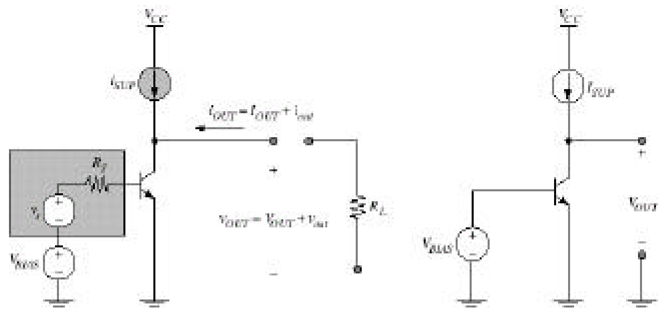
npn BJT Amplifier Stages: Common-Emitter (CE)

1. Bias amplifier in high-gain region

Note that the source resistor R_S and the load resistor R_L are *removed* for determining the bias point; the small-signal source is ignored, as well.

Use the load-line technique to find $V_{BIAS} = V_{BE}$ and $I_C = I_{SUP}$.

2. Determine two-port model parameters



Small-Signal Model of CE Amplifier

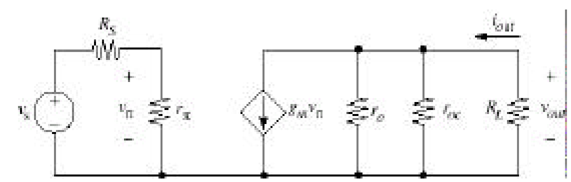
- The small-signal model is evaluated at the bias point; we assume that the current gain is β_o = 100 and the Early voltage is V_{An} = 25 V:

$$g_m = I_C / V_{th} \text{ (at room temperature)}$$

$$r_\pi = \beta_o / g_m = 10\ \mathrm{k}\Omega$$

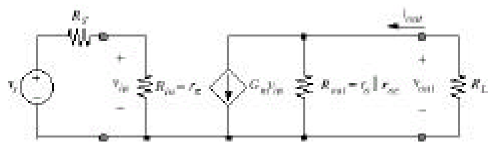
$$r_o = V_{An} / I_C = 100\ \mathrm{k}\Omega$$

- Substitute small-signal model for BJT; V_{CC} and V_{BIAS} are short-circuited for small-signals



Two-Port Model: CE Amplifier

- Use transconductance amplifier form for model (*not* mandatory)
- $R_{in} = r_\pi$, $R_{out} = r_o \parallel r_{oc}$, $G_m = g_m$ by inspection



- Compare with CS amplifier

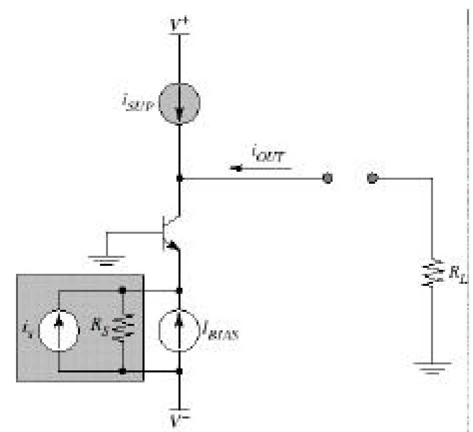
  inferior input resistance

  superior transconductance

  about the same output resistance (assuming r_o dominates)

Common-Base Amplifier

Input current is applied to the emitter (with a bias current source) and the output current is taken from the collector

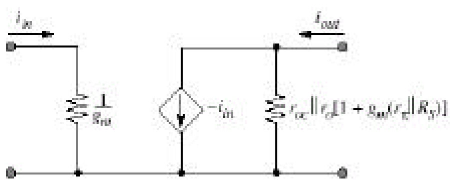


Common Base Two-Port Model

- See text for details of nodal analysis

  $R_{in} \cong 1/g_m$, $R_{out} \cong r_{oc}\|[r_o(1+g_m(r_\pi\|R_S))]$, $A_i = -\beta_o/(1+\beta_o) \cong -1$

- CB stage is an excellent current buffer



Comparison with the CG stage:

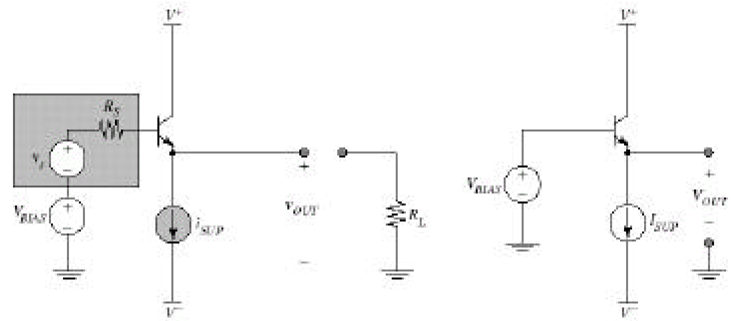
note the effect of the source resistance on the output resistance

if R_S is much greater than r_π, then the output resistance is approximately:

$$\boxed{R_{out} \approx r_{oc}\|[\beta r_o]}$$

Common-Collector Amplifier

- Circuit configuration



- Biasing: if transistor is “on” (i.e., not cutoff), then

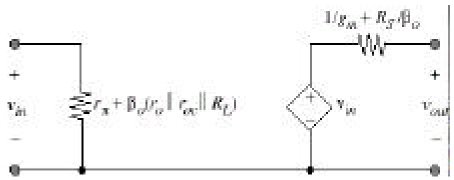
  V_{BIAS} - V_{OUT} = 0.7 V. Plot --

Alternative name ... emitter follower

Common Collector Two-Port Model

- Two-port model:

  presence of r_π makes the analysis more involved than for a common drain



Note 1: both the input and the output resistances depend on the load and source resistances, respectively (note typo in Fig. 8.47 in text)

Note 2: this model is approximate and can give erroneous results for extremely low values of R_L. However, it is very convenient for hand analysis.

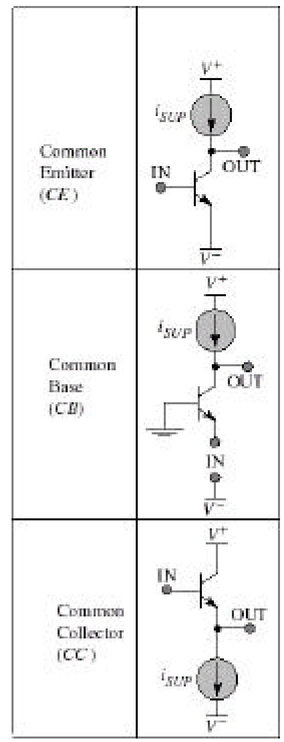
Comparison with CD stage:

CC's input resistance: high but not infinity

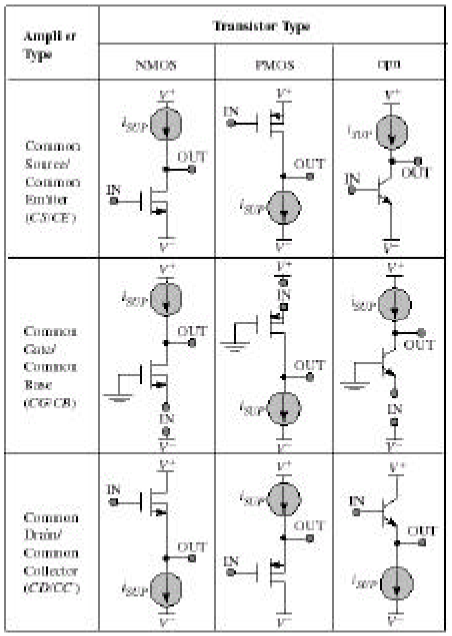
CC's output resistance: generally lower (but watch out for large R_S)

Summary of BJT Single-Stage Amplifiers

Why no pnp's?



Single-Stage MOS and BJT Amplifier

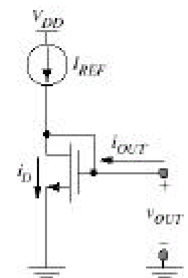


DC Voltage and Current Sources

- Output characteristics of a BJT or MOSFET look like a family of current sources ... how do we pick one?

  specify the gate-source *voltage* V_{GS} in order to select the desired current level for a MOSFET (specify V_{BE} exactly for a BJT)

  how do we generate a precise voltage? ... we use a current source to set the current in a "diode-connected" MOSFET



(wait a minute ... where do we find I_{REF}? Assume that one is available!)

$$i_D = I_{REF} + i_{OUT} \cong \left(\frac{W}{2L}\right)\mu_n C_{ox}(v_{OUT} - V_{Tn})^2$$

DC Voltage Sources

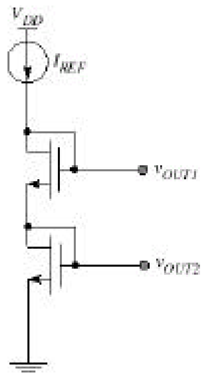
- Solving for the output voltage

$$v_{OUT} = V_{Tn} + \sqrt{\frac{I_{REF} + i_{OUT}}{\left(\frac{W}{2L}\right)\mu_n C_{ox}}}$$

If $I_D = 100\ \mu A$, $\mu_n = 50\ \mu AV^{-2}$, $(W / L) = 20$, $V_{Tn} = 1$ V, then

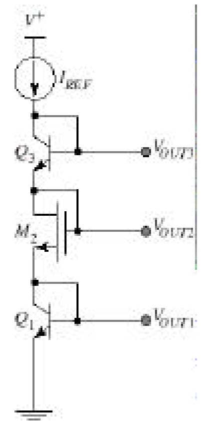
$V_{OUT} = 1.45$ V for $I_{OUT} = 0$ A.

- Stack up two diode-connected MOSFETs



Totem Pole Voltage Sources

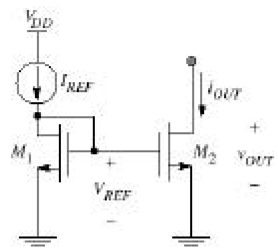
- Define a series of bias voltages between the positive and the negative supply voltages.



- In practice, output currents are small (or zero), so that the DC bias voltages are set by I_{REF}

MOSFET Current Sources

- Bias the n-channel MOSFET with a MOSFET DC voltage source!



- Intuitively, V_{REF} is set by I_{REF} and determines the output current of M_2

$$V_{REF} = V_{Tn} + \sqrt{\frac{I_{REF}}{\left(\frac{W}{2L}\right)_1 \mu_n C_{ox}}} = V_{GS1} = V_{GS2}$$

Substituting into the drain current of M_2 (and neglecting $(1 + \lambda_n V_{DS2})$ term)

$$i_{OUT} = i_{D2} = \left(\frac{W}{2L}\right)_2 \mu_n C_{ox} (V_{GS2} - V_{Tn})^2$$

MOSFET Current Sources (cont.)

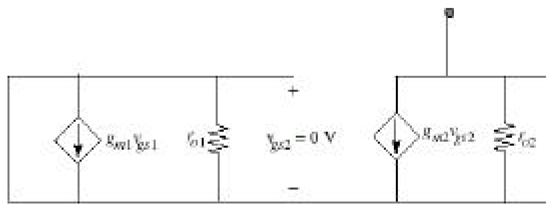
- Output current is scaled from I_{REF} by a geometrical ratio:

$$i_{OUT} = i_{D2} = \left(\frac{W}{2L}\right)_2 \mu_n C_{ox} \left(V_{Tn} + \sqrt{\frac{I_{REF}}{\left(\frac{W}{2L}\right)_1 \mu_n C_{ox}}} - V_{Tn} \right)^2$$

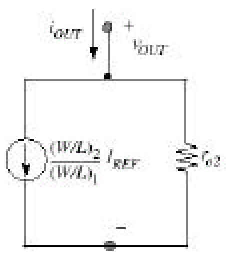
$$\boxed{I_{OUT} = \left(\frac{(W/L)_2}{(W/L)_1}\right) I_{REF}}$$

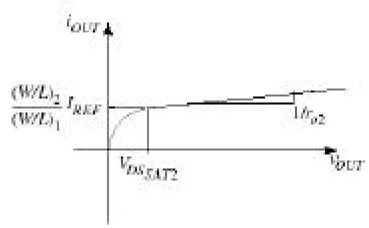
MOSFET Current Source Equivalent Circuit

- Small-signal model: source resistance is r_{o2} by inspection



- Combine output resistance with DC output current for approximate equivalent circuit ... actual i_{OUT} vs. v_{OUT} characteristics are those of M_2 with $V_{GS2} = V_{REF}$

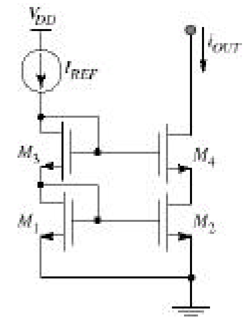




The model is only valid for $v_{DS} = v_{OUT} > v_{DS(SAT)} = V_{GS} - V_{Tn}$

The Cascode Current Source

- In order to boost the source resistance, we can study our single-stage building blocks and recognize that a common-gate is attractive, due to its high output resistance



- Adapting the output resistance for a common gate amplifier, the cascode current source has a source resistance of

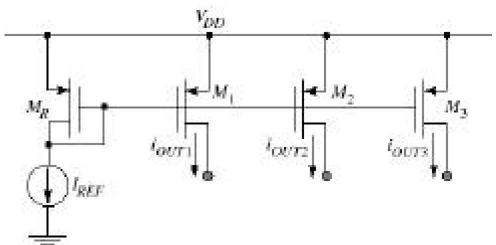
$$r_{oc} = (1 + g_{m4}r_{o2})r_{o4} \approx g_{m4}r_{o4}r_{o2}$$

- Penalty for cascode:

  needs larger V_{OUT} to function

MOSFET Current "Mirrors"

- n-channel current source *sinks* current to ground ... how do we *source* current from the positive supply? Answer: p-channel current sources...?



- By mixing n-channel and p-channel diode-connected devices, we can produce current sinks and sources from a reference current connected to V_{DD} or ground.

Two-Port Parameters for Single-Stage Amplifiers

Amplifier Type	Controlled Source	Input Resistance R_{in}	Output Resistance R_{out}
Common Emitter	$G_m = g_m$	r_π	$r_o \parallel r_{oc}$
Common Source	$G_m = g_m$	infinity	$r_o \parallel r_{oc}$
Common Base	$A_i = -1$	$1 / g_m$	$r_{oc} \parallel [(1 + g_m(r_\pi \parallel R_S))\, r_o]$, for $g_m r_o >> 1$
Common Gate	$A_i = -1$	$1 / g_m$, $(v_{sb} = 0)$ -otherwise- $1 / (g_m + g_{mb})$	$r_{oc} \parallel [(1 + g_m R_S) r_o]$, $(v_{sb}=0)$ -otherwise- $r_{oc} \parallel [(1+ (g_m + g_{mb})R_S)\, r_o]$ both for $r_o >> R_S$
Common Collector	$A_v = 1$	$r_\pi + \beta_o(r_o \parallel r_{oc} \parallel R_L)$	$(1 / g_m) + R_S / \beta_o$
Common Drain	$A_v = 1$ if $v_{sb} = 0$, -otherwise- $g_m / (g_m + g_{mb})$	infinity	$1 / g_m$ if $v_{sb} = 0$, -otherwise- $1 / (g_m + g_{mb})$

Note: appropriate two-port model is used, depending on controlled source

Sinusoidal Function Review

Sinusoidal functions are important is analog signal processing

$$v(t) = v\cos(\omega t + \phi)$$

amplitude (half of peak-to-peak)

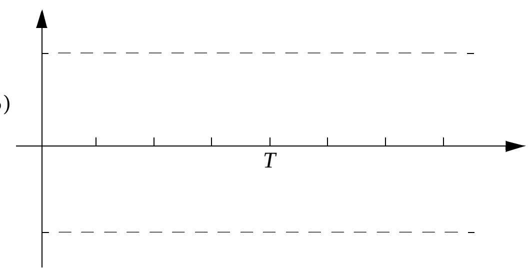
frequency (radian) ... $\omega = 2\pi f = 2\pi\,(1/T)$

phase (degrees or radians)

$v_1(t) = v\cos(\omega t)$

$v_2(t) = v\cos(\omega t - 45)$

$\omega = \dfrac{2\pi}{T}$



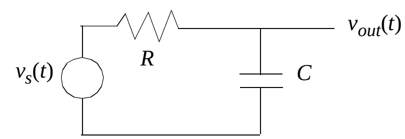
1. EECS 20/120: periodic functions can be represented as sums of sinusoids functions at different frequencies.

2. The response of a circuit to a sinusoidal input signal, as a function of the frequency, leads to insights into the behavior of the circuit.

Frequency Response

Key concept: small-signal models for amplifiers are *linear* and therefore, cosines and sines are solutions of the linear differential equations which arise from R, C, and controlled source (e.g., G_m) networks.

- The problem: finding the solutions to the differential equations is TEDIOUS and provides little insight into the behavior of the circuit!



Phasors

It is much more efficient to work with *imaginary exponentials* as “representing” the sinusoidal voltages and currents ... since these functions are solutions of linear differential equations and

$$\frac{d}{dt}(e^{j\omega t}) = j\omega(e^{j\omega t})$$

How to connect the exponential to the measured function *v(t)*? Conventionally, *v(t)* is the real part of the of the imaginary exponential

$$v(t) = v\cos(\omega t + \phi) \rightarrow \mathrm{Re}(ve^{(j\omega t + \phi)}) = \mathrm{Re}(ve^{j\phi}e^{j\omega t})$$

where v is the amplitude and ϕ is the phase of the sinusoidal signal *v(t)*.

The ***phasor V*** is defined as the complex number

$$\boxed{V = ve^{j\phi}}$$

Therefore, the measured function is related to the phasor by

$$\boxed{v(t) = \mathrm{Re}(Ve^{j\omega t})}$$

Circuit Analysis with Phasors

- The current through a capacitor is proportional to the derivative of the voltage:

$$i(t) = C\frac{\mathrm{d}}{\mathrm{d}t}v(t)$$

We assume that all signals in the circuit are represented by sinusoids. Substitution of the phasor expression for voltage leads to:

$$v(t) \rightarrow Ve^{j\omega t} \qquad \ldots \qquad Ie^{j\omega t} = C\frac{\mathrm{d}}{\mathrm{d}t}(Ve^{j\omega t}) = j\omega CVe^{j\omega t}$$

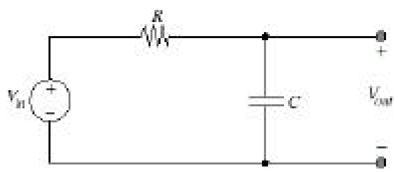
which implies that the ratio of the phasor voltage to the phasor current through a capacitor (the ***impedance***) is

$$\boxed{Z(j\omega) = \frac{V}{I} = \frac{1}{j\omega C}}$$

- Implication: the phasor current is *linearly proportional* to the phasor voltage, making it possible to solve circuits involving capacitors and inductors as rapidly as resistive networks ... as long as all signals are sinusoidal.

Phasor Analysis of the Low-Pass Filter

- Voltage divider with impedances --



Replacing the capacitor by its impedance, 1 / ($j\omega C$), we can solve for the ratio of the phasors V_{out} / V_{in}

$$\frac{V_{out}}{V_{in}} = \frac{1/j\omega C}{R + 1/j\omega C}$$

multiplying by $j\omega C/j\omega C$ leads to

$$\frac{V_{out}}{V_{in}} = \frac{1}{1 + j\omega RC}$$

Frequency Response of LPF Circuits

The phasor ratio V_{out} / V_{in} is called the transfer function for the circuit

How to describe V_{out} / V_{in}?

complex number ... could plot Re(V_{out} / V_{in}) and Im(V_{out} / V_{in}) versus frequency

polar form translates better into what we measure on the oscilloscope ... the magnitude (determines the amplitude) and the phase

- ***"Bode plots"***:

magnitude and phase of the phasor ratio: V_{out} / V_{in}

range of frequencies is very wide (DC to 10^{10} Hz, for some amplifiers)
therefore, plot frequency axis on log scale

range of magnitudes is also very wide:
therefore, plot magnitude on log scale

Convention: express the magnitude in decibels "dB" by

$$\left|\frac{V_{out}}{V_{in}}\right|_{dB} = 20\log\left|\frac{V_{out}}{V_{in}}\right|$$

phase is usually expressed in degrees (rather than radians):

$$\angle\frac{V_{out}}{V_{in}} = \operatorname{atan}\left[\frac{Im(V_{out}/V_{in})}{Re(V_{out}/V_{in})}\right]$$

Complex Algebra Review

* Magnitudes:

$$\left|\frac{Z_1}{Z_2}\right| = \frac{|Z_1|}{|Z_2|} = \frac{\sqrt{X_1^2 + Y_1^2}}{\sqrt{X_2^2 + Y_2^2}}, \text{ where}$$

$$Z_1 = X_1 + jY_1 \qquad Z_2 = X_2 + Y_2$$

* Phases:

$$\angle\frac{Z_1}{Z_2} = \angle Z_1 - \angle Z_2 = \operatorname{atan}\frac{Y_1}{X_1} - \operatorname{atan}\frac{Y_2}{X_2}$$

* Examples: