Lecture 9: Midterm Review

Administrivia — CHEAT SHEET.

TWO 8.5 x 11 inch sheets (a total of FOUR PAGES), both sides, any font size/type.

1) TAs took 20 - 30 minutes

2) Makesh midturn yesterday [25% of that test was different]

student still finished 30 - 40 minutes, got an A-/A? p [25, 25, 25, 12, 10 + 3 points for your name]

midterm has 5 questions, worth 100 points — Bart thinks difficult (?)

Continue from last time, p. 7.33:

$t = 0^+$ Tip: (U) Try to draw three circuits for RC/RL problems: $t < 0$

$t = 0^+$

$t > 0$

$A^{+}$

(2) Circuit variables $[v, i]$ after switch movement will follow exponential. *Except independent sources
Important: However, only voltage across a capacitor (not current through an inductor) remains constant across the discontinuity (i.e., switch movement).

\[ i_L(t=0^-) = -I_2 = \frac{v_{80}}{10} = -5 \, \text{A} \]

\[ v_L(t=0^-) = 0 \, \text{V} \]

\[ v_o(t=0^-) = 0 \, \text{V} \]

\[ t>0^+ \]

\[ v_L(0^+) = 20 \, \text{V} \, \text{(X)} \]

\[ v_L(0^+) = 60 \, \text{V} \, \text{V} \]
\[ i_L(t) = i_{L_{\text{final}}} + (i_{L_{\text{initial}}} - i_{L_{\text{final}}}) e^{-t/\tau} \]

where

\[ i_L(t = 0^+) = \left[ \frac{L}{R_m} \right] \]

\[ \tau = \frac{L}{R_m} \]

\( R_m \to \infty \): Steady-state \( \Rightarrow \) inductor is modelled as short circuit.

\[ i_L(t + \infty) = -i_3 \]

\[ = -\left[ \frac{80 - 40}{16 + 4} \right] = \frac{-40}{20} = -2 \text{ A} \]

\( \tau = \frac{L}{R_m}, \quad R_m > 3 \)
"Kill" all sources ⇒ replace voltage sources with short circuits!

\[ R_m = 20 \Omega \]

\[ c = \frac{4 \text{ mH}}{20 \Omega} = 0.2 \text{ ms} \]

\[ i_L(t) = i_{L_{\text{final}}} + (i_{L_{\text{initial}}} - i_{L_{\text{final}}}) e^{-\frac{t}{c}} \]

\[ i_L(t=0^+)= \]

\[ = -2 + \left[ -5 - (-2) \right] e^{-\frac{t}{0.2 \text{ ms}}} \]

\[ \Rightarrow \quad i_L(t) = -2 - 3e^{-\frac{t}{0.2 \text{ ms}}} \text{ A} \]

\[ \text{Incorrect!} \]

Shade of graphs
Let's find $V_0(t)$:

$t = 0^+$

\[ V_0(t) = i_0(t) \cdot (1b) \]

\[ = [i_L(t + 5)] (1b) \]

\[ = (3 - 3e^{-\frac{t}{0.2ns}}) 1b \]

\[ b_0(t) = 48 \left(1 - e^{-\frac{t}{0.2ns}} \right) \text{ volts} \]

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From Chapter 7:

1. No section 7.5? 7.6, 7.7 on the test.

2. Don't worry about multiple switches.
(Q1) Why do you replace independent voltage sources with short circuit & independent current sources with open circuit for calculating $R_m$?

(A1)

[Diagram]

But, ideally, $R_{source} \rightarrow 0$

(Q2) Circuit with RC circuit,

(p. 728) on p. 311: Switch in circuit has been in position 1 for a long time, before moving to position 2 at $t=0$. Find $i(t)$ for $t > 0$. 
If you are really careful, you can do:

$$i_o(t) = i_o(t_{\text{final}}) + (i_o(t_{\text{initial}}) - i_o(t_{\text{final}})) e^{-\frac{t}{C}}$$

$$C = RmC$$

But $i_o(t_{\text{initial}}) = i_o(t_{\text{zero}})$ may not be equal to $i_o(t_{\text{final}})$ because only capacitor voltage remains unchanged across discontinuities.

Safer way: $i_o = \frac{C}{dt} \frac{dV_o}{dt}$
\[ V_o = V_{o,\text{final}} + (V_{o,\text{initial}} - V_{o,\text{final}}) e^{-t/\tau} \]

but: \[ V_{o,\text{initial}} = V_o(t=0^+) = V_o(t=0^-) \]

I will do first method:

\[ t < 0 \]

\[ V_o(t=0^-) = 15 \text{ V} \]

\[ t = 0^+ \]

\[ V_o(t=0^+) = V_o(t=0^-) = 15 \text{ V} \]
\[\text{Voltage: } -V_R - 5i_o - V_o = 0\]
\[\Rightarrow -V_R - 5i_o - \left(15\right) = 0\]
\[\Rightarrow - \left[10, 15\right] - 5i_o - 15 = 0\]
\[\Rightarrow i_o = -3/4 \text{ Amps}\]

\[t \to \infty,\]

\[\text{Thevenin equivalent}\]

\[\text{Notice the Thevenin equivalent at } ab \text{ is}\]

Simply a resistor! \[\Rightarrow R_m\]

\[\Rightarrow \text{Capacitor is fully discharged as } t \to \infty\]

\[\Rightarrow i_o \text{ final } \to 0 \text{ Amps}\]
\[ R_m = \frac{V_{\text{test}}}{I_{\text{test}}} \]

\[ V_r = -15 I_{\text{test}} \]

**kV:** \[ V_{\text{test}} + 5i_0 + V_2 = 0 \]

\[ = \]

\[ V_{\text{test}} = -V_r - 5i_0 \]

\[ = 15 I_{\text{test}} - [5 (-I_{\text{test}})] \]

\[ = 20 I_{\text{test}} \]

\[ R_m = \frac{V_{\text{test}}}{I_{\text{test}}} = \frac{20 I_{\text{test}}}{I_{\text{test}}} = 20 \Omega \]

\[ i_0(t) = i_{0\text{ind}} + (i_{\text{initial}} - i_{0\text{ind}}) e^{-t/\tau} \]

\[ = \left( \frac{3}{4} - \frac{t}{4} \right) A \]