

## EE100 lecture 5- Capacitors & Inductors

- Administrivia
- Waiting list → sorted out by the end of this week?
  - H.W #1 → Solutions up by the end of this week (?)
  - Get back graded Hwks in lab on Thursday & next Tuesday
    - [grade correction deadline: my off]
    - on next Wednesday

T<sub>b</sub>day: Chapter 3 → 3.1 ] Capacitors      | 3.4 ] Inductors  
                                3.2 ]

READ: 3.3, 3.6 || Skip: 3.7

Chapter 4 → 4.1 through 4.4, skip 4.5

In circuits, we deal with current:

$$i \triangleq \frac{d\varphi}{dt} = \frac{d}{dt}(CV) = C \frac{dv}{dt} + \cancel{V} \frac{de}{dt} \xrightarrow{0} [C \text{ is constant}]$$

$$\Rightarrow i = C \frac{dv}{dt}$$

Convention:

$$\begin{array}{c} \downarrow \\ C \xrightarrow{T} \end{array} \quad \begin{array}{c} i \\ + \\ V \\ - \end{array} \Leftrightarrow i = C \frac{dv}{dt} \quad \left| \begin{array}{c} \uparrow \\ i \\ C \xrightarrow{T} \\ - \end{array} \right. \Leftrightarrow i = -C \frac{dv}{dt}$$

Cap. is  charging

Cap. is  discharging

Bottomline: Capacitors store energy in the form of electric field between the plates!

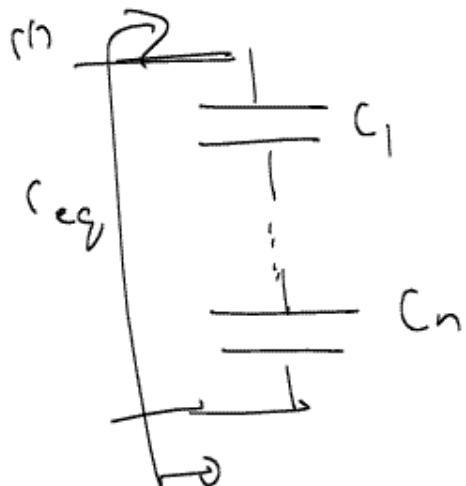
Note:

(1)  $\frac{\psi i}{\frac{1}{T} i_0}$   $\rightarrow$  displacement current  
 $\downarrow i$  (maxwell's equations)

(2)  $E = \int p dt = \int v i dt$   $c \frac{\psi i}{T} \downarrow$

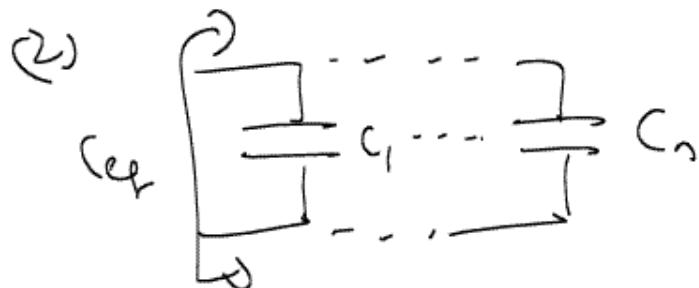
$$= \int v \left( \frac{cdv}{dt} \right) dt$$
$$= c \int v dv$$
$$= c \frac{v^2}{2} \Rightarrow \boxed{E = \frac{1}{2} cv^2}$$

## Section 3-2: Capacitors in series & parallel



$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots + \frac{1}{C_n}$$

[i.e., Capacitors in series combine like resistors in parallel]



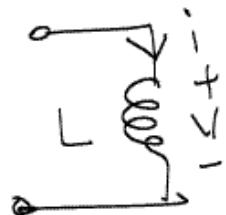
$$C_{eq} = C_1 + C_2 + \dots + C_n$$

---

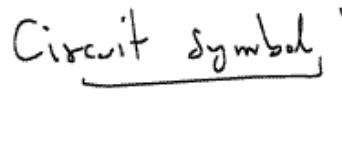
READ Section 3-3 ] useful for lab!

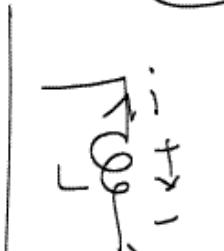
## Section 3.4 : Inductors

(1) Think about inductors as "ducks" of capacitors.



$$V = L \frac{di}{dt}$$

Circuit symbol:  inductance  
(units: Henrs)



$$V = C \frac{di}{dt}$$

$$\begin{aligned} i &= C \frac{dy}{dt} \\ V &= C \frac{dy}{dt} \end{aligned}$$

(2) Energy:  $E = \frac{1}{2} Li^2$

(3) Inductors in series:  $L_{eq} = l_1 + l_2 + \dots + l_n$

Inductors in parallel:  $\frac{1}{L_{eq}} = \frac{1}{l_1} + \frac{1}{l_2} + \dots + \frac{1}{l_n}$

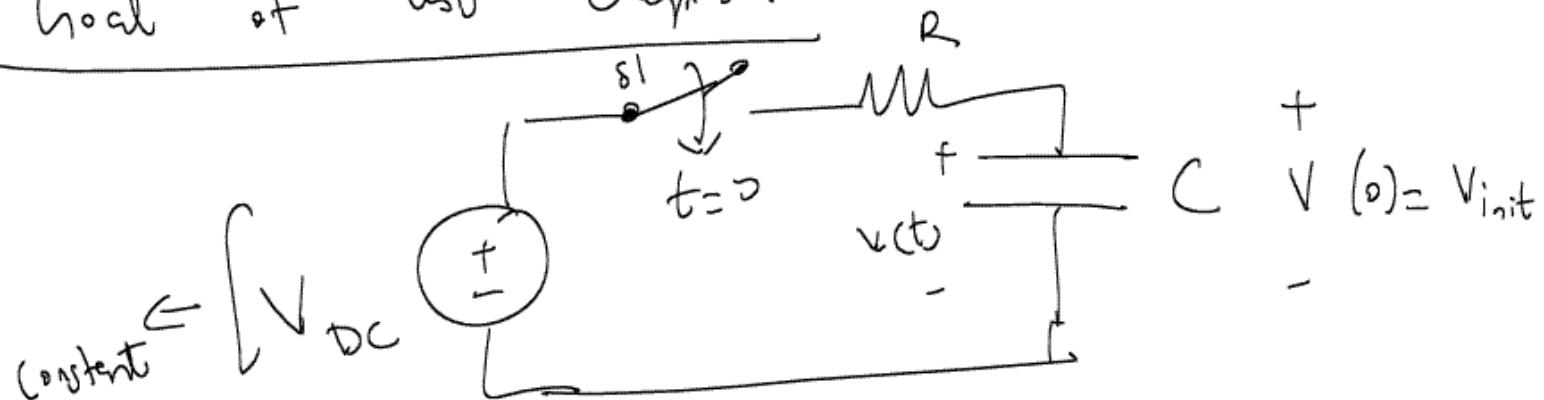
Recd 3-6, skip 3-7

---

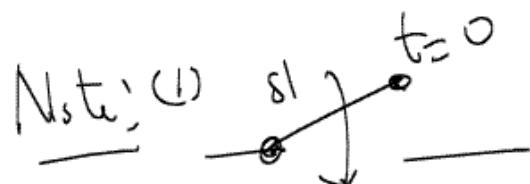
---

## Chapter 4 - Transients

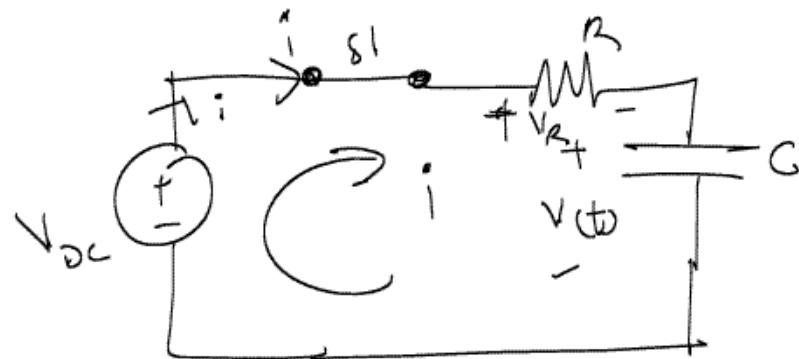
Goal of this chapter:



(Q:.) Find  $\underline{I}$  sketches  $\downarrow v(t)$  ( $t \geq 0$ ) ?



means: "switch  $S_1$  closes at  $t=0$ "



Circuit diagram  
for  $t \geq 0^+$

Note:  $0^+$  means  
the "instant" after  
switch  $S_1$  closes!

$$\text{KVL: } V_{DC} = V_R + V$$

$$\Rightarrow V_{DC} = iR + V \quad (\text{Ohm's law})$$

$$\Rightarrow V_{DC} = \left( C \frac{dV}{dt} \right) R + V$$

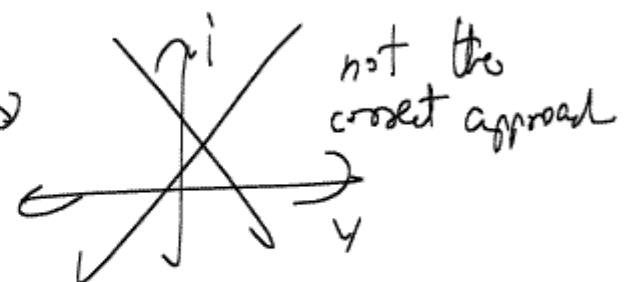
$$\Rightarrow \boxed{V_{DC} = RC \frac{dV}{dt} + V} \quad - (1)$$

Note(s): (1) units of  $RC$  in eqn ①: time  $\rightarrow$  explain  
significance (atcr.) (second)

(2) Eqn. ① is a linear first-order ordinary differential equation with constant coefficients.

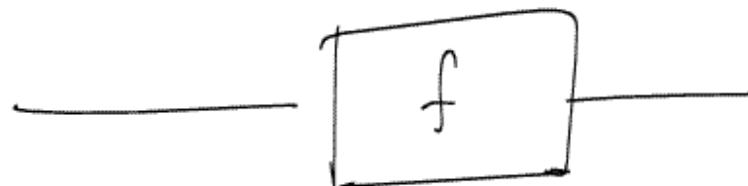
Sub-note: (1) Linearity: Is a capacitor a linear device?

$$C \frac{dv}{dt} + v \Leftrightarrow i = C \frac{dv}{dt}$$



(A':) Yes, capacitors are linear because the  $\frac{d}{dt}$  is

a linear operator. Mathematical definition of a linearity.



If function  $f$  is linear;  $\alpha, \beta \in$  domain of  $f$ .

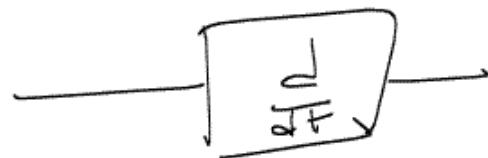
$$\alpha, \beta \in R$$

$$f(\alpha\alpha + \beta\beta) = \alpha f(\alpha) + \beta f(\beta)$$

Ex:

(1) 
$$f(x_1 + x_2) = (\alpha + \beta)^2 \quad [\alpha = \beta = 1]$$
$$= x_1^2 + x_2^2 + 2x_1 x_2$$
$$\neq f(x_1) + f(x_2) = x_1^2 + x_2^2$$

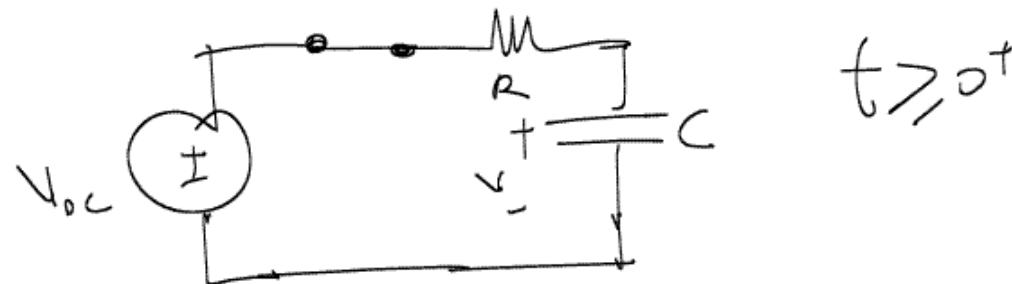
(2)



$$\frac{d}{dt} (\alpha f + \beta g) = \alpha \frac{df}{dt} + \beta \frac{dg}{dt}$$



Giving here:



$$V_{C(t)} = RC \frac{dv}{dt} + V \quad \text{--- (2)}$$

Read book for separation of variables ! I will

solve (2) by guessing

Guess:  $V(t) = A + Be^{-t/\tau}$  ③

Verify my guess by plugging ③

in ② and trying to solve

for  $A, B, \tau$ :

$$V_{oc} = RC \frac{d}{dt} (A + Be^{-t/\tau}) + (A + Be^{-t/\tau})$$

$$\frac{dy}{dt} = -\frac{V}{RC} t V_{oc}$$

Basicells  
const

$$\frac{dv}{dt} = -v$$

↓

solution is exponential

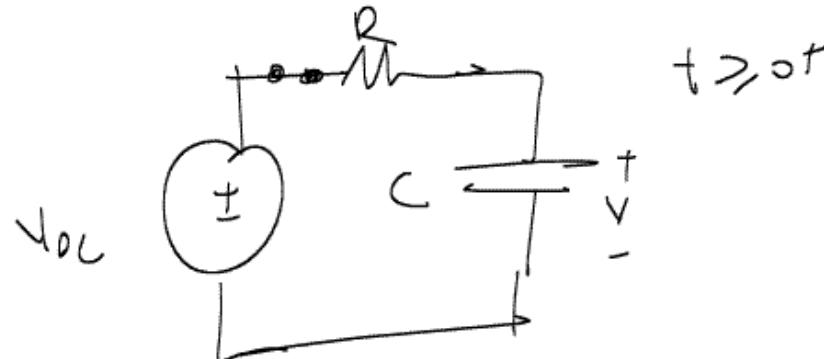
$$\Rightarrow V_{oc} = RC \left( -\frac{Be^{-t/\tau}}{\tau} \right) + A + Be^{-t/\tau}$$

$$\Rightarrow V_{DC} = A + \left[ B - \frac{B}{\tau} RC \right] e^{-t/\tau} \quad (3)$$

$$V = A + B e^{-t/\tau} : \text{guess}$$

From (3):  $A = ?$

$\tau = ?$



Note:  $V(0) = V_{initial}$

Comparing coefficients on both sides in initial condition (3):

$$\boxed{V_{DC} = A}, \quad 0 = B - \cancel{\frac{B}{\tau} RC} \quad \begin{array}{l} \text{Note: Intuitively } \\ \text{is not true} \end{array}$$

$$\Rightarrow \boxed{\tau = RC}$$

$$\therefore V(t) = V_{DC} + Be^{-t/\tau_{RC}}$$

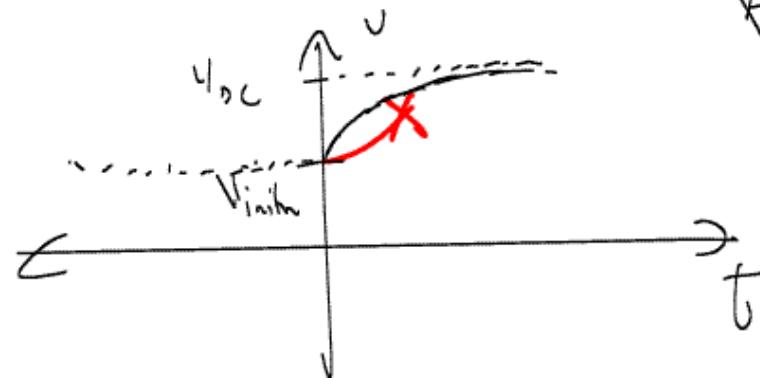
Initial condition  $V(0) = V_{DC} + B$

$$\Rightarrow B = -V_{DC} + V_{initial}$$

$$\therefore \boxed{V(t) = V_{DC} + (V_{initial} - V_{DC}) e^{-t/\tau_{RC}}}$$

Note: (1)  $V_{DC} \rightarrow$  steady-state resp. as  $t \rightarrow \infty$ ,  
 $(V_{initial} - V_{DC}) e^{-t/\tau_{RC}} \rightarrow 0$  (transient part "dies out").

(2) Sketch v(t):

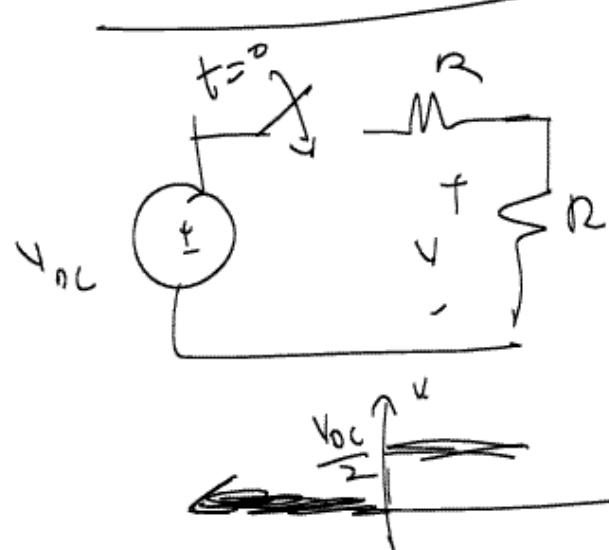


Assume:

$$V_{initial} > 0$$

$$\Delta V_{DC} > V_{initial}$$

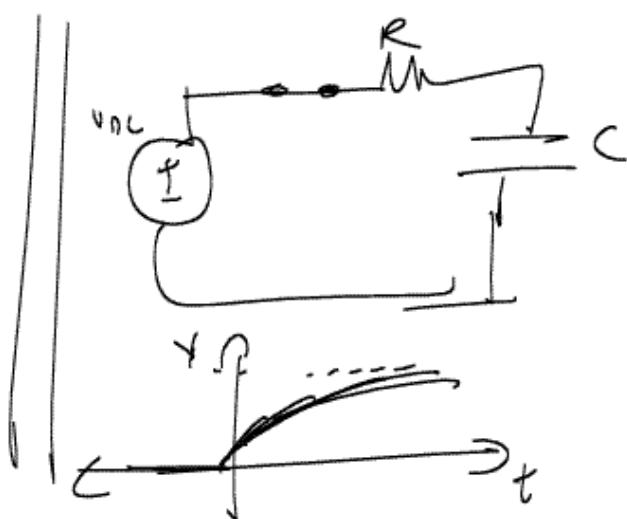
Note the difference:



Non-dynamic

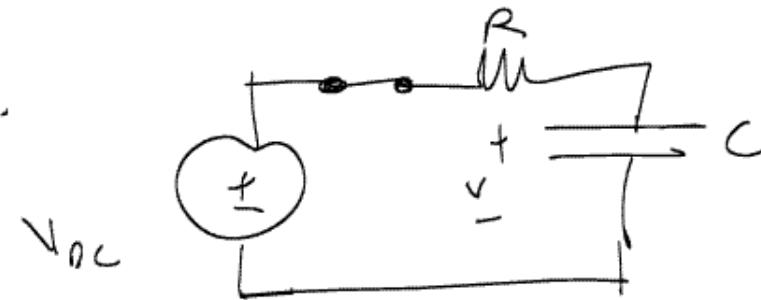
(assume  $V_{DC} > 0$ )

Dynamic



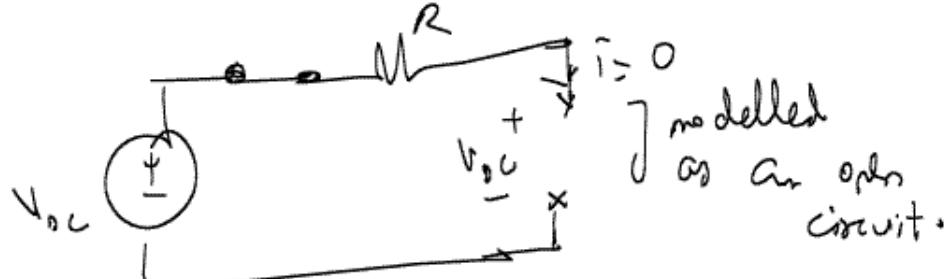
(3) Natural question: How long does the transient last? Technically:  $t \rightarrow \infty$ . Rule of thumb:  $[5\tau = t_1]$   
 transient has died out.

(4) Circuit Concepts:



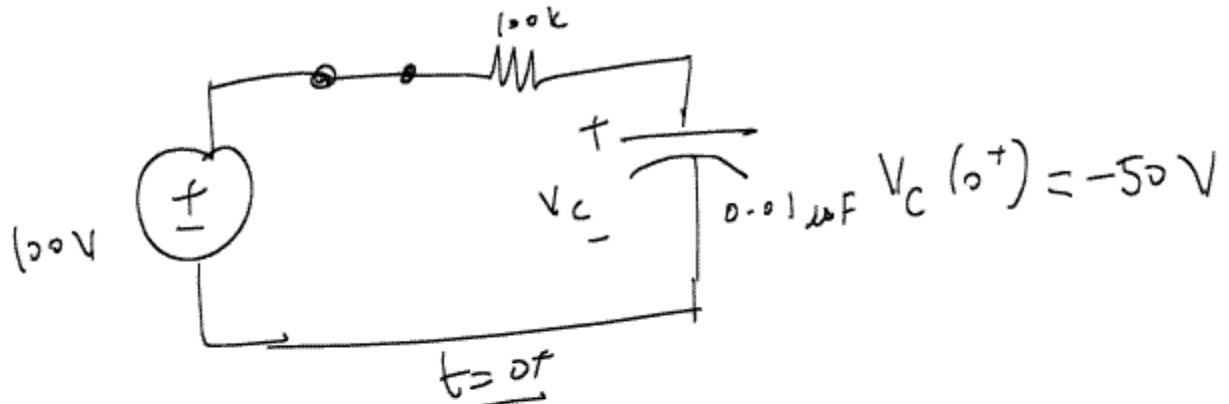
$$V_D = V_{DC} + (V_{initial} - V_{DC}) e^{-\frac{t}{RC}} \quad t \rightarrow \infty$$

$\downarrow$   
 $V_{final}$

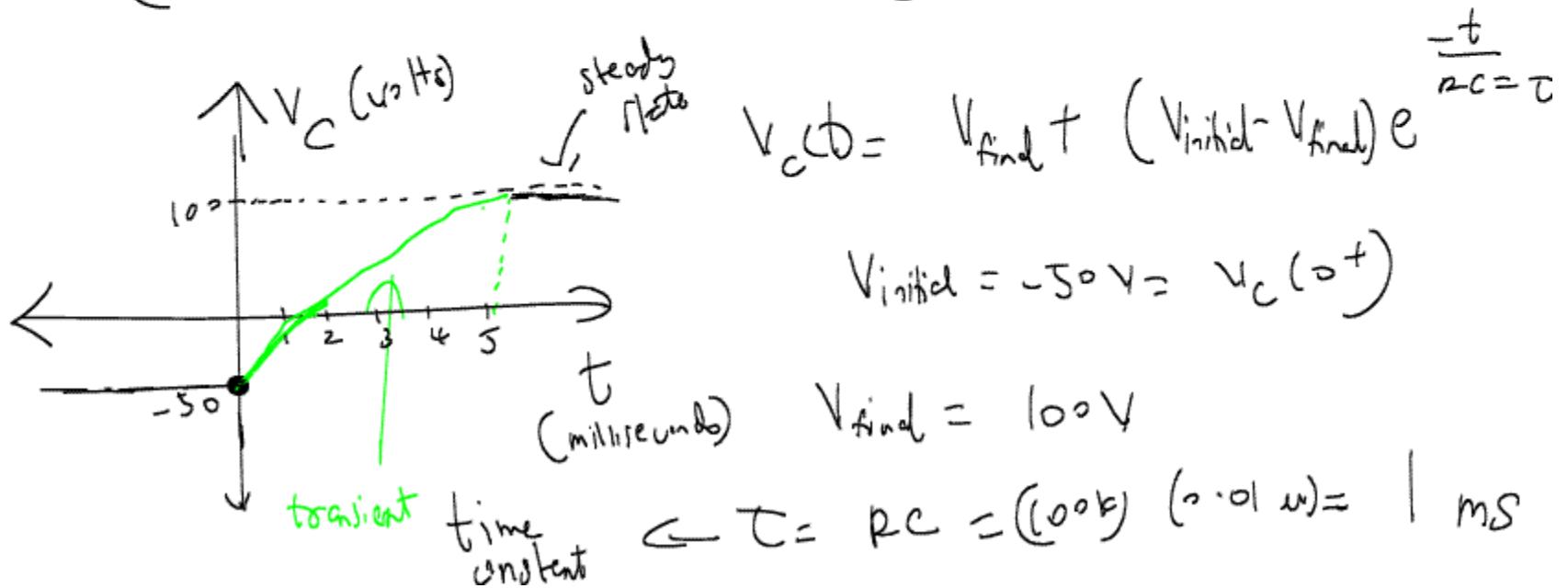


Examps:-

Q4.4:

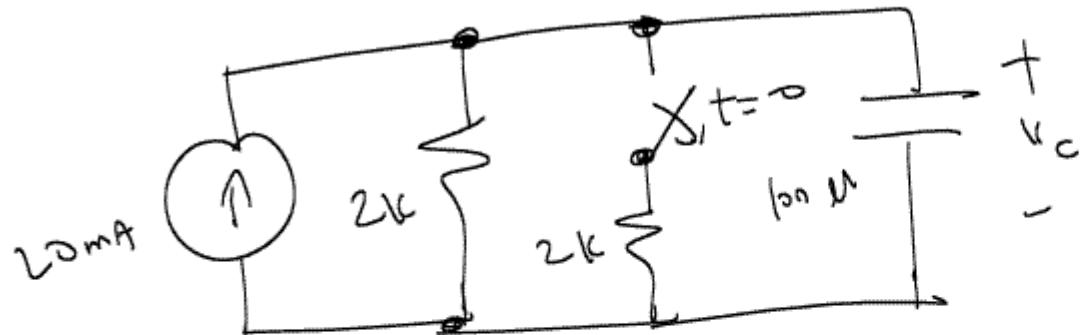


(Q4.) Find & sketch  $v(t)$ :



More complicated example:

p. 4.19: (p. 184)



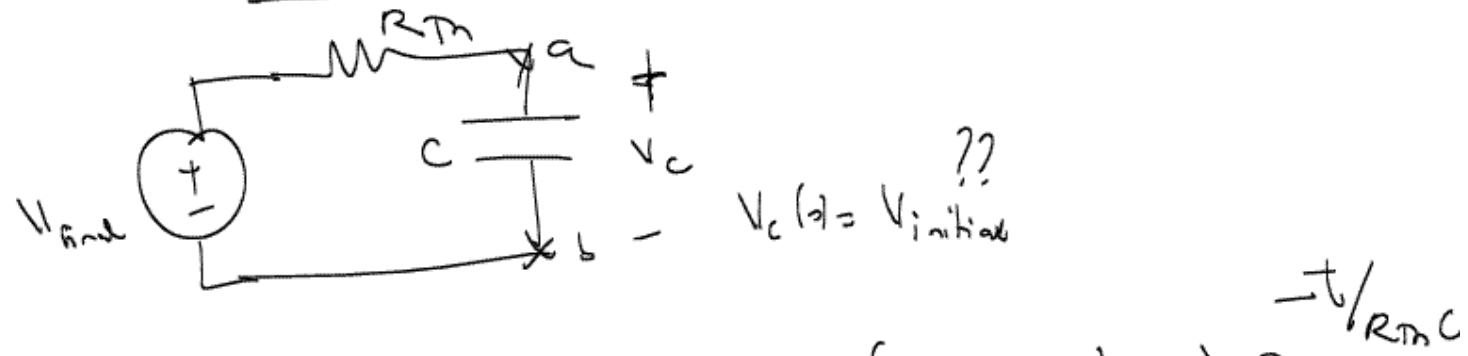
(Q:) Switch has been closed for a long time before opening at  $t=0$ . Determine:

(a)  $V_c(t=0^-)$ ,  $V_c(t=0^+)$

(b)  $V_c(t \rightarrow \infty)$  (c)  $V_c(t)$ ,  $t \geq 0^+$

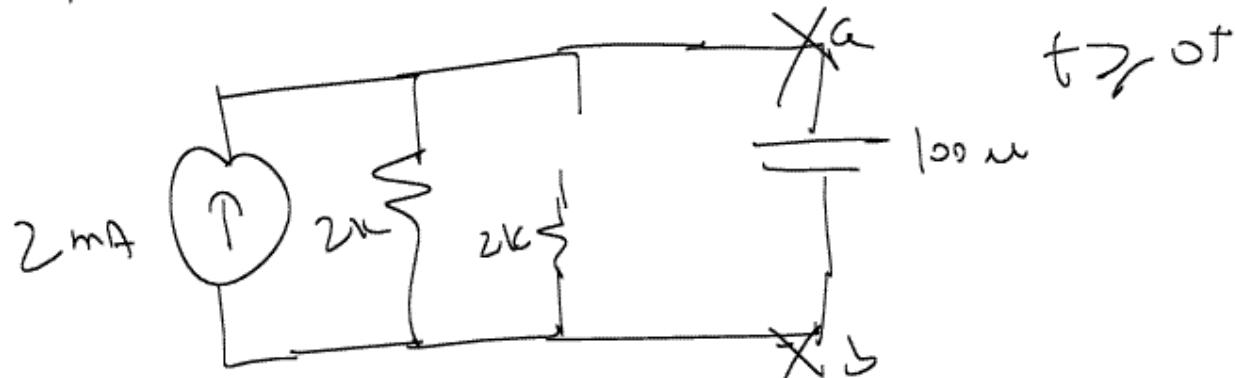
(d) sketch  $V_c(t)$ .

Solution: One way to do this problem:



$$V_c(t) = V_{\text{final}} + (V_{\text{initial}} - V_{\text{final}}) e^{-t/R_m C}$$

Therefore, we find the Thevenin equivalent:



Goal: Find  $V_{final}$  &  $V_{initial}$  using circuit analysis

(1)  $V_{final}$ : model capacitor as an open circuit as  $t \rightarrow \infty$ !

(2)  $V_{initial}$ :

$$\frac{dV}{dt} + \frac{i}{C} = 0 \Rightarrow i = C \frac{dy}{dt}$$

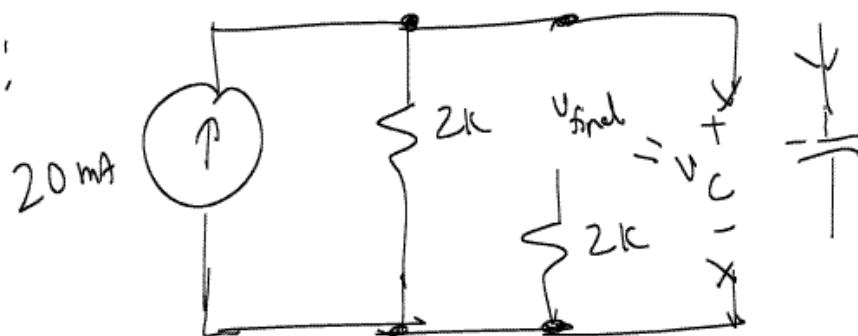
If voltage across a capacitor changes 'instantaneously'  
(i.e.,  $\frac{dy}{dt} \rightarrow \infty$ )  $\Rightarrow i \rightarrow \infty$ ] Physically not possible!

↳ Voltage across a capacitor cannot change 'instantaneously'

$$V_c(0^-) = V_c(0^+), \text{ in our case}$$

In our problem: we want:  $V_c(t) = V_{\text{final}} + (V_{\text{initial}} - V_{\text{final}}) e^{\frac{-t}{R_m C}}$

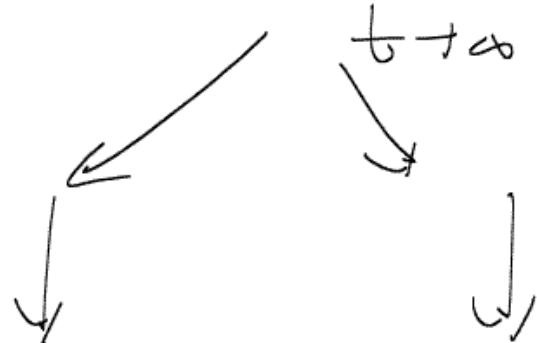
$V_{\text{final}}$ :

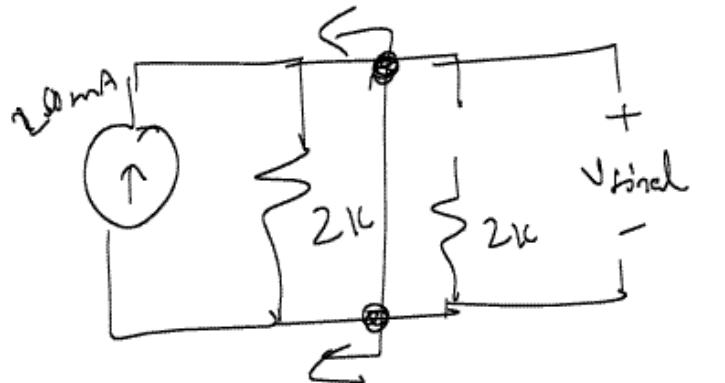


$$i_c = C \frac{dV_c}{dt}$$

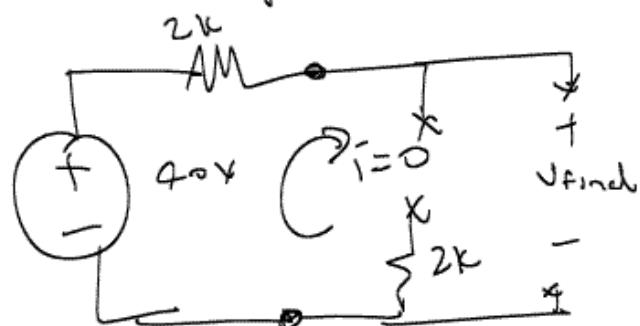
$\Rightarrow i_c = 0$   
(if  $V_c$  is constant)

$V_c = \text{constant}$ : open circuit

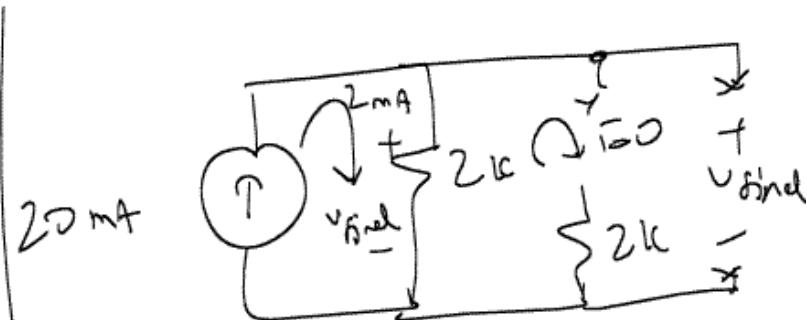




Source  
bottom



$$V_{\text{find}} = 40 \text{ V}$$



$$V_{\text{find}} = (20 \text{ mA}) (2 \text{ k})$$

$$V_{\text{find}} = 40 \text{ V}$$

- On Friday: (1) Finish up example  
(2) More Examples (2b examples as well)