(b) Consider an LTI filter $G : \mathbb{Z} \to \mathbb{R} \to \mathbb{Z} \to \mathbb{R}$ having impulse response $g$, where

$$\forall n \quad \exists N, \quad g(n) = f(n + N).$$

The transfer function $\hat{G}$ of the filter $G$ has exactly one zero at $z = 0$.

(i) Determine $N$ and provide a well-labeled pole-zero diagram for the transfer function $\hat{G}$. Be sure to specify the multiplicity of every pole and zero, whether finite or infinite.

$$F(z) = z^{N - \frac{z}{z - 2}} \frac{z^{2} - z}{(z + 1)(z - 1)}$$

Thus $N = 2$

Zeros: $0, \frac{1}{2}$

Poles: $-1, \frac{1}{2}$

(ii) Explain whether $G$ is a causal filter, and determine the numerical value of $g(0)$.

$G(2)$ was stable, $\hat{G}(z)$ was not causal, $f(n) \neq 0$ for some $N_0 < 0$.

$g(n) = f(n+1)$, there is a value $g(N_0 - 2) = f(N_0) \neq 0$. Since

$N_0 < 0, N_0 - 2 < 0$. Thus,

$G$ cannot be causal.

$$g(10) = \lim_{z \to 0} G(z) = \lim_{z \to 0} \frac{z^2 - 2z}{z^2 + \frac{z}{2} + \frac{z}{2}} = \lim_{z \to 0} \frac{2z - 2}{2z + \frac{z}{2} + \frac{z}{2}}$$

$$g(10) = \frac{1}{4}$$