Magnitude Bode Plot for the Low Pass Filter

- Transfer function is

$$\frac{V_{out}}{V_{in}} = \frac{1}{1 + j\omega RC}$$

"The magnitude of the ratio is the ratio of the magnitudes:"

$$\left|\frac{V_{out}}{V_{in}}\right|_{dB} = 20\log\left|\frac{1}{1 + j\omega RC}\right| = 20\log\left(\frac{1}{\sqrt{1 + (\omega/\omega_o)^2}}\right)$$

- $\omega_o = 1 / RC$ is the "break frequency" or "-3 dB frequency"

  $\omega \ll \omega_o$ results in a magnitude of 20 log (1/1) = 0 dB

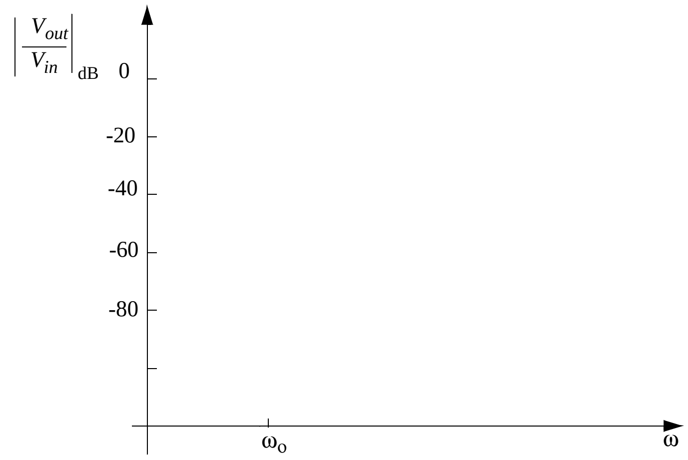
  $\omega \gg \omega_o$ results in a magnitude of

$$\left|\frac{V_{out}}{V_{in}}\right|_{dB} = 20\log\left(\frac{1}{\sqrt{1 + (\omega/\omega_o)^2}}\right) \cong 20\log\left(\frac{1}{(\omega/\omega_o)}\right) = -20\log\left(\frac{\omega}{\omega_o}\right)$$

- substitute $\omega = 10\ \omega_o$, $100\ \omega_o$, $1000\ \omega_o$

Approximate Magnitude Bode Plot

- Sketch asymtotes above and below the break frequency



At $\omega = \omega_o$, the exact magnitude is:

$$\left|\frac{V_{out}}{V_{in}}\right|_{dB} = 20\log\left|\frac{1}{1 + j(\omega_o/\omega_o)}\right| = 20\log\left[\frac{1}{\sqrt{1+1}}\right] = -3dB$$

Phase Bode Plot for Low Pass Filter

- From the definition of the phase,

$$\angle\frac{V_{out}}{V_{in}} = \angle\left(\frac{1}{1+j\omega RC}\right) = \angle 1 - \angle(1 + j(\omega/\omega_o)) = -\angle(1 + j(\omega/\omega_o))$$

- Substituting the arctangent,

$$\angle\frac{V_{out}}{V_{in}} = -\mathrm{atan}(\omega/\omega_o)$$

- Look at asymtotes, again:

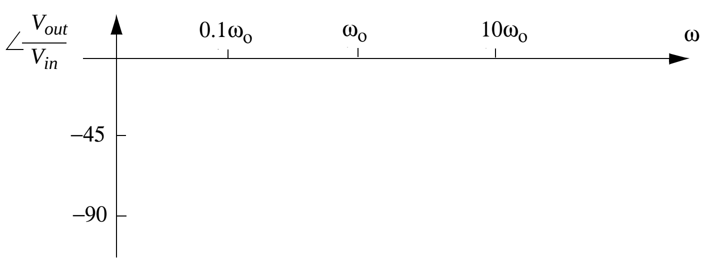
$\omega << \omega_o$ results in a phase of - atan(0) = 0

$\omega >> \omega_o$ results in a phase of - atan(infinity) = - 90^o

$\omega = (1/10)\omega_o$ results in a phase of - atan(0.1) = - 6^o

$\omega = (10)\omega_o$ results in a phase of - atan(10) = - 84^o

$\omega = \omega_o$ results in a phase of - atan(1) = - 45^o



Finding the Output Waveform from the Bode Plot

- Suppose that $v_{in}(t)$ = 100 mV cos ($\omega_o t + 0^o$)

note that the input signal frequency is equal to the break frequency and that the phase is 0^o ... the input signal phase is arbitrary and is generally selected to be 0.

the output phasor is:

$$V_{out} = V_{in}\left[\frac{1}{1+j(\omega_o/\omega_o)}\right] = V_{in}\left[\frac{1}{1+j}\right]$$

magnitude:

$$\left|\frac{V_{out}}{V_{in}}\right|_{dB} = -3\text{dB} \qquad \ldots \qquad |V_{out}| = \frac{|V_{in}|}{\sqrt{2}} = \frac{100\text{mV}}{\sqrt{2}} = 71\text{mV}$$

phase:

$$\angle\frac{V_{out}}{V_{in}} = \angle 1 - \angle(1+j) = 0 - 45° \qquad \angle V_{out} = -45°$$

$$V_{out} = (71\text{mV})e^{-j45°}$$

output waveform $v_{out}(t)$ is given by:

$$v_{out}(t) = Re\left(V_{out}e^{j\omega_o t}\right) = Re(71\text{mV}e^{-j45°}e^{j\omega_o t})$$

$$v_{out}(t) = 71\text{ mV}\cos(\omega_o t - 45^o)$$

Bode Plots of General Transfer Functions

- Procedure is to identify standard forms in the transfer functions, apply asymptotic techniques to sketch each form, and then combine the sketches graphically

$$H(j\omega) = \frac{Aj\omega(1 + j\omega\tau_2)(1 + j\omega\tau_4)...(1 + j\omega\tau_n)}{(1 + j\omega\tau_1)(1 + j\omega\tau_3)...(1 + j\omega\tau_{n-1})}$$

where the τ_i are time constants -- $(1/\tau_i)$ are the break frequencies, which are called *poles* when in the denominator and *zeroes* when in the numerator

- From complex algebra, the factors can be dealt with *separately* in the magnitude and in the phase and the results *added up* to find $|H(j\omega)|$ and phase $(H(j\omega))$

Three types of factors:

1. poles (binomial factors in the denominator)
2. zeroes (binomial factors in the numerator)
3. $j\omega$ in the numerator (or denominator)

(note: we aren't going to consider complex poles)

Rapid Sketching of Bode Plots

- Poles: - 3 dB and -45^{o} at break frequency

  0 dB below and -20 dB/decade above

  0^{o} for low frequencies and -90^{o} for high frequencies; width of transition is between 10 and (1/10) break frequency

- Zeros: +3 dB and +45^{o} at break frequency

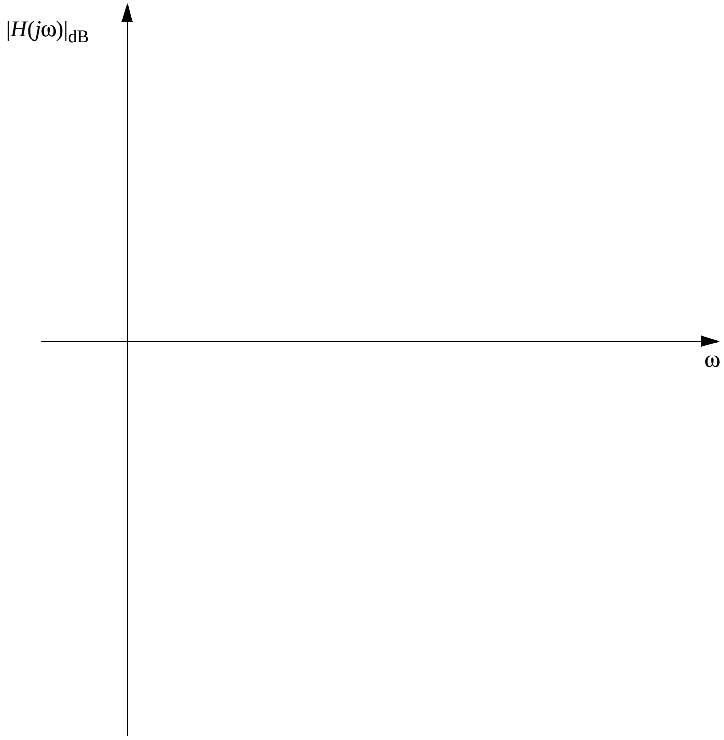
  0 dB below and + 20 dB/decade above

  0^{o} for low frequencies and +90^{o} for high frequencies; width of transition is between 10 and (1/10) break frequency

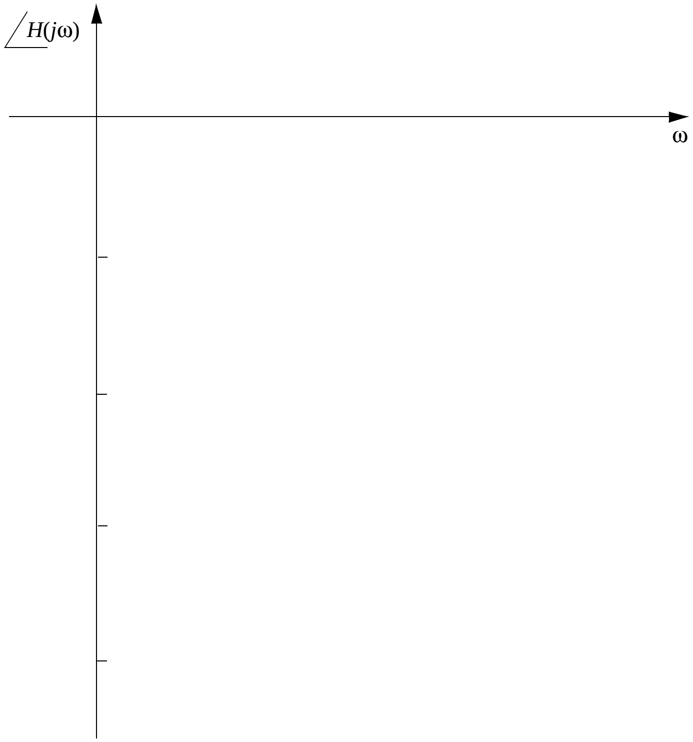
- $j\omega$: +20 dB/decade (0 dB at ω = 1 rad/s) and +90^{o} contribution to phase

Example I:

Magnitude and Phase Bode Plot Sketches



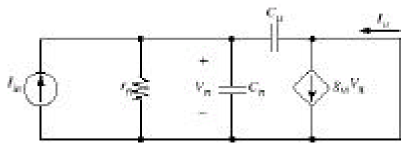
Phase Bode Plot Sketches



Device Models and Frequency Response

- "Classic" cases: give insight into the connection between device model and frequency response

  1. CE short-circuit current gain $A_i(j\omega)$ as a function of frequency



  Kirchhoff's current law at the output node:

$$I_o = g_m V_\pi - V_\pi j\omega C_\mu$$

  Kirchhoff's current law at the input node:

$$I_s = \frac{V_\pi}{Z_\pi} + V_\pi j\omega C_\mu \qquad \text{where} \qquad Z_\pi = r_\pi \| \left(\frac{1}{j\omega C_\pi}\right)$$

- Solving for V_π at the input node:

$$V_\pi = \frac{I_s}{(1/Z_\pi) + j\omega C_\mu}$$

Short-Circuit Gain Frequency Response

- Substituting V_π into the output node equation--

$$\frac{I_o}{I_s} = \frac{g_m Z_\pi \left(1 - \frac{j\omega C_\mu}{g_m}\right)}{1 + j\omega C_\mu Z_\pi}$$

- Substituting for Z_π and simplifying --

$$\frac{I_o}{I_s} = \frac{g_m r_\pi \left(1 - \frac{j\omega C_\mu}{g_m}\right)}{1 + j\omega r_\pi (C_\pi + C_\mu)} = \frac{\beta_o \left(1 - \frac{j\omega C_\mu}{g_m}\right)}{1 + j\omega r_\pi (C_\pi + C_\mu)} = \beta_o \left[\frac{1 - j\frac{\omega}{\omega_z}}{1 + j\frac{\omega}{\omega_p}}\right]$$

  Current gain has one pole:

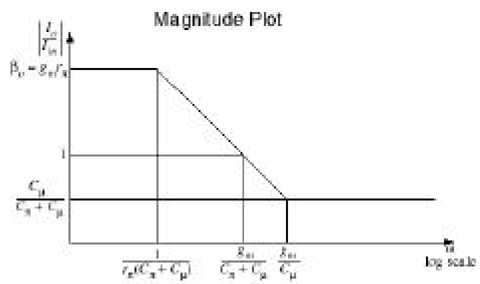
$$\omega_p = (r_\pi (C_\pi + C_\mu))^{-1}$$

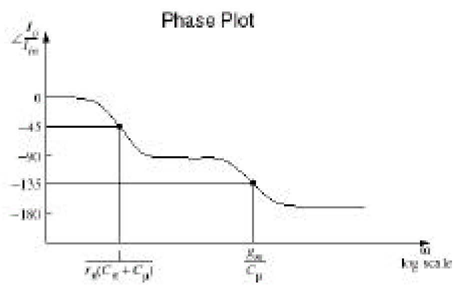
  and one zero

$$\omega_z = (g_m^{-1} C_\mu)^{-1} » \omega_p$$

Bode Plot of Short-Circuit Current Gain

- Note low frequency magnitude of gain is β_o





- Frequency at which current gain is reduced to 0 dB is defined as the **transition frequency** ω_T. Neglecting the zero,

$$\boxed{\omega_T = \frac{g_m}{(C_\pi + C_\mu)}}$$

Transition Frequency of the Bipolar Transistor

- Dependence of transition time $\tau_T = \omega_T^{-1}$ on the bias collector current I_C:

$$\tau_T = \frac{1}{\omega_T} = \frac{C_\pi + C_\mu}{g_m} = \frac{g_m \tau_F + C_{jE} + C_\mu}{g_m}$$

$$\tau_T = \tau_F + \left(\frac{C_{jE} + C_\mu}{g_m}\right) = \tau_F + \frac{V_{th}}{I_C}(C_{jE} + C_\mu)$$

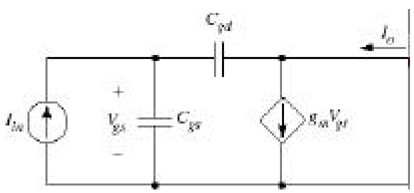
- If the collector current is increased enough to make the second term negligible, then the minimum τ_T is the base transit time, τ_F.. In practice, the ω_T decreases at very high values of I_C due to other effects and the minimum τ_T may not be achieved.

- Numerical values of $f_T = (1/2\pi)\omega_T$ range from 10 MHz for lateral pnp's to 75 GHz for submicron, oxide-isolated, SiGe heterojunction npn's

Note that the small-signal model is not valid above f_T (due to distributed effects in the base) and the zero in the current gain is not actually observed

Common-Source Current Gain

- CS amplifier has a non-infinite input *impedance* for $\omega > 0$ and we can measure its small-signal current gain.



- Analysis is similar to CE case; result is

$$\frac{I_o}{I_{in}} = \frac{g_m\left(1 - \frac{j\omega C_{gd}}{g_m}\right)}{j\omega(C_{gs} + C_{gd})} \approx \frac{g_m}{j\omega(C_{gs} + C_{gd})}$$

- Transition frequency for the MOSFET is

$$\boxed{\omega_T \approx \frac{g_m}{C_{gs} + C_{gd}}}$$

Transition Frequency of the MOSFET

- Substitution of gate-source capacitance and transconductance:

$$C_{gs} = \frac{2}{3}WLC_{ox} \gg C_{gd} \text{ and } g_m = \frac{W}{L}\mu_n C_{ox}(V_{GS} - V_{Tn})$$

$$\omega_T \approx \frac{g_m}{C_{gs}} = \frac{\frac{W}{L}\mu_n C_{ox}(V_{GS} - V_{Tn})}{\frac{2}{3}WLC_{ox}} = \frac{3}{2}\mu_n\left[\frac{(V_{GS} - V_{Tn})}{L}\right]\left(\frac{1}{L}\right)$$

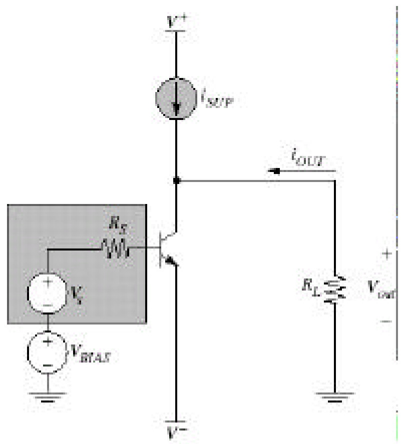
- The transition time is the inverse of ω_T and can be written as the average time for electrons to drift from source to drain

$$\tau_T = \frac{L}{\mu_n\left[\frac{3}{2}\frac{(V_{GS} - V_{Tn})}{L}\right]} = \frac{L}{|v_{dr}|}$$

- velocity saturation causes τ_T to decrease linearly with L; however, submicron MOSFETs have transition frequencies that are approaching those for oxide-isolated BJTs

Frequency Response of Voltage Amplifiers

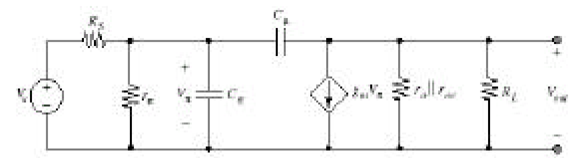
- Common-emitter amplifier:

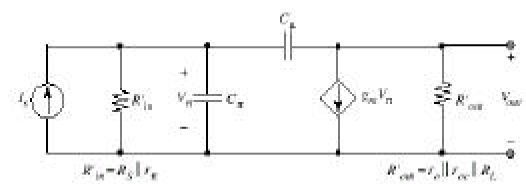


Procedure: substitute small-signal model at the operating point and perform phasor analysis

"Brute Force" Phasor Analysis

- "Exact" analysis: transform into Norton form at input to facilitate nodal analysis





Note that C_{cs} is omitted, along with r_b