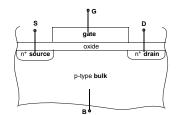
## The MOS Field Effect Transistor

Cross-section:



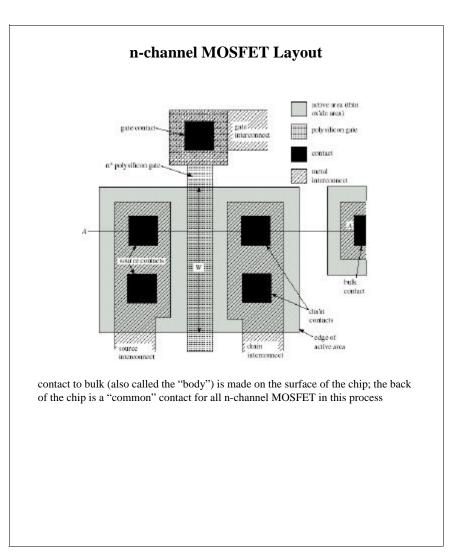
Basic idea: add n-type regions adjacent to the MOS capacitor so that current can flow between them only when the surface is inverted

Invented 1930s, demonstrated 1960s, mass produced digital ICs 1970s

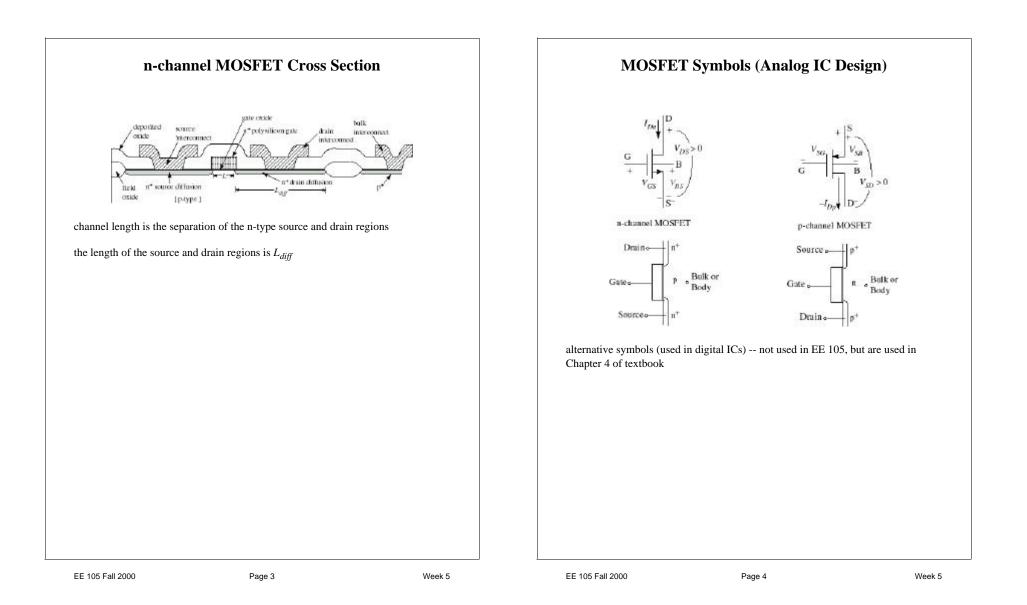
What's hard about making a MOSFET?

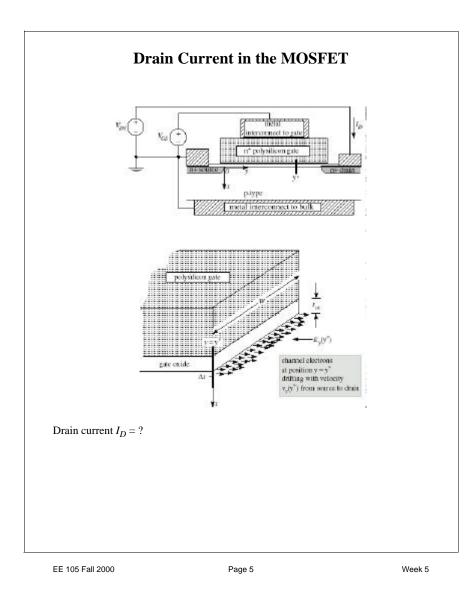
Oxide can contain a large amount of fixed and (worse) slightly mobile charge ... making the threshold voltage drift during use

Four terminals control the electrical properties of the MOSFET



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# **Drift Current Equation**

• Drift current for electrons in the channel:

$$J_y(x, y) = -qn(x, y)v_y(y)$$

The drain current at position y is the integral of the drift current density across the cross section. Since the conventional direction of  $I_D$  is *opposite* to the direction of the y axis, we insert a minus sign:

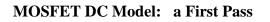
$$I_D = -W \int_0^{\Delta x} J_y(x, y) dx = W v_y(y) \left( \int_0^{\Delta x} qn(x, y) dx \right)$$

• The integral is the negative of the electron charge in the channel, per unit area, at point y. The symbol for this quantity is -  $Q_N(y)$ :

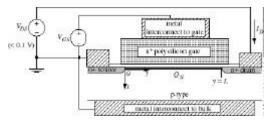
$$I_D = -Wv_y(y)Q_N(y)$$

Note that  $I_D$  isn't a function of the position in the channel

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• Start simple -- small  $V_{DS}$  makes the channel uniform; bulk and source are shorted together



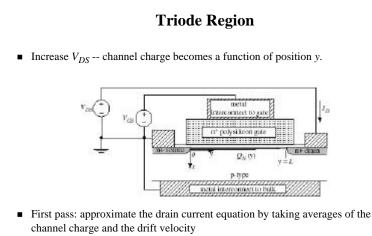
• Channel charge: MOS capacitor in inversion, with  $V_{GB} = V_{GS}$ .

$$Q_N = -C_{ox}(V_{GB} - V_{Tn}) = -C_{ox}(V_{GS} - V_{Tn})$$

- Drift velocity: electric field is just  $E_y = -V_{DS} / L$  so  $v_y = -\mu_n (-V_{DS} / L)$
- Drain current equation for  $V_{DS}$  "small" ... say, less than 0.1 V.

$$I_D = \mu_n C_{ox} \left(\frac{W}{L}\right) (V_{GS} - V_{Tn}) V_{DS}$$

Note that  $I_D$  is proportional to  $V_{DS}$  with channel resistance under gate control. This voltage controlled resistor region is sometimes useful.



 $I_D \approx -W\overline{Q_N}\overline{V_V}$ 

(Second pass: section 4.4 (not assigned))

• Average drift velocity: still use  $\mu_n (V_{DS} / L)$  -- which is a very rough approximation.

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## **Triode Region (Cont.)**

• Next, approximate the average channel charge by averaging  $Q_N(y=0)$  at the source end and  $Q_N(y=L)$  at the drain end of the channel:

$$Q_N(y=0) = -C_{ox}(V_{GS} - V_{Tn})$$

At the drain end, the positive drain voltage *reduces* the magnitude of the channel charge ... why? The effect can be approximated by using  $V_{GD}$  (the drop from drain to channel, at y = L) --

$$Q_N(y=L) = -C_{ox}(V_{GD} - V_{Tn}) = -C_{ox}(V_{GS} - V_{DS} - V_{Tn})$$

Note that  $V_{GD} = V_{GS} - V_{DS} > V_{Tn}$  in order for there to be a channel left at the drain end.

• Substituting, we derive the equation for the triode region, which is defined by  $V_{GS} - V_{DS} > V_{Tn}$  and  $V_{GS} > V_{Tn}$ .

$$I_D = \mu_n C_{ox} \left(\frac{W}{L}\right) (V_{GS} - V_{Tn} - V_{DS}/2) V_{DS}$$

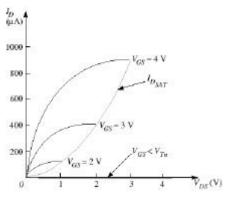
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## **Drain Characteristics**

• Example:  $\mu_n C_{ox} (W/L) = 50 \ \mu A/V^2$ ,  $V_{Tn} = 1$  V, and (W/L) = 4.



• What happens when  $V_{DS} > V_{GS} - V_{Tn} = V_{DS(SAT)}$ ?  $|Q_N(y = L)| = 0$ ! Initial thought is that the lack of a channel at the drain end means that  $I_D$  must drop to zero ... WRONG!

Drain terminal loses control over channel, so the drain current "saturates" and remains constant (to first approximation) at the value given by  $V_{DS} = V_{DS(SAT)}$ .

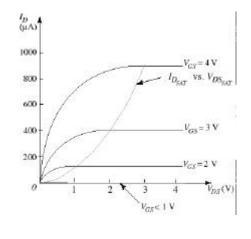
,

## **Saturation Region**

• When  $V_{GS} > V_{Tn}$  and  $V_{DS} > V_{DS(SAT)} = V_{GS} - V_{Tn}$ , the drain current is:

$$I_D = I_{D_{SAT}} = \mu_n C_{ox} \left(\frac{W}{2L}\right) (V_{GS} - V_{Tn})^2$$

n-channel MOSFET drain characteristics:



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## **MOSFET Circuit Models**

## • n-channel MOSFET drain current in cutoff, triode, and saturation:

$I_D = 0 \text{ A}$	$(V_{GS} \le V_{Tn})$
$I_D = \mu_n C_{ox} (W/L) [V_{GS} - V_{Tn} - (V_{DS}/2)] (1 + \lambda_n V_{DS}) V_{DS}$	$(V_{GS} \geq V_{Tn}, V_{DS} \leq V_{GS} - V_{Tn})$
$I_{D} = \mu_{n} C_{ox} (W/(2L)) (V_{GS} - V_{Tn})^{2} (1 + \lambda_{n} V_{DS})$	$(V_{GS} \ge V_{Tn}, V_{DS} \ge V_{GS} - V_{Tn})$

### Numerical values:

 $\mu_n$  is a function of  $V_{GS}$  along the channel and is much less than the mobility in the bulk (typical value 200 cm<sup>2</sup>/(Vs)) -- therefore, we consider that  $\mu_n C_{ox}$  is a measured parameter. Typical value:  $\mu_n C_{ox} = 50 \ \mu \text{AV}^{-2}$ 

 $\lambda_n$ , sometimes called the channel length modulation parameter, increases as the channel length *L* is reduced:

$$\lambda_n \approx \frac{0.1 \mu \text{mV}^{-1}}{L}$$

The triode region  $I_D$  equation has  $(1 + \lambda_n V_{DS})$  added in order to avoid a jump at the boundary with the saturation region. For hand calculation of DC voltages and currents, this term is usually omitted from  $I_D$ .

 $V_{Tn}$  = threshold voltage = 0.7 - 1.0 V typically for an n-channel MOSFET.

## **Backgate Effect**

- The MOSFET has four (G, S, D, and B) electrical terminals
- If  $V_{BS} \neq 0$ , then  $X_d$  increases, for example  $X_{DMAX}$ :

$$\therefore \sqrt{\frac{2\varepsilon(-2\phi_p)}{qNa}} \to \sqrt{\frac{2\varepsilon(-2\phi_p + V_{SB})}{qNa}}$$

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$$\underbrace{\underbrace{V_{FB}-2\phi_{p}+\frac{\sqrt{2\epsilon qNa(-2\phi_{p})}}{C_{ox}}}_{V_{TO}} \rightarrow$$

Define 
$$\gamma = \frac{\sqrt{2q\varepsilon_s Na}}{C_{ox}}$$

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**Backgate Effect** 

• The threshold voltage is a function of the bulk-to-source voltage  $V_{BS}$  through the backgate effect.

$$V_{Tn} = V_{TOn} + \gamma_n (\sqrt{-V_{BS} - 2\phi_p} - \sqrt{-2\phi_p})$$

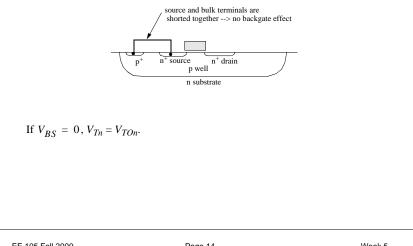
where  $V_{TO}$  is the threshold voltage with  $V_{BS} = 0$  and  $\gamma$  is the backgate effect parameter

$$\gamma_n = (\sqrt{2q\epsilon_s N_a})/C_{ox}$$

• Physical origin:  $V_{BS}$  (a negative voltage to avoid forward biasing the bulk-tosource pn junction) increases the depletion width, which increases the bulk charge and thus, the threshold voltage.

$$I_D = I_D(V_{GS}, V_{DS}, V_{BS})$$
 since  $V_{Tn} = V_{Tn}(V_{BS})$ 

Common situation is that  $V_{BS} = 0$  by electrically shorting the source to the bulk (either the substrate or a deep diffused region called a *well*)



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## **MOSFET Small-Signal Model**

- Concept: find an equivalent circuit which interrelates the incremental changes in i<sub>D</sub>, v<sub>GS</sub>, v<sub>DS</sub>, etc. Since the changes are small, the small-signal equivalent circuit has linear elements only (e.g., capacitors, resistors, controlled sources)
- Derivation: consider for example the relationship of the increment in drain current due to an increment in gate-source voltage when the MOSFET is saturated-- with *all other voltages held constant*.

$$v_{GS} = V_{GS} + v_{gs}$$
,  $i_D = I_D + i_d$  -- we want to find  $i_d = (?) v_{gs}$ 

We have the functional dependence of the total drain current in saturation:

$$i_D = \mu_n C_{ox} (W/2L) (v_{GS} - V_{Tn})^2 (1 + \lambda_n v_{DS}) = i_D (v_{GS}, v_{DS}, v_{BS})$$

Do a Taylor expansion around the DC operating point (also called the quiescent point or Q point) defined by the DC voltages  $Q(V_{GS}, V_{DS}, V_{BS})$ :

$$i_D = I_D + \frac{\partial i_D}{\partial v_{GS}} \bigg|_Q (v_{gs}) + \frac{1}{2} \frac{\partial^2 i_D}{\partial v_{GS}^2} \bigg|_Q (v_{gs})^2 + \dots$$

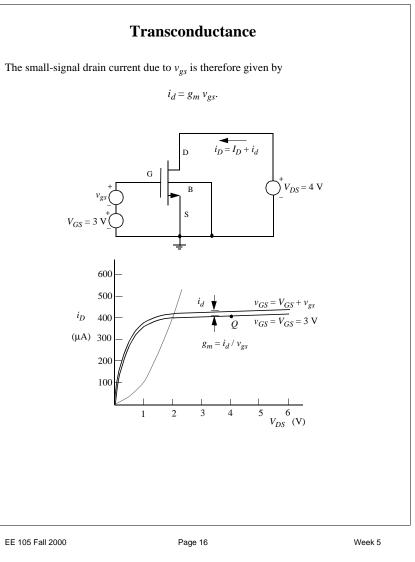
If the small-signal voltage is really "small," then we can neglect all everything past the linear term --

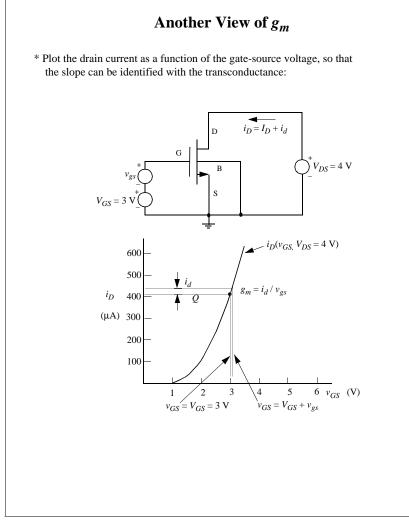
$$i_D = I_D + \frac{\partial i_D}{\partial v_{GS}} \left|_O (v_{gs}) = I_D + g_m v_{gs} \right|_O$$

where the partial derivative is defined as the *transconductance*,  $g_m$ . That is,

$$g_{m_{SAT}} \equiv \frac{\partial i_{DSAT}}{\partial v_{GS}}$$







## **Transconductance (Cont.)**

• Evaluating the partial derivative:

$$g_m = \mu_n C_{ox} \left(\frac{W}{L}\right) (V_{GS} - V_{Tn}) (1 + \lambda_n V_{DS})$$

Note that the transconductance is a function of the operating point, through its dependence on  $V_{GS}$  and  $V_{DS}$  -- and also the dependence of the threshold voltage on the backgate bias  $V_{BS}$ .

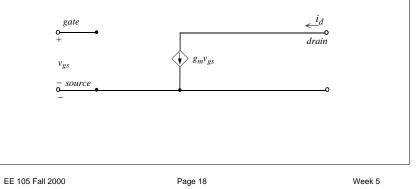
In order to find a simple expression that highlights the dependence of g<sub>m</sub> on the DC drain current, we neglect the (usually) small error in writing:

$$g_m = \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right) I_D} =$$

For typical values (*W/L*) = 10,  $I_D = 100 \,\mu\text{A}$ , and  $\mu_n C_{ox} = 50 \,\mu\text{AV}^{-2}$  we find that

$$g_m = 320 \,\mu \text{AV}^{-1} = 0.32 \,\text{mS}$$

• How do we make a circuit which expresses  $i_d = g_m v_{gs}$ ? We need a voltagecontrolled current source:



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# **Output Conductance**

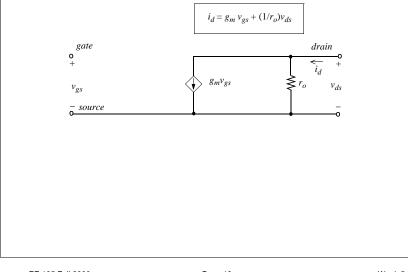
 We can also find the change in drain current due to an increment in the drainsource voltage:

$$g_o = \frac{\partial i_D}{\partial v_{DS}} \bigg|_O = \mu_n C_{ox} \left(\frac{W}{2L}\right) (V_{GS} - V_T)^2 \lambda_n \cong \lambda_n I_D$$

The output resistance is the inverse of the output conductance

$$r_o = \frac{1}{g_o} = \frac{1}{\lambda_n I_D}$$

The small-signal circuit model with  $r_o$  added looks like:



# **Backgate Transconductance**

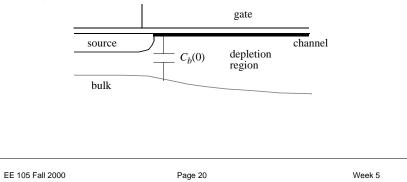
• We can find the small-signal drain current due to a change in the backgate bias by the same technique. The chain rule comes in handy to make use of our previous result for  $g_m$ :

$$\begin{split} g_{mb} &= \left. \frac{\partial i_D}{\partial v_{BS}} \right|_Q = \left. \frac{\partial i_D}{\partial V_T} \right|_Q \frac{\partial V_T}{\partial v_{BS}} \right|_Q \\ g_{mb} &= (-g_m) \frac{\partial V_T}{\partial v_{BS}} \right|_Q = (-g_m) \left( \frac{-\gamma_n}{2\sqrt{-2\phi_p - V_{BS}}} \right) = \frac{\gamma_n g_m}{2\sqrt{-2\phi_p - V_{BS}}} \end{split}$$

The ratio of the "front-gate" transconductance  $g_m$  to the backgate transconductance  $g_{mb}$  is:

$$\frac{g_{mb}}{g_m} = \frac{\sqrt{2q\varepsilon_s N_a}}{2C_{ox}\sqrt{-2\phi_p - V_{BS}}} = \frac{1}{C_{ox}}\sqrt{\frac{q\varepsilon_s N_a}{2(-2\phi_p - V_{BS})}} = \frac{C_b(y=0)}{C_{ox}}$$

where  $C_b(y=0)$  is the depletion capacitance at the source end of the channel --



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## **MOSFET** Capacitances in Saturation

- There are *lots* of capacitances built into the MOSFET
- Gate-source capacitance:

the "wedge" channel charge in saturation: less charge than for a MOS capacitor see text:  $C_{gs} = (2/3) C_{ox} WL +$  "overlap" capacitance

$$C_{ov} = C_{ox} \left( W\Delta \right)$$

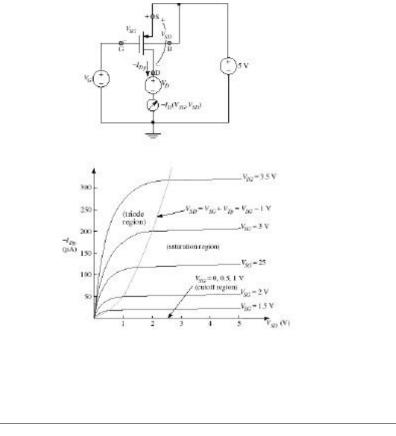
• Gate-drain capacitance:

 $C_{gd}$  = overlap capacitance

- Parasitic depletion capacitances: drain-to-bulk, source-to-bulk
- "Complete" model:

## p-channel MOSFET

Test circuit for measuring drain characteristics



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## p-channel MOSFET Models

• DC drain current in the three operating regions:  $-I_{Dp} > 0$ 

$$\begin{split} -I_{Dp} &= 0 \text{ A} & (V_{SG} \leq -V_T) \\ -I_{Dp} &= \mu_p C_{ox} (W/L) [V_{SG} + V_{Tp} - (V_{SD}/2)] (1 + \lambda_p V_{SD}) V_{SD} & (V_{SG} \geq -V_{Tp}, V_{SD} \leq V_{SG} + V_{Tp}) \\ -I_{Dp} &= \mu_p C_{ox} (W/(2L)) (V_{SG} + V_{Tp})^2 (1 + \lambda_p V_{SD}) & (V_{SG} \geq -V_{Tp}, V_{SD} \geq V_{SG} + V_{Tp}) \end{split}$$

• The threshold voltage with backgate effect is given by:

$$V_{Tp} = V_{TOp} - \gamma_p ((\sqrt{-V_{SB} + 2\phi_n}) - \sqrt{2\phi_n})$$

Numerical values:

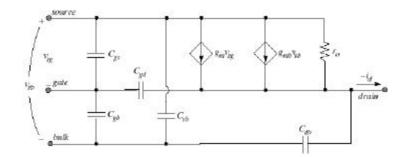
 $\mu_p C_{ox}$  is a measured parameter. Typical value:  $\mu_p C_{ox} = 25 \ \mu \text{AV}^{-2}$ 

$$\lambda_p \approx \frac{0.1 \mu \mathrm{mV}^{-1}}{L}$$

 $V_{Tp}$  = -0.7 to -1.0 V, which should be approximately  $-V_{Tn}$  for a well-controlled CMOS process

# PMOS Small-Signal Model

Same functional forms as for NMOS ... control voltages are  $v_{sg}$ ,  $v_{sd}$ ,  $v_{sb}$ 



Source is located at the top (corresponding to its position on the schematic as the higher potential compared to the drain); however, the source is often connected to a DC voltage --> a short for small signals.

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## **Circuit Simulation**

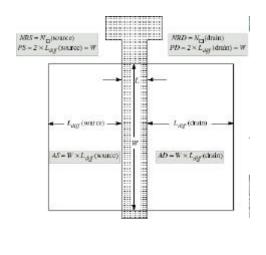
- Objectives:
  - fabricating an IC costs \$1000 ... \$100,000 per run
  - ---> nice to get it "right" the first time
  - check results from hand-analysis
  - (e.g., validity of assumptions)
  - evaluate functionality, speed, accuracy, ... of large circuit blocks or entire chips
- Simulators:
  - **SPICE**: invented at UC Berkeley circa 1970-1975
  - commercial versions: HSPICE, PSPICE, I-SPICE, ... (same core as Berkeley SPICE, but add functionality, improved user interface, ...)
  - EE 105: student version of PSPICE on PC, limited to about 15 transistors
  - other simulators for higher speed, special needs (e.g. SPLICE, RSIM)
- Limitations:
  - simulation results provide no insight (e.g. how to increase speed of circuit)
    results sometimes wrong (errors in input, effect not modeled in SPICE)
  - ===> always do hand-analysis first and COMPARE RESULTS

## **MOSFET Geometry in SPICE**

• Statement for MOSFET ... *D*,*G*,*S*,*B* are node numbers for drain, gate, source, and bulk terminals

Mname D G S B MODname L= W= AD= AS= PD= PS=

MODname specifies the model name for the MOSFET



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## **MOSFET Model Statement**

### .MODEL MODname {NMOS or PMOS} VTO=\_KP=\_GAMMA=\_PHI=\_ LAMBDA=\_RD=\_RS=\_RSH=\_CBD=\_CBS=\_CJ=\_MJ=\_CJSW=\_ MJSW=\_PB=\_IS=\_CGDO=\_CGSO=\_CGBO=\_TOX=\_LD=\_

Parameter name (SPICE / this text)	SPICE symbol	Analytical symbol	Units
channel length	$L_{eff}$	L	m
polysilicon gate length	L	Lgate	m
lateral diffusion/ gate-source overlap	LD	L <sub>D</sub>	m
transconductance parameter	KP	$\mu_n C_{ox}$	$A/V^2$
threshold voltage / zero-bias threshold	VTO	V <sub>TnO</sub>	v
channel-length modulation parameter	LAMBDA	$\lambda_n$	V-1
bulk threshold / backgate effect parameter	GAMMA	$\gamma_n$	V <sup>1/2</sup>
surface potential / depletion drop in inversion	РНІ	-	v

## NMOS SPICE DC Drain Current Equations:

$$\begin{split} I_{DS} &= 0 & (V_{GS} \leq -V_{TH}) \\ I_{DS} &= \frac{KP}{2} (W/L_{eff}) V_{DS} [2(V_{GS} - V_{TH}) - V_{DS}] (1 + LAMBDA \cdot V_{DS}) & (0 \leq V_{DS} \leq V_{GS} - V_{TH}) \\ I_{DS} &= \frac{KP}{2} (W/L_{eff}) (V_{GS} - V_{TH})^2 (1 + LAMBDA \cdot V_{DS}) & (0 \leq V_{GS} - V_{TH} \leq V_{DS}) \end{split}$$

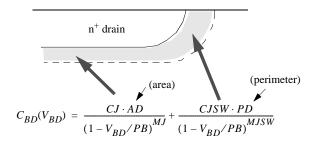
$$V_{TH} = V_{TO} + GAMMA(\sqrt{2 \cdot PHI - V_{BS}} - \sqrt{2 \cdot PHI})$$

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## Capacitances

SPICE includes the "sidewall" capacitance due to the perimeter of the source and drain junctions --



Gate-source and gate-bulk overlap capacitance are specified by *CGDO* and *CGSO* (units: F/m).

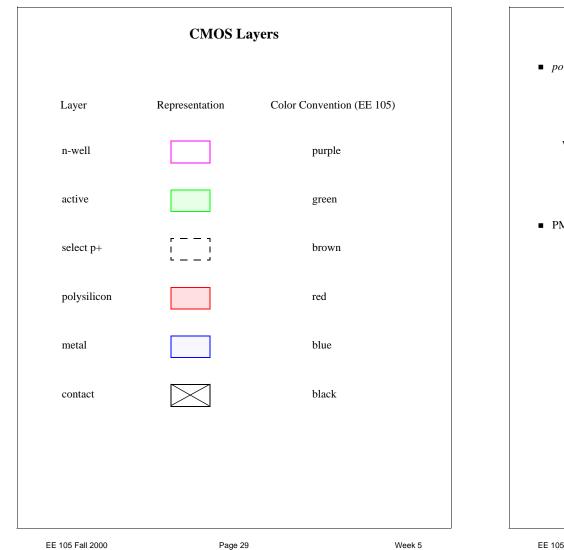
Level 1 MOSFET model (for 3 µm channel length NMOS device)

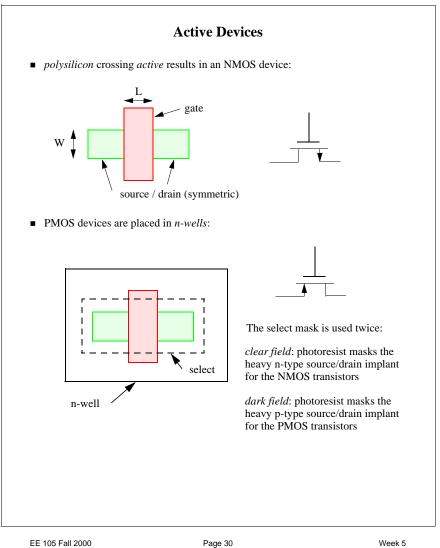
.MODEL MODN NMOS LEVEL=1 VTO=1 KP=50U LAMBDA=.033 GAMMA=.6 + PHI=0.8 TOX=1.5E-10 CGDO=5E-10 CGSO= 5e-10 CJ=1E-4 CJSW=5E-10 + MJ=0.5 PB=0.95

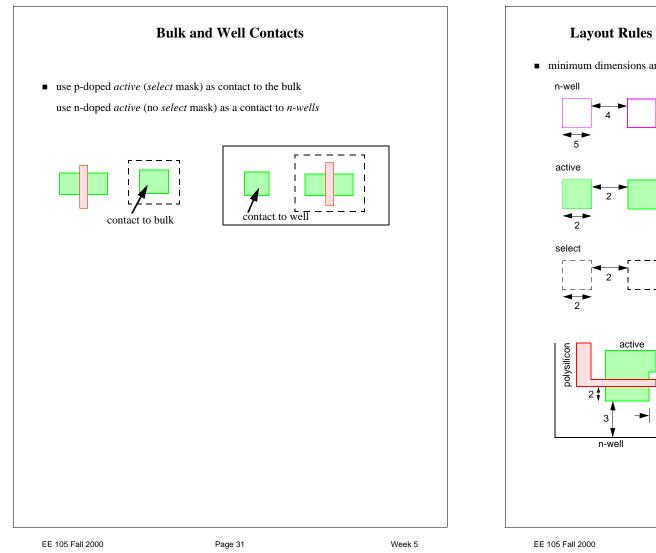
The Level 1 model is adequate for channel lengths longer than about 1.5  $\mu m$  -- see the MOSFET experiment for an example of its limitations

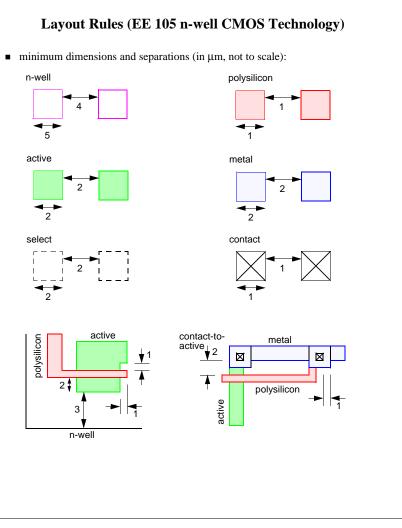
For sub- $\mu$ m MOSFETs, BSIM = "Berkeley Short-Channel IGFET Model" is the industry-standard SPICE model.

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