

## EE102 lecture 8 - Chapter 5 (finish it)

Administrivia:  $\rightarrow$  Better solution for lecture videos on  
the way (?)

Recall: Phase:

$$v(t) = V_m \cos(\omega t + \phi) \quad \longleftrightarrow \quad \tilde{V} = V_m e^{j\phi}$$

Time domain

Complex domain

Ex 5.4  
Cp. (99)

$$i_1(t) = 10 \cos(\omega t + 30^\circ) + 5 \sin(\omega t + 30^\circ)$$

(Q.) Reduce  $i_1(t) = V_m \cos(\omega t + \phi)$  using phasors.

A.:  $i_1(t) = 10 \cos(\omega t + 30^\circ) + 5 \cos(\omega t + 30^\circ - 90^\circ)$

$$= 10 \cos(\omega t + 30^\circ) + 5 \cos(\omega t - 60^\circ)$$

$$\Rightarrow \overline{I_1} = 10 \angle 30^\circ + 5 \angle -60^\circ$$

$$= 10 e^{j30^\circ} + 5 e^{-j60^\circ}$$

↓ Simplify using a calculator

$$\approx 11.18 \cos(\omega t + 3.43^\circ) \left[ \begin{array}{l} \text{TI-89 saves} \\ \text{the day} \end{array} \right]$$

Note: (1) Good reference for TI-89 complex math.

<http://www.acad.sunytccc.edu/instruct/sbrown/ti83/complx89.htm>

(2) Simplifying by hand.

$$\begin{aligned} I_1 &= 10 \left( \cos 30^\circ + j \sin 30^\circ \right) + 5 \left( \cos 60^\circ - j \sin 60^\circ \right) \\ &= 10 \left[ \frac{\sqrt{3}}{2} + j \frac{1}{2} \right] + 5 \left[ \frac{1}{2} - j \frac{\sqrt{3}}{2} \right] \\ &= \underbrace{5\sqrt{3}}_{\text{real part}} + j \underbrace{5 + \frac{5}{2}}_{\text{imaginary part}} - j \underbrace{5\frac{\sqrt{3}}{2}}_{\text{negative imaginary part}} \end{aligned}$$

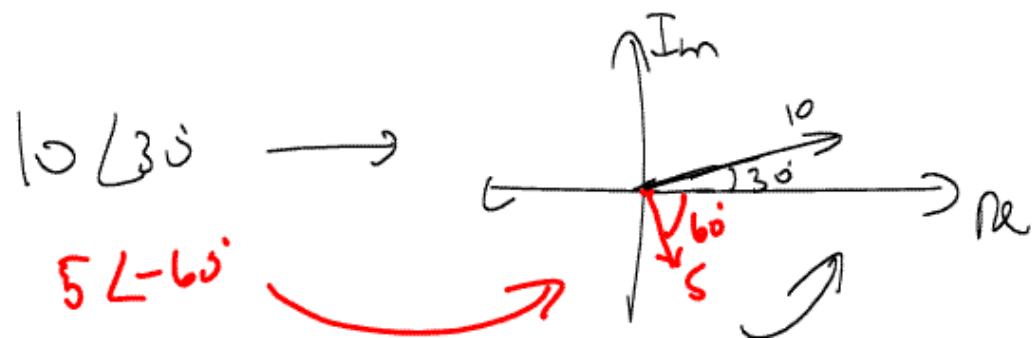
$$= \left( 5\sqrt{3} + \frac{5}{2} \right) + j \left( 5 - \frac{5\sqrt{3}}{2} \right)$$

↓                              ↓  
 $\alpha$                              $\beta$

$$\therefore \text{mag.} = \sqrt{\alpha^2 + \beta^2}, \quad \Theta = \tan^{-1} \left( \frac{\beta}{\alpha} \right)$$

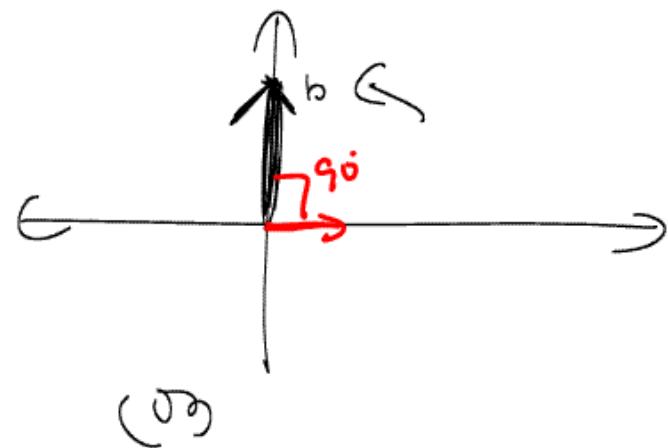

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Another way of visualizing phasor is using phasor diagrams:



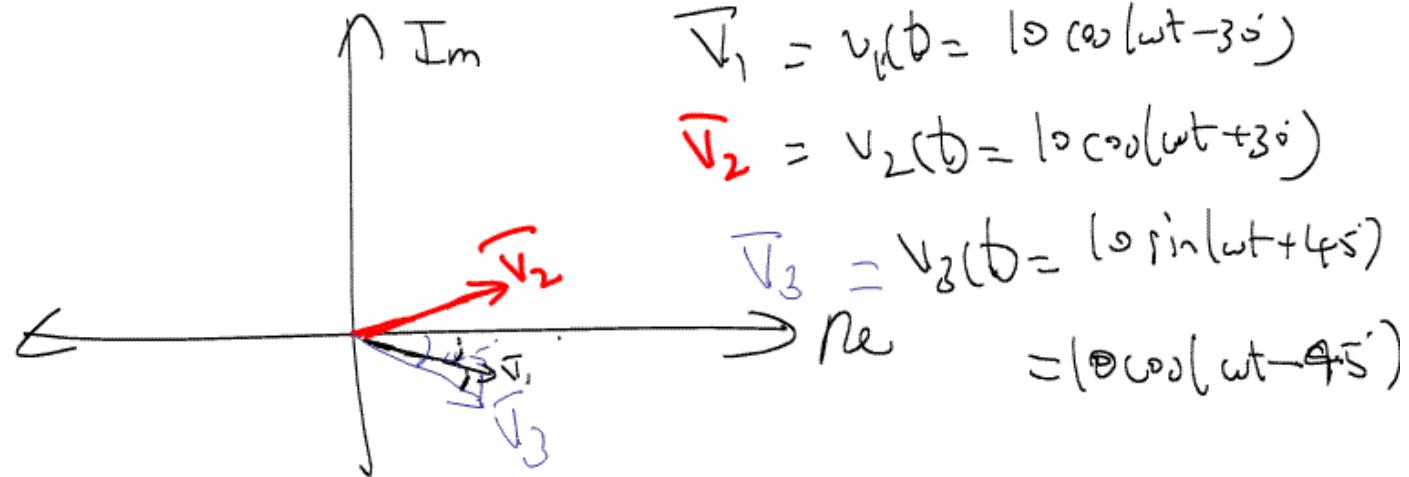
A common terminology is leading & lagging:

10/3s leading 5L-6s by 90°;



5L-6s lags 10/3s by 90°.

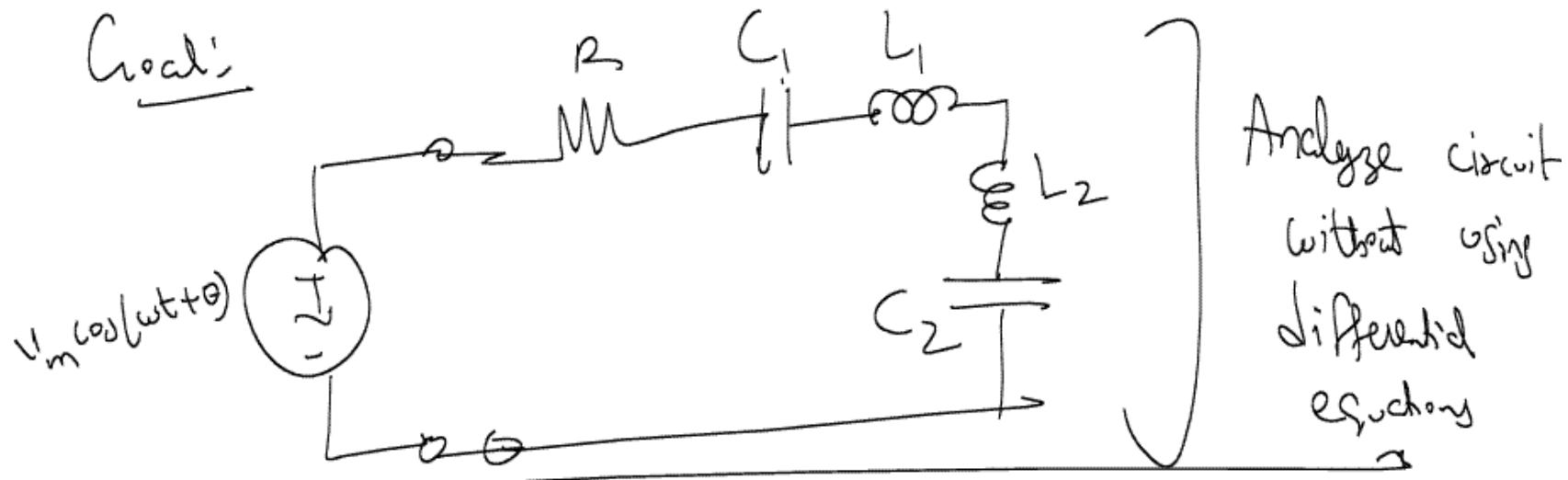
$$\left\{ \begin{array}{l} x \\ y \end{array} \right. \begin{array}{l} 5.5 \\ (p, 20) \end{array}$$



$\vec{V}_1$  leading  $\vec{V}_3$  by  $15^\circ$

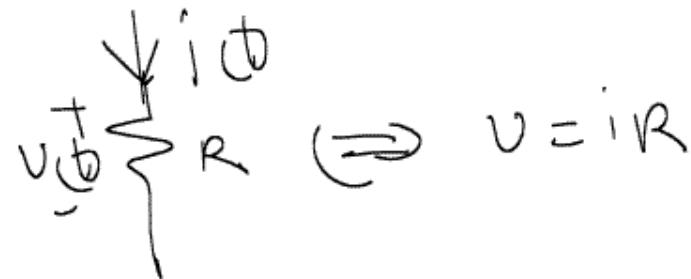
$\vec{V}_1$  lags  $\vec{V}_2$  by  $60^\circ$   
 $\vec{V}_3$  lags  $\vec{V}_2$  by  $75^\circ$

### S-3: Complex Impedances



How? Use phasors. We are going to derive a phasor i-v relationship for R, C & L.

(1)



Voltage &  
current phasor are in  
phase for a  
resistor

Phase form:

$$i(t) = I_m \cos(\omega t + \phi)$$
$$V(t) = \left[ I_m \cos(\omega t + \phi) \right] R$$
$$\Rightarrow \boxed{V = IR}$$

$\downarrow$

$$Z_R = R + 0j$$

(2) Inductor :



$$V_L = L \frac{di_L}{dt}$$

Let  $i_L(t) = I_m \sin(\omega t + \phi)$  Phase:  $\bar{I}_L = I_m \angle (\theta - 90^\circ)$

$$\therefore V_L(t) = L \frac{di_L}{dt} = L I_m \omega \cos(\omega t + \phi), \omega$$

$$V_L(t) = \omega L I_m \cos(\omega t + \phi)$$

Phase:  $\bar{V}_L = \omega L I_m \angle 0$

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Note:  $\omega L \angle 90^\circ I_m \angle (\theta - 90^\circ)$   
 $= \omega L e^{j90^\circ} \cdot I_m e^{j(\theta-90^\circ)}$   
 $= j\omega L \cdot I_m e^{j\theta} e^{-j90^\circ}$

$$= \omega L \angle 90^\circ \cdot I_m \angle (\theta - 90^\circ)$$

$$\Rightarrow j\omega L \cdot \bar{I}_L = \bar{V}_L \quad \text{---(1)}$$

$$\begin{aligned} & \left. \begin{aligned} & \text{Left: } \omega L \cdot I_m e^{j\theta} \cdot j \\ & \text{Right: } \omega L \cdot I_m e^{j\theta} \end{aligned} \right| \theta = 0^\circ \quad \boxed{\text{From (1), notice } V_L \text{ leads}} \\ & \Rightarrow \boxed{Z_L(\omega) = j\omega L} \end{aligned}$$

$\bar{I}_L$  by  $90^\circ$ .

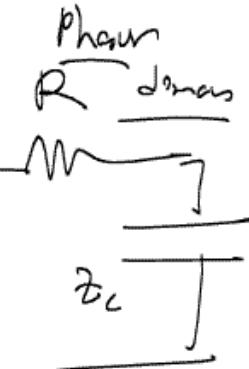
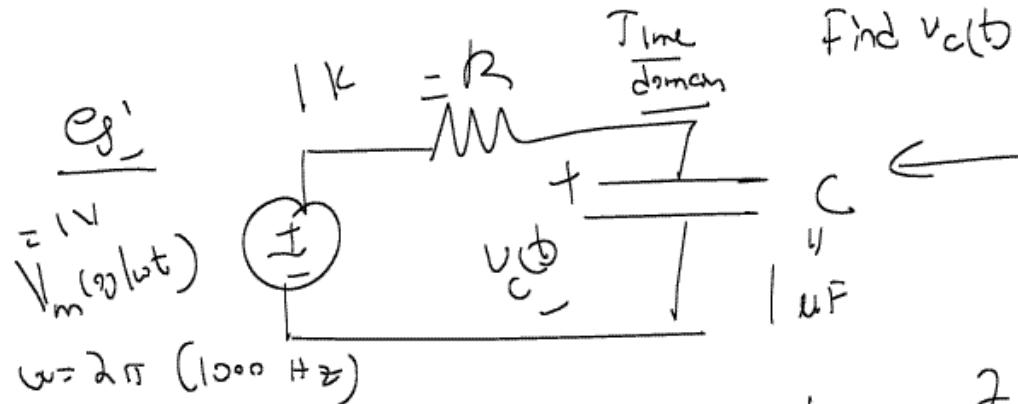
FEEs say:  $V_C$  lags current in an inductor by  $90^\circ$

$$\text{Cap: } \frac{V_C}{+} \xrightarrow{\text{---}} \frac{\bar{I}_C}{\frac{1}{j\omega C}} \quad \therefore \quad \bar{V}_C = \left( \frac{1}{j\omega C} \right) \bar{I}_C$$

$\Rightarrow Z_C(\omega)$

5.4-

Circuit analysis using phasors



$$Z_C = \frac{1}{j\omega C} \quad \sim$$

Note:  $KCL$  &  $KVL$   
are easily transferred to

phasor domain.

eg:  $KVL:$

$$\sum_{k=1}^4 v_k(t) \leftarrow \begin{array}{l} \text{Phasor:} \\ \ddot{v}_1 + \ddot{v}_2 + \ddot{v}_3 + \ddot{v}_4 = 0 \end{array}$$

$$= \frac{1}{j(2 \times 10^{-3}) \pi}$$

$$\begin{aligned}
 & \text{Circuit Diagram:} \\
 & \text{A voltage source } V_s = 0.5 \times 10^3 \text{ is connected in series with a } 1k\Omega \text{ resistor. This combination is in parallel with a } -\frac{500j}{\pi} \text{ ohm capacitor. The output voltage } V_C \text{ is measured across the capacitor.} \\
 & \text{Calculation:} \\
 & \frac{0.5 \times 10^3}{j\pi} = -\frac{500j}{\pi}
 \end{aligned}$$

$$\begin{aligned}
 V_C &= \left( \frac{-500j}{\pi} \right) \downarrow k_s \\
 &\quad \left[ \begin{array}{l} \text{Voltage} \\ \text{divider in} \\ \text{the freq.} \\ \text{domain} \end{array} \right]
 \end{aligned}$$

$$= \frac{-500j}{100\pi - 500j} \cdot 1 \angle 0^\circ$$

$100\pi - 500j$   
 $\downarrow$ , calculate

$$= 0.15 \angle -80^\circ, 1 \angle 0^\circ$$

$$\Rightarrow \bar{v}_c = 0.15 \angle -80^\circ \Rightarrow \boxed{v_c(t) = 0.15 \cos(2\pi 1000t - 80^\circ)}$$

Notice:  $v_b$  has a very small amplitude  $\Rightarrow$  it has been attenuated. Notice also how phase difference is approaching  $90^\circ$ !

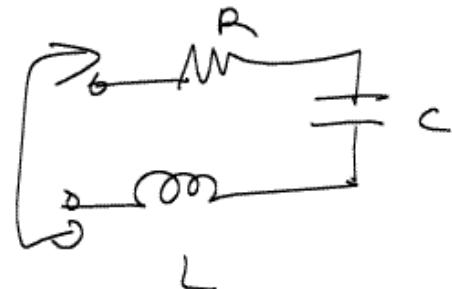
Notes: (1) AC analysis in PSpice  $\Rightarrow$  p. 891, Appendix D.

If all else fails, google search.

(2) Rest of Section 5.4  $\Rightarrow$  nodal analysis using phasors.

Review:

$\hookrightarrow$  Example:  $Z_{eq}$



$$Z_{eq} = R + \frac{1}{j\omega C} + j\omega L$$

simplify

$\downarrow$  Read Section 5.4, skip 5.5, read 5.6

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Next week  $\rightarrow$  midterm review & midterm ] Start learning!  
 $\rightarrow$  Review less  $\Rightarrow$  easy, have fun