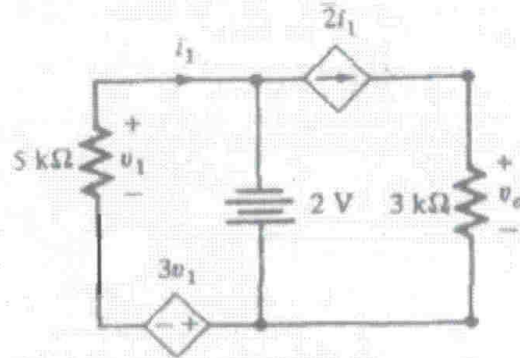
I'm too lazy to actually graph it. But you can graph the above if you want.

11.) Let's do KVL for all 3 circuits we make.



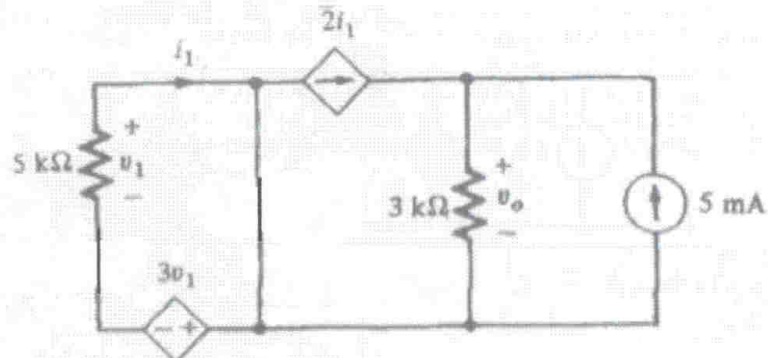
KVL around left loop:  $10 + v_1 - 3v_1 = 0 \rightarrow 2v_1 = 10 \rightarrow v_1 = 5 \text{ V}$

$$i_1 = \frac{10 - 3v_1}{5000} = -1 \text{ mA} \rightarrow v_o = 3000(2i_1) = -6 \text{ V}$$



KVL around the left loop:  $v_1 + 2 - 3v_1 = 0 \rightarrow 2v_1 = 2 \rightarrow v_1 = 1 \text{ V}$

$$i_1 = \frac{0 - v_1}{5000} = -0.2 \text{ mA} \rightarrow v_o = 3000(2i_1) = -1.2 \text{ V}$$



No dependent sources on the left:  $v_1 = 0 \text{ V}; i_1 = 0 \text{ mA} \rightarrow v_o = 3000(5 \text{ mA}) = 15 \text{ V}$

$$v_o = -6 + -1.2 + 15 = 7.8 \text{ V}$$