

EE128 Lecture 12 - Op-amps (Nonlinear version)

Administration → Schedule changes online, check

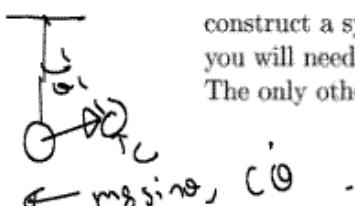
Today: → Nonlinear intro to op-amps] Thanks to Chua!

(1) Nonlinear systems example: from EE128 Fall 04, lab 1:
Using the equation of motion for a *damped* pendulum given by:

$$\ddot{\theta} + \frac{c}{ml}\dot{\theta} + \frac{g}{l}\sin\theta = \frac{T_c}{ml^2},$$

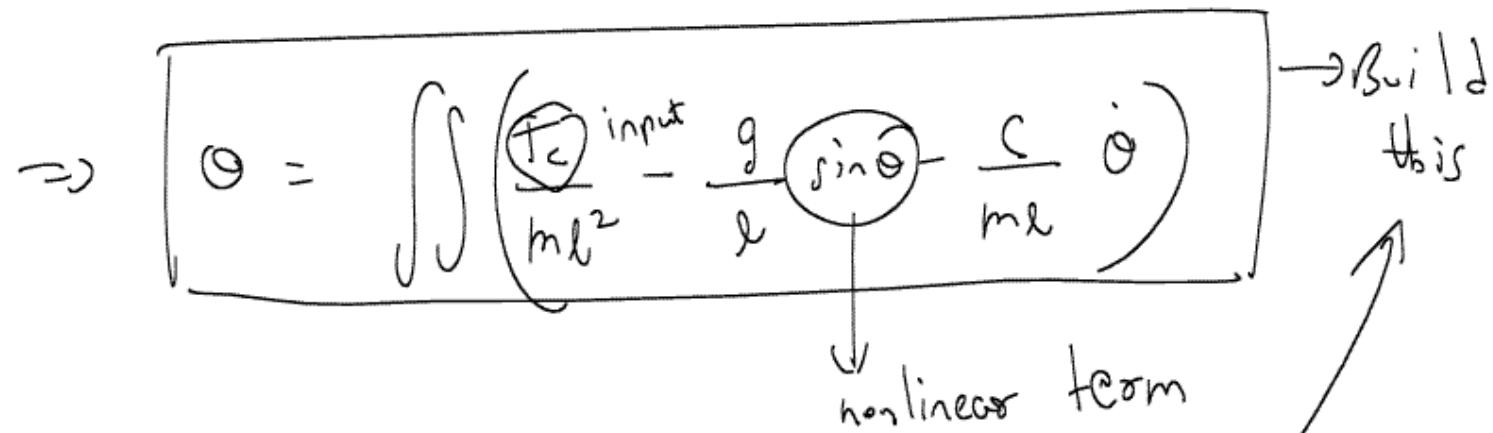
(Feedback control)

construct a system with input T_c and output θ . Choose $l = 2.5$, $m = 0.75$, and $c = 0.15$. To construct this system, you will need to use the "trigonometric function" component which is found in the "Nonlinear library" of Simulink. The only other components you will need are: a summer, constant gains, and integrators.



We will build the above system in simulink.

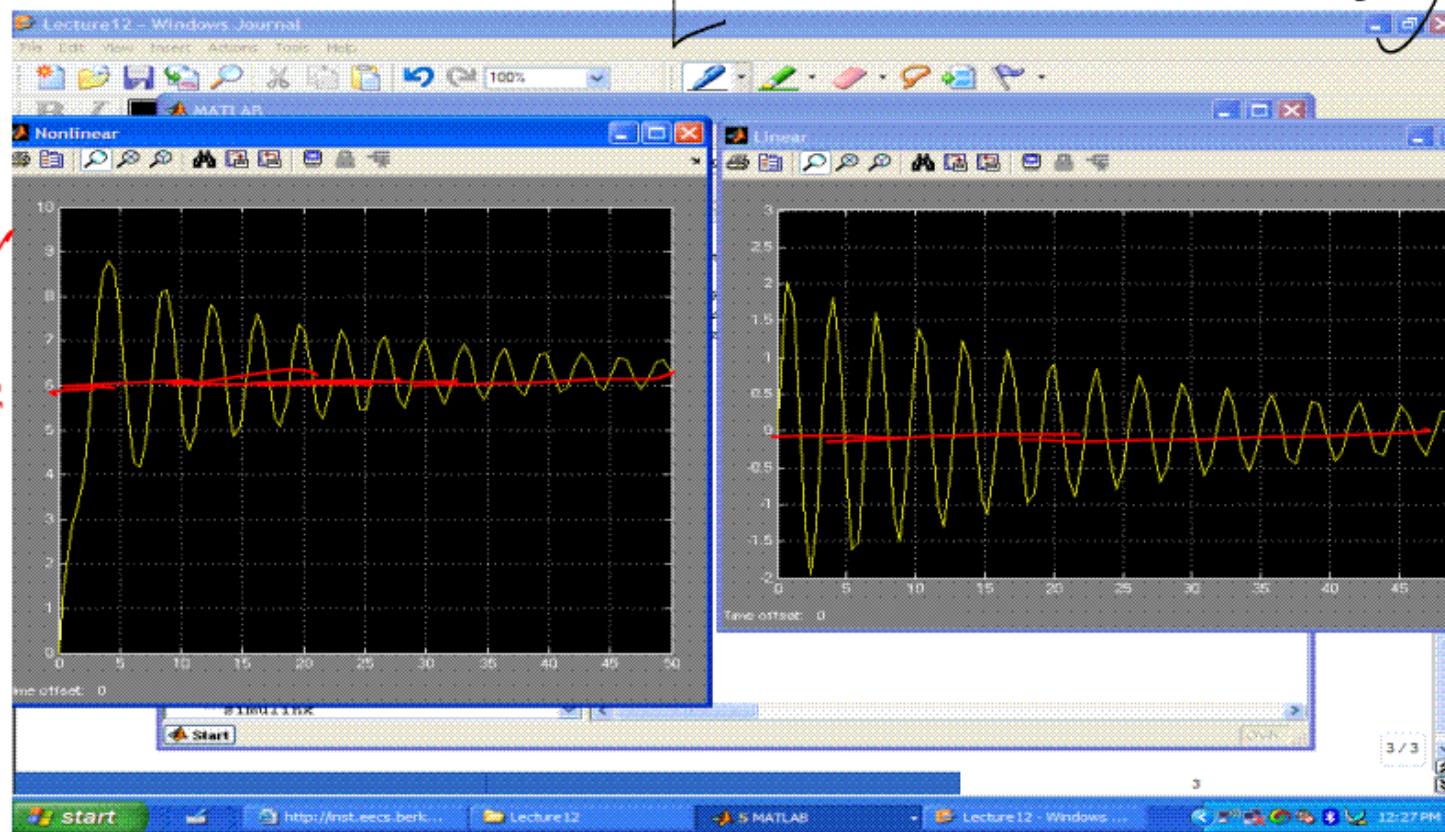
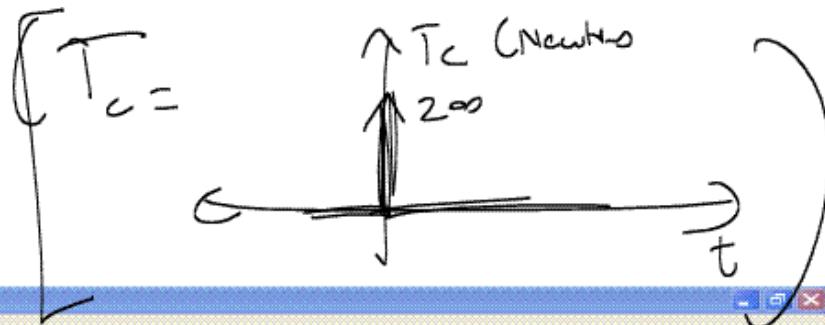
$$\ddot{\theta} = \frac{T_c}{ml^2} - \frac{g}{l} \sin\theta - \frac{c}{ml} \dot{\theta}$$



Let us look at linearized version.

$$\theta = \int \left(\frac{T_c}{ml^2} - \frac{g}{l} \theta - \frac{c}{ml} \dot{\theta} \right) \quad \left\{ \begin{array}{l} \text{valid for} \\ \text{small } \theta, \\ \text{since } \sin\theta \approx \theta \end{array} \right\}$$

From Simulink diagram:



More difficult to answer: How to find equilibrium
points?

Def: $f^{(n)}(t) = \frac{d^n f(t)}{dt^n}$

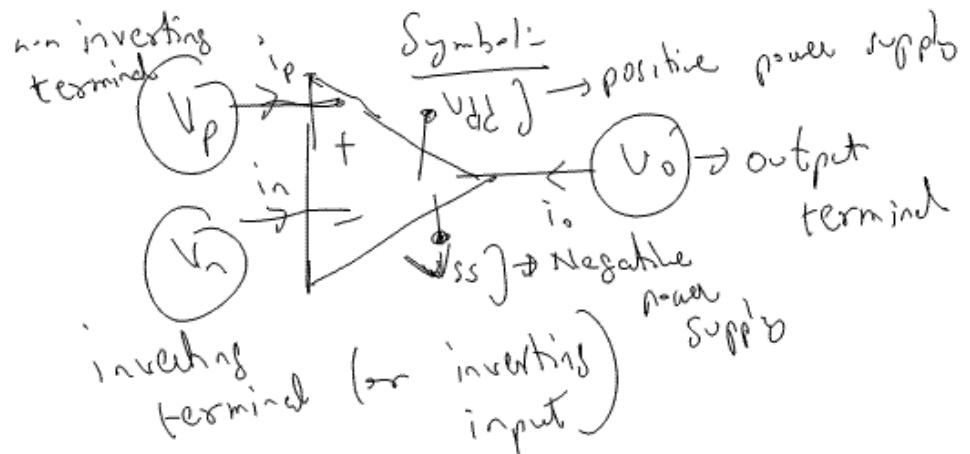
Equilibrium points exist when derivative is zero.

Nonlinear pendulum: $\sin \theta = 0 \Rightarrow \theta = 2n\pi$

linear pendulum: $\theta = 0$

(2) Nonlinear model of an op-amp (operational amplifier)

Recall from Monday:



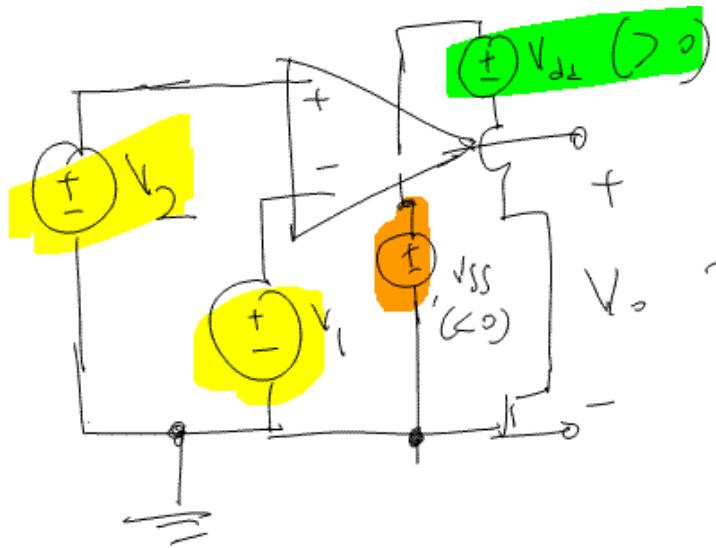
An op-amp does work by using energy from the power supply.

i.e. it is an active device

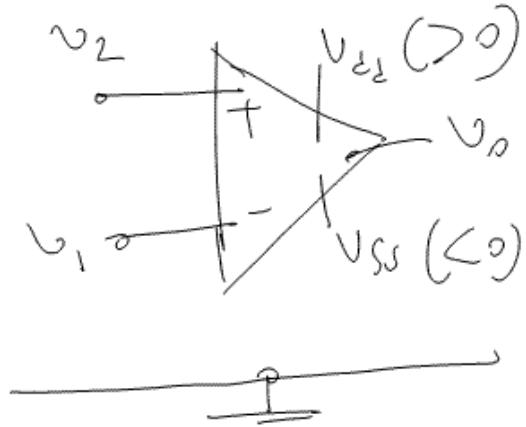
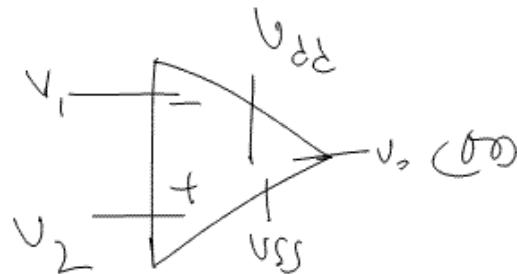
unlike a resistor which is passive

Mathematical description:

Op-amps are described by a transfer characteristic (i.e. $\frac{V_o}{V_{in}}$)



Input signals



→ positive power supply
 → positive rail
 → Negative power supply
 → Negative rail

Short-hand
circuit
schematic

Set used
to this!

Math:

$$v_o = \begin{cases} V_{dd} & \text{if } v_p > v_n \\ A(v_p - v_n) & \text{if } v_p \approx v_n \\ V_{ss} & \text{if } v_p < v_n \end{cases}$$

$A \rightarrow$ open loop gain
($\approx 10^6$)

Note:

$$V_o = A(v_p - v_n)$$

↓
can't change this! Physical property of

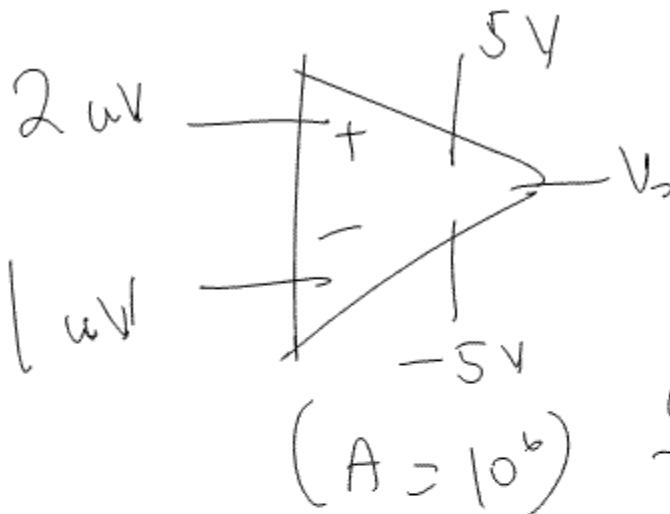
an op-amp.

In other words an op-amp is a very high gain differential amplifier.

But, real op-amps do have physical limitations HUGE!
e.g. If $v_p - v_n = 2 \text{ V}$, $V_o = 2 \times 10^6 \text{ V}$

This is physically not possible → Op-amp will
sail,
 $V_o = V_{dd}$

e8'



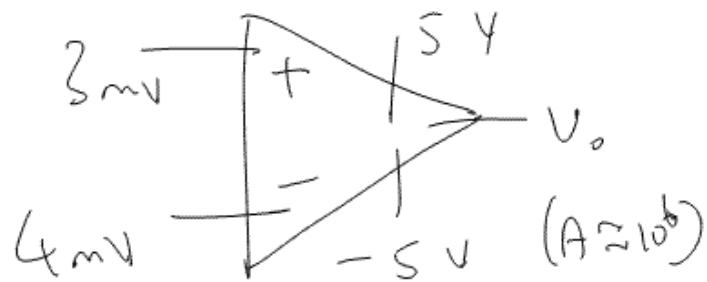
$$V_o = \begin{cases} 5 \\ A(V_p - V_n) \\ -5 \end{cases} (?)$$

Assume op-amp does not (rail) means
 $V_o = A(V_p - V_n)$ (V_o does not hit any

$$= 10^6(2 \text{ mV} - 1 \text{ uV}) \quad \text{(power supply)} \\ = 1 \text{ V} \in [-5, 5]$$

$\therefore \boxed{V_o = 1 \text{ V}}$

Ex:



$$\text{Assume } V_o = A(V_p - V_n)$$

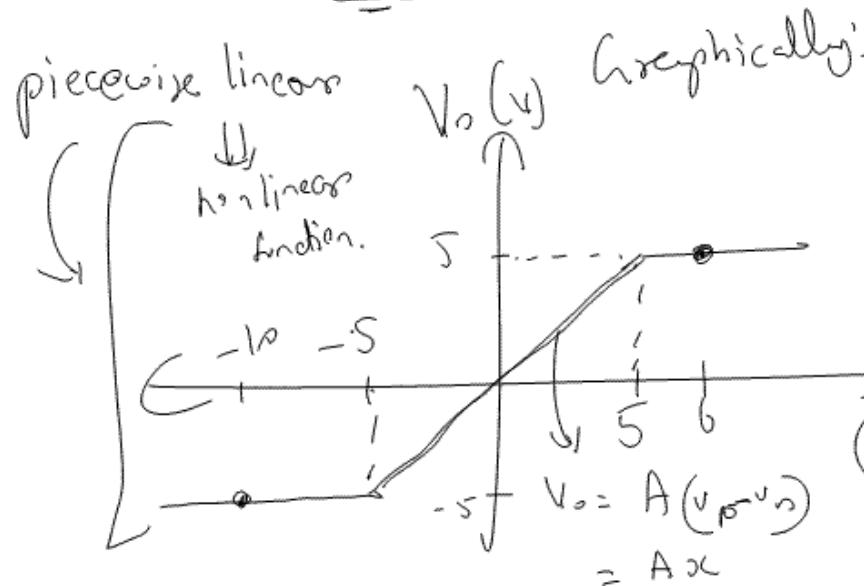
$$= 10^5(3\text{mV} - 4\text{mV})$$

$$= -1000 \text{V} \angle -5^\circ$$

op-amp rail. (or saturates)

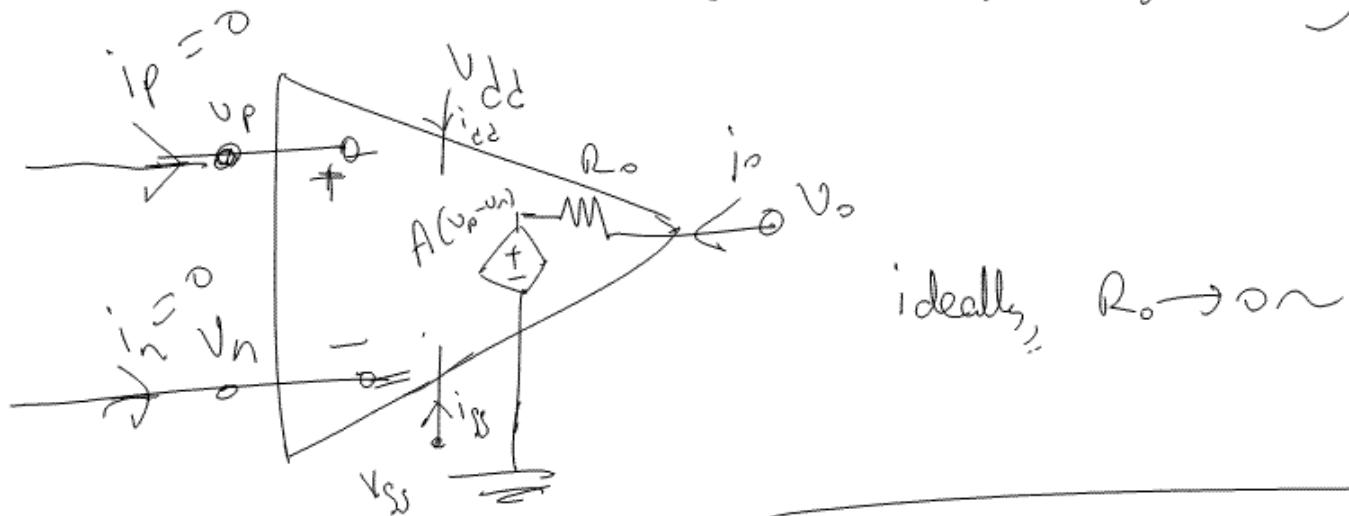
$$\boxed{V_o = -5 \text{V}}$$

$$\boxed{\begin{array}{l} V_{dd} = 5 \text{V} \\ V_{ss} = -5 \text{V} \end{array}}$$



$$(v)$$

(2) (b) An equivalent circuit model of an op-amp
(similar as fig. 14.2)

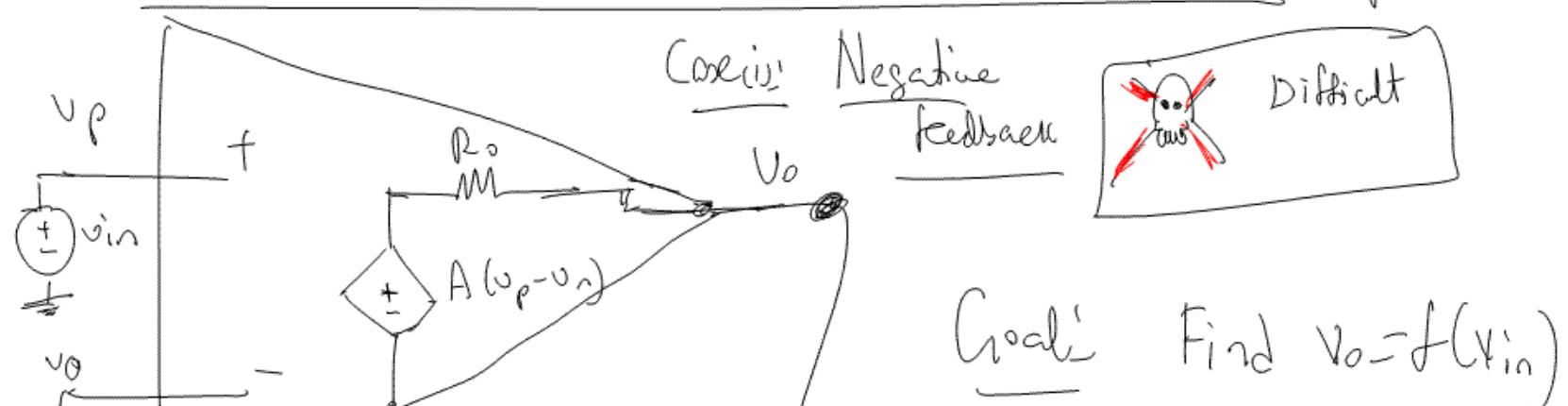


Note: In any op-amp

$$i_p = i_n = 0 \text{ A clearly!}$$

but, i_o , i_{dd} , i_{SS} are not-zero (usually)

(C) Negative vs. positive feedback



Case (ii) Negative feedback

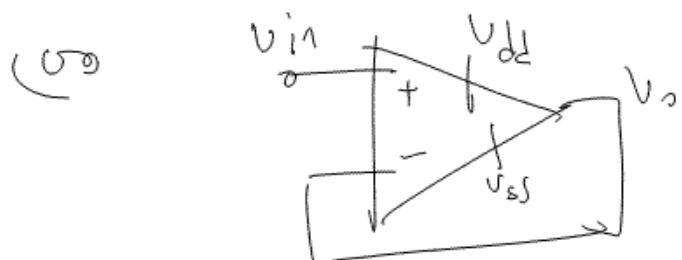


Goal: Find $v_o = f(v_{in})$

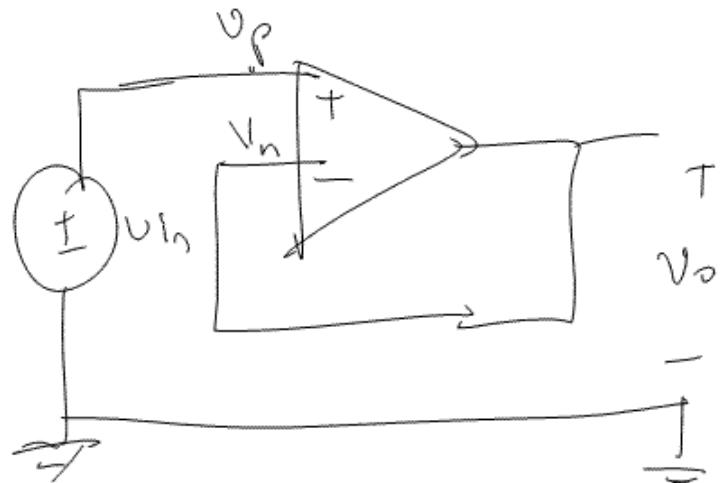
Sketch $v_o = f(v_{in})$

x^{v_o}

$\curvearrowleft v_{in}$



(C)



(Assume Op-amp does not rail)

$$V_o = A(V_p - V_n)$$

Key

$$V_o = A(V_{in} - V_o)$$



$$\Rightarrow V_o = A V_{in} - A V_o$$

$$\Rightarrow V_o(1 + A) = A V_{in}$$

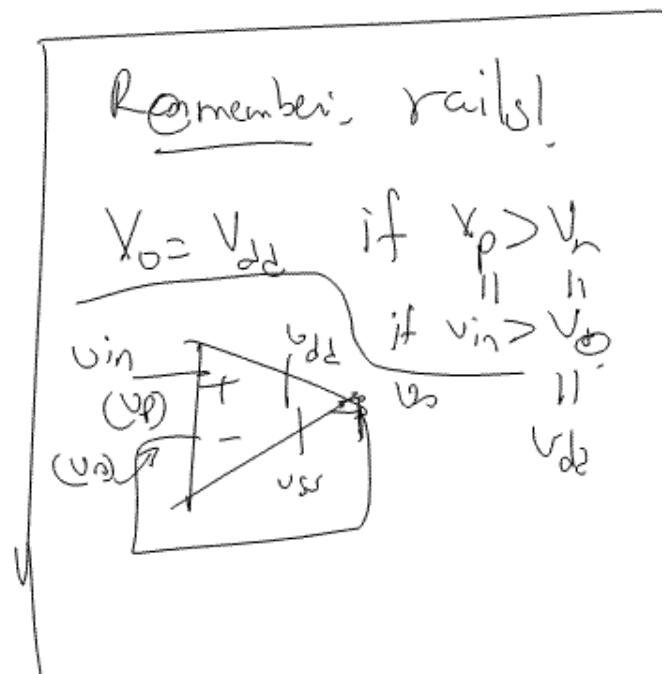
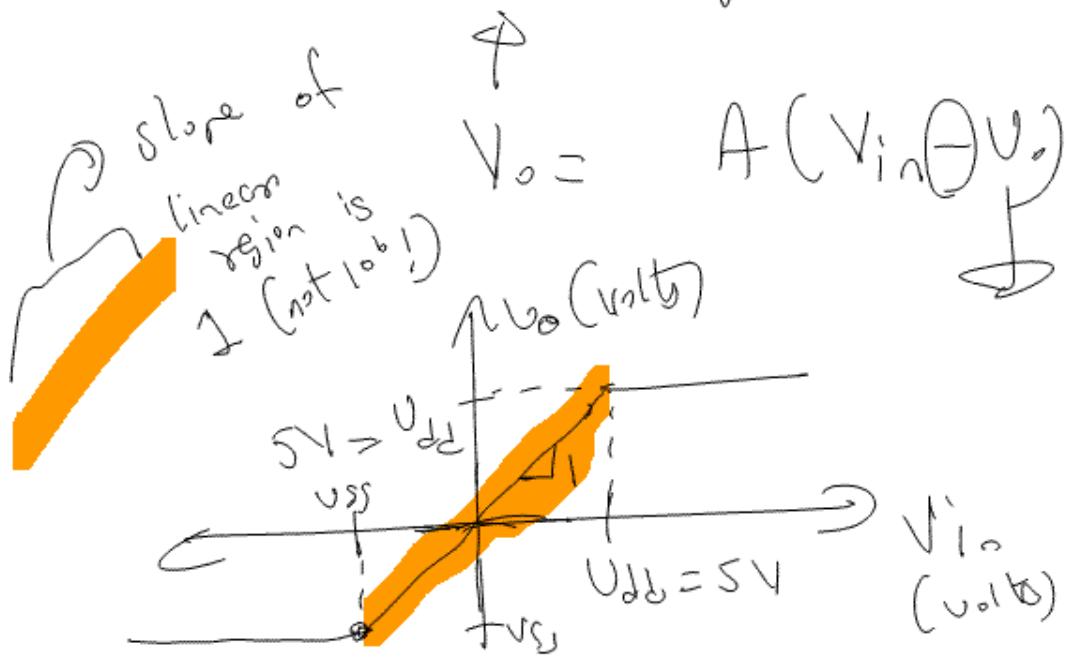
$$\Rightarrow V_o = \frac{A V_{in}}{1 + A}$$

$$\Rightarrow V_o = \frac{V_{in}}{1 + V_A}$$

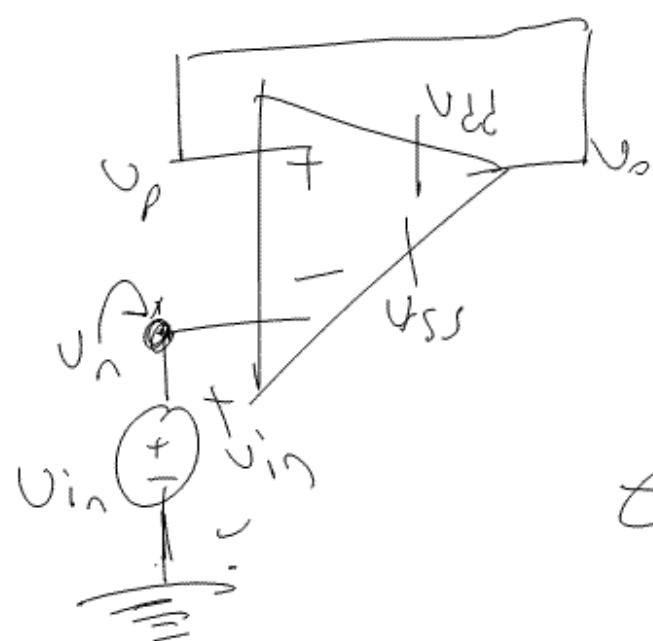
Remember $A \approx 10^6$ (pretty much $\rightarrow \infty$)

$$\Rightarrow \boxed{V_o = V_{in}}$$

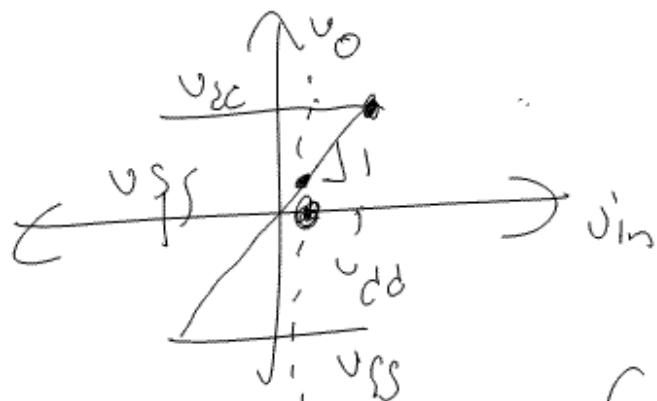
Let's look at physically what happens:



(an(ii)) Positive feedback:



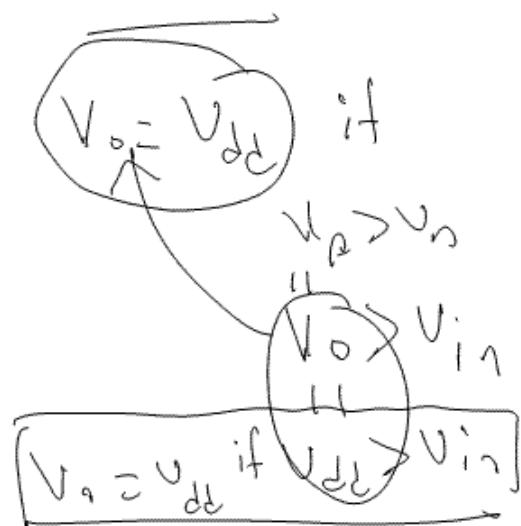
(Q.) Find $V_o = f(V_{in})$ &
Sketch $V_o = f(V_{in})$



Assume $V_o = A(V_P - V_o) = A(V_o - V_{in})$ ($V_P \approx V_o$)

$$\Rightarrow V_o = AV_o - AV_{in}$$

Rails:



$$\Rightarrow A v_{in} = V_o (A - 1)$$

$$\Rightarrow V_o = \frac{A v_{in}}{A - 1}$$

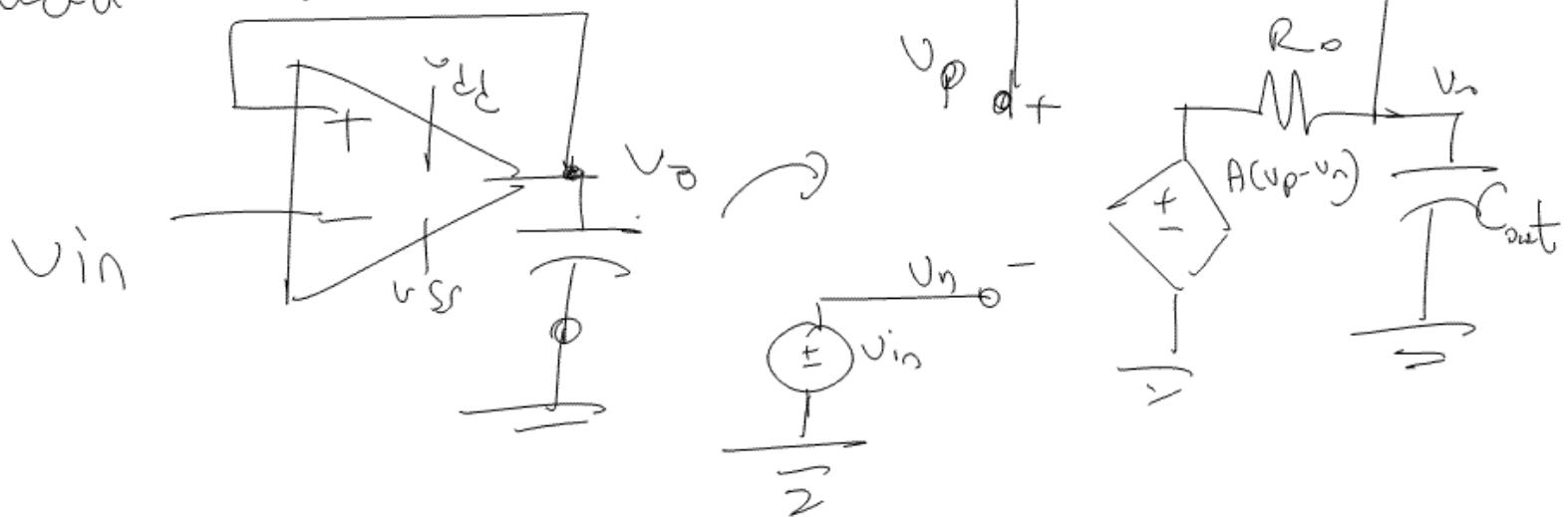
$$V_o = \frac{v_{in}}{1 - 1/A} \Rightarrow [V_o = v_{in} \quad (A \approx 1.5)]$$

Comment (i) $V_o = A(v_p - v_n)$

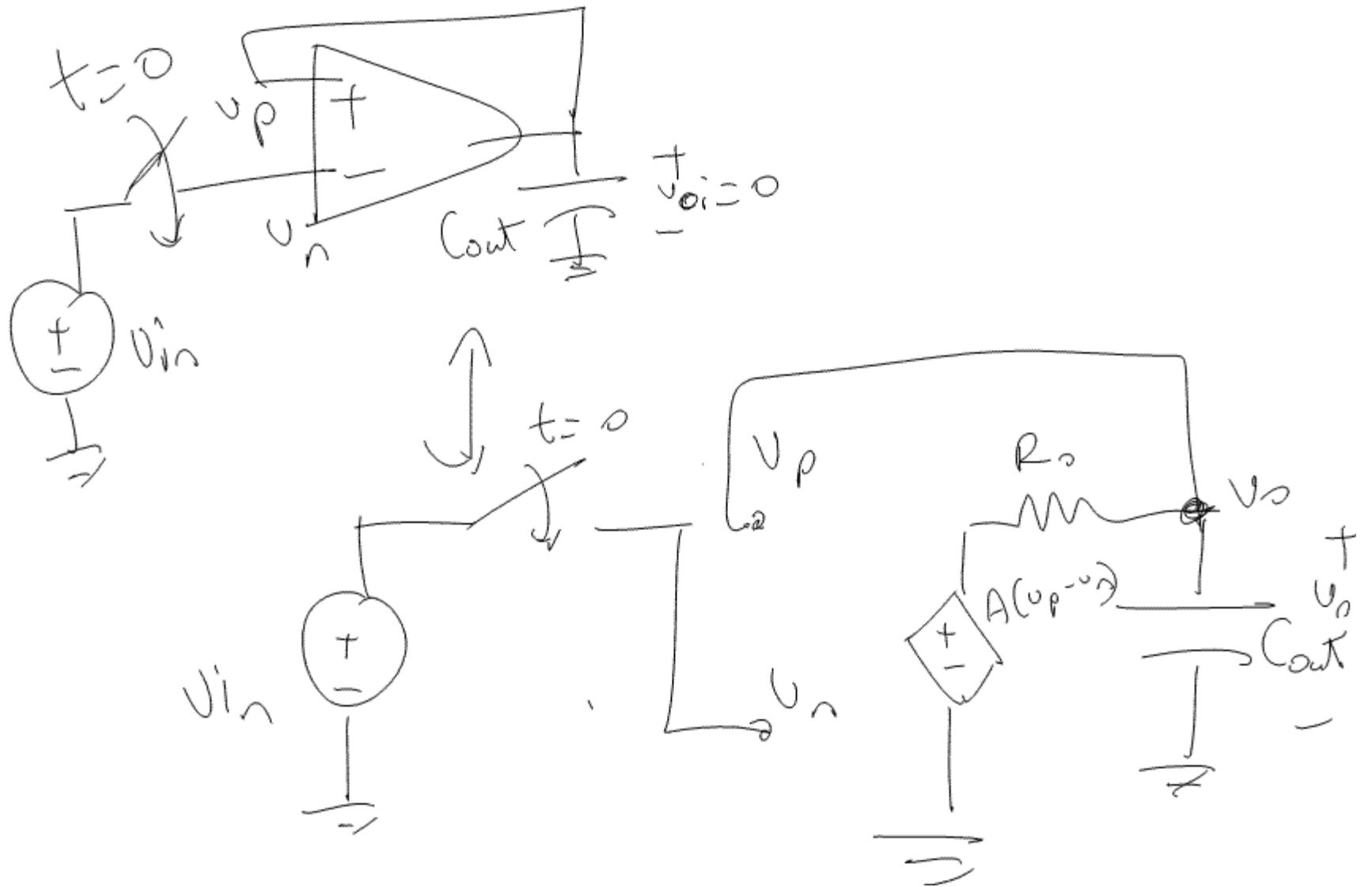
$$\boxed{\uparrow V_o = A(\uparrow V_o - v_{in})}$$

mathematically

(2) To see what happens in positive feedback, need to improve op-amp model
 (add C_{out})

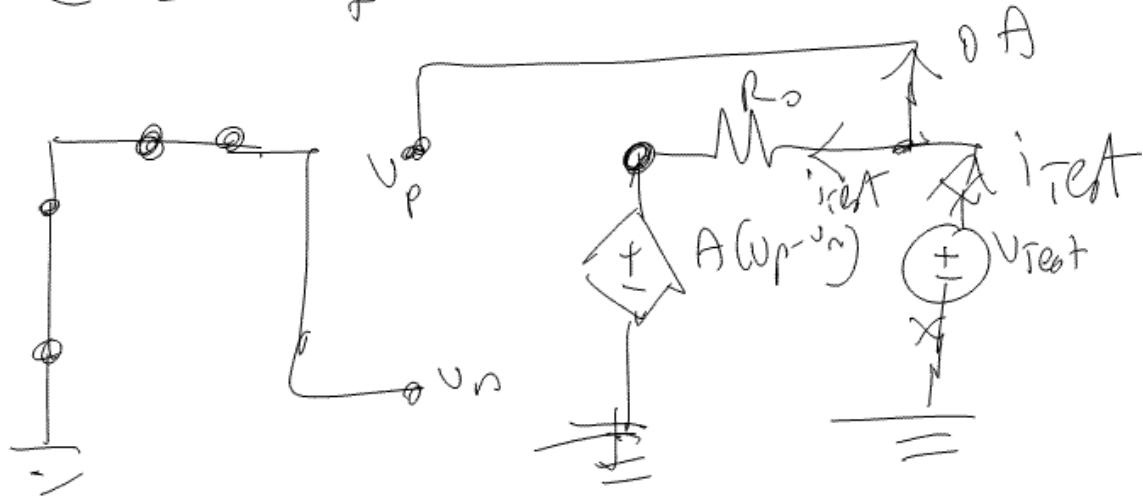


Let's setup an RC problem & find $v_o(t)$



$$V_o(t) = V_{of} + (V_{oi} - V_{of}) e^{-t/R_m C}$$

$R_m = ?$



$$R_m = \frac{V_{test}}{i_{test}} = \frac{\frac{V_{test+}}{V_{test+} - A(V_p - V_n)}}{R_0}$$

$$= \frac{V_{\text{test}}}{V_{\text{test}} - A(v_p - v_s)}$$

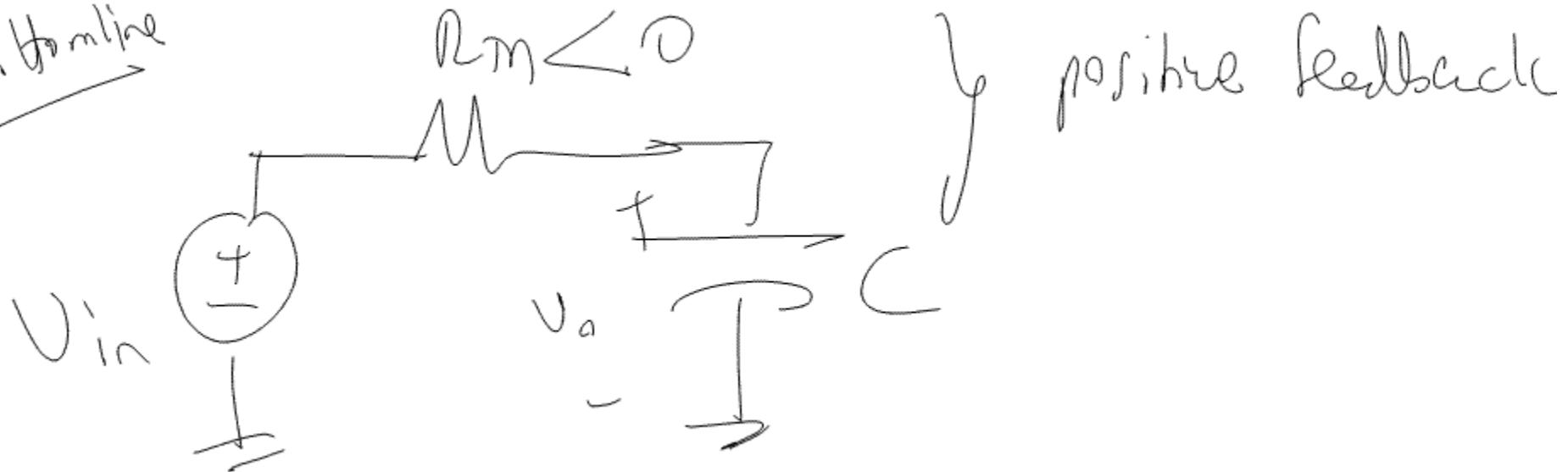
R_s ← really small

$$= \frac{V_{\text{test}}}{V_{\text{test}} - A(v_p - 0)} R_s$$

$$R_m = \frac{V_{\text{test}} R_s}{V_{\text{test}} - A V_{\text{test}}}$$

$\Rightarrow R_m = \frac{R_s}{1-A} \Rightarrow R_m < 0$ (really small negative)

β_{bottom}



$$V_{out} = V_{oi} + (V_{oi} - V_{of}) e^{-t/R_f C}$$

$$= V_{oi} + (V_{oi} - V_{of}) e^{\frac{-t}{R_f C}}$$

Note system is unstable } USE this
} later to
make oscillations