pn Junction Under Bias

- Reverse bias ($V_D < 0$ V adds to the barrier ϕ_B)
  - > Depletion region widens

(e.g., n^+ p junction: $X_d = \sqrt{\dfrac{2\varepsilon_s(\phi_B - V_D)}{qN_a}} > X_{do}$)

* Tiny negative I_D --

Why? The drift current is increased over diffusion current in the

depletion region in reverse bias due to the higher $|E(x)| > |E_o(x)|$ and the lower gradient in carrier concentration due to the wider depletion region $X_d > X_{do}$

* Hole and electron concentrations in the depletion regions are *lower* than in thermal equilibrium (so the depletion region is even more depleted!)

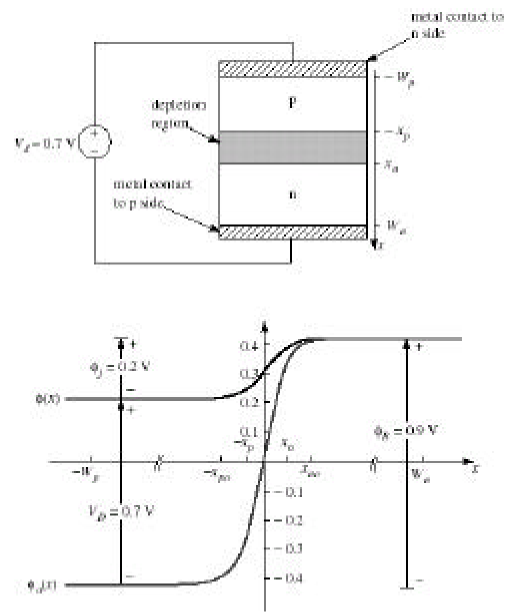
  - > $n, p \to 0$ in depletion region $n(x)p(x) \ll n_i^2$

- Forward bias ($V_D > 0$ V reduces ϕ_B)
  - > Depletion region narrows: $|E(x)| < |E_o(x)|$ and $X_d < X_{do}$ so diffusion current exceeds drift current ... get increasing I_D
  - > In forward bias, the product of the electron and hole concentrations in the depletion region $n(x)p(x) > n_i^2$

The pn Junction under Forward Bias

- $V_D > 0$ ... what happens?

neglect the Ohm's Law voltage drops across the p and n regions; the drop across the junction is reduced to $\phi_j = \phi_B - V_D$



ϕ_B = thermal equilibrium barrier height = $\phi_{no} - \phi_{po}$

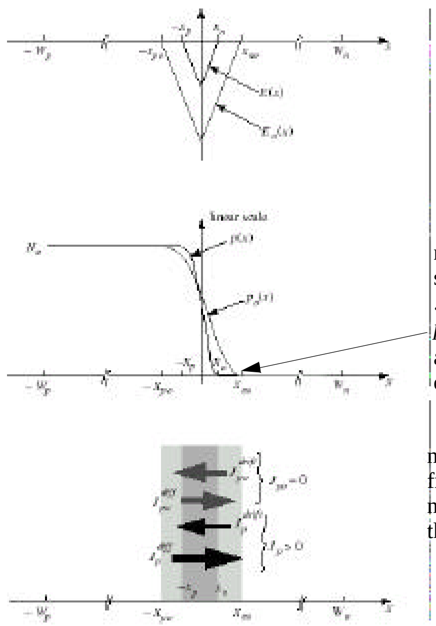
Physical Reasoning

- thermal equilibrium --> balance between drift and diffusion:

  $J = J^{drift} + J^{diff} = 0$ for holes and electrons separately

- forward bias upsets balance

  Both the change in $E(x)$ and in $\frac{dn}{dx}$ work together (latter is most important)



note that the scale is linear ... the value of $p(x_n) >> p_o(x_{no})$ as we will shortly discover

net hole transport from p region to n region across the depletion region

Modeling Forward-Bias Diode Currents

Goal: we want to find $I_D = f(V_D)$ and its dependence on the device parameters.

basic observation: current is carried both by diffusion of *injected* minority carriers away from the depletion region and across the undepleted n and p regions and drift of majority carriers from the ohmic contacts to supply this injection

Complicated!

- **Step 1**: find how minority carrier concentrations at the edges of depletion region change with forward bias V_D

- **Step 2**: what happens to the minority carrier concentration at the ohmic contacts under forward bias? *Answer*: no change from equilibrium.

- **Step 3**: find the minority carrier concentrations $n_p(x)$ in the p region and $p_n(x)$ in the n region.

- **Step 4**: find the minority carrier diffusion currents.

- **Step 5**: conclude what the majority carrier currents must be and find the total current density J.

Step 1: Carrier Concentrations in Thermal Equilibrium at the Junction Edges

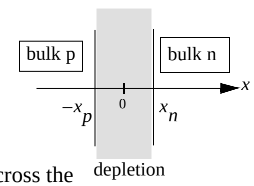
- What happens under an applied forward bias?

  the voltage drop still appears almost completely across the depletion region, but it is reduced to $\phi_B - V_D$

  Boltzmann still governs distribution of electrons over energy (that is, exponentially less likely at higher energy, i.e., proportional to $e^{(-\Delta\phi)/V_{th}}$)

  moreover, the Boltzmann rule "*reaches across*" the narrow depletion region

$$n(x=-x_p) = n(x=x_n)e^{\overbrace{\left(\frac{\phi(-x_p)-\phi(x_n)}{V_{th}}\right)}^{-\Delta\phi}}$$



but $\phi(-x_p) - \phi(x_n)$ is the negative of the drop across the depletion region (known)

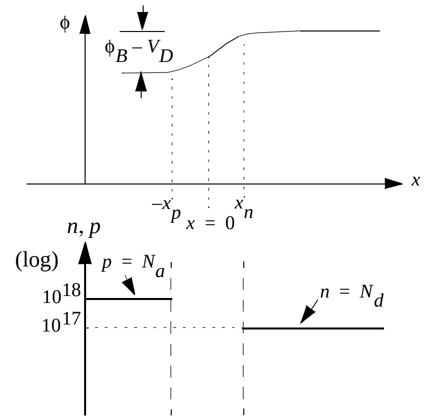
$$n(x=-x_p) = n(x=x_n)e^{\left(\frac{-(\phi_B - V_D)}{V_{th}}\right)}$$

- What is $n(x = x_n)$?

  we start by considering a "small" forward bias, which means that the electron concentration on the n-side of the depletion region is still the donor concentration --

$$n(x = x_n) = N_d$$

Law of the Junction



Suppose:

$$\left.\begin{aligned} N_A &= 10^{18} \\ N_D &= 10^{17} \end{aligned}\right\} \phi_B = 0.9\text{ V}$$

$\phi_B - V_D = 0.18$ V; i.e.,

$V_D = 0.72$ V

The barrier voltage is a function of the doping concentrations:

$$\phi_B = \phi_n - \phi_p = V_{th}\left(\ln\left(\frac{N_d}{n_i}\right) - \left[-\ln\left(\frac{N_a}{n_i}\right)\right]\right) = V_{th}\ln\left(\frac{N_d N_a}{n_i^2}\right)$$

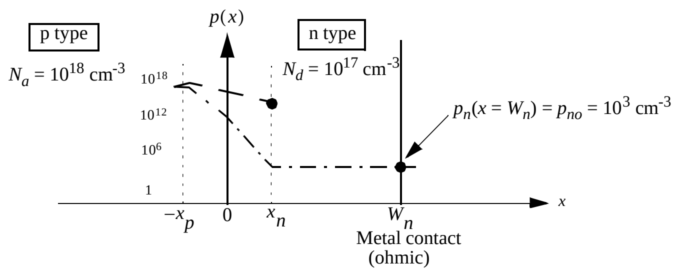
$$n_n(x = x_n)e^{-\phi_B/V_{th}} = N_d e^{-\ln\left(\frac{N_d N_a}{n_i^2}\right)} = \frac{N_d n_i^2}{N_d N_a} = \frac{n_i^2}{N_a} = n_{po}$$

$$\boxed{\begin{aligned} n_p(-x_p) &= \left(N_d e^{-\phi_B/V_{th}}\right)e^{V_D/V_{th}} = n_{po}e^{V_D/V_{th}} \\ p_n(x_n) &= \left(N_a e^{-\phi_B/V_{th}}\right)e^{V_D/V_{th}} = p_{no}e^{V_D/V_{th}} \end{aligned}}$$

The Law of the Junction

Step 3: Find the Minority Carrier Concentrations

- Current flows because we raise the minority carrier concentration (through “Law of the Junction”) at depletion layer boundaries. These “excess carriers” diffuse across the bulk.

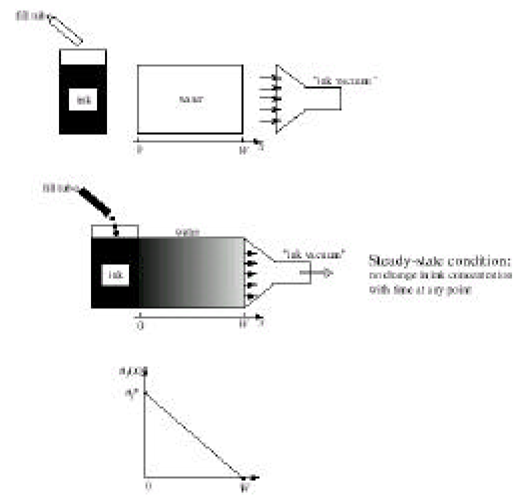


- Boundary conditions on minority hole concentration in n-type bulk region are found from the law of the junction and the fact that an ohmic contact maintains the carrier concentrations at their thermal equilibrium values

Diffusion Analogy

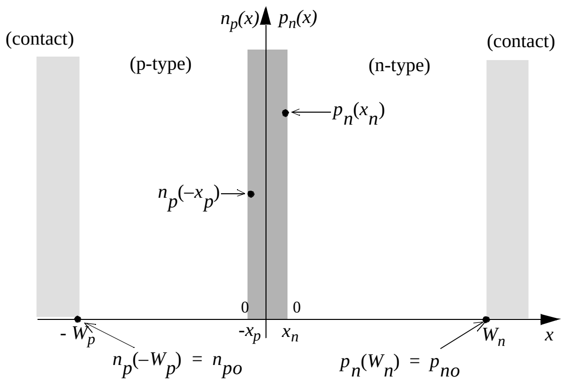
Ink concentration in steady state is similar to minority holes diffusing across the n-type bulk region

Note that none of the ink molecules are lost along the way ...



Step 3: Carrier Concentrations under Forward Bias

- Apply the Law of the Junction at the edges of the depletion region



- Numerical values: $N_a = 10^{18}$ cm^{-3}, $N_d = 10^{17}$ cm^{-3}

  $V_D = 0.72$ V = 720 mV, $V_{th} = 26$ mV (warm room) ... example values; note that V_D must be known *precisely* to substitute into exp[V_D/V_{th}].

$$n_p(-x_p) = 10^2 \text{ cm}^{-3} \exp[720/26] = 10^{14} \text{ cm}^{-3}$$

$$p_n(x_n) = 10^3 \text{ cm}^{-3} \exp[720/26] = 10^{15} \text{ cm}^{-3}$$

- The minority carrier concentration is maintained at thermal equilibrium at the ohmic contacts

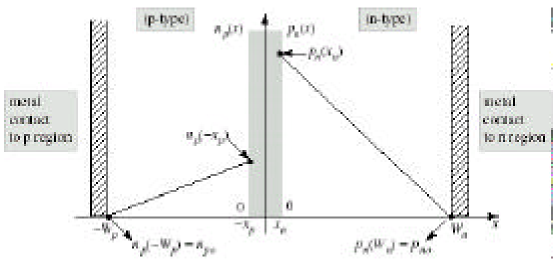
The Short-Base Solution

- Carrier concentrations: linear solutions if we assume that the p-type and n-type regions are so short that all of the diffusing minority carriers “make it” across to the ohmic contacts

  In n-type region: $J_p^{diff} = -qD_p dp_n / dx$ = constant --> $p_n(x)$ is *linear*

  In p-type region: $J_n^{diff} = qD_n dn_p / dx$ = constant --> $n_p(x)$ is *linear*

- If negligible hole loss in transit (due to recombination), then



$$\frac{dp_n}{dx} \cong \frac{p_n(x_n) - p_n(W_n)}{W_n - x_n}$$

Current Densities

- Minority carrier diffusion currents

$$J_n^{diff} = qD_n \frac{dn_p}{dx} = qD_n \frac{n_p(-x_p) - n_p(-W_p)}{W_p - x_p} = .$$

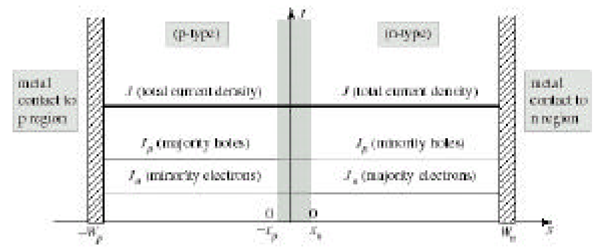
$$J_p^{diff} = -qD_p \frac{dp_n}{dx} = -qD_p \frac{p_n(W_n) - p_n(x_n)}{W_n - x_n} = .$$

Total Current Density

- The total current density is the sum of the minority electron and hole diffusion current densities at the junction ... and is constant through the diode

$$J = J_n^{diff} + J_p^{diff}$$

$$J = q\left(\frac{D_n n_{po}}{W_p - x_p} + \frac{D_p p_{no}}{W_n - x_n}\right)(e^{V_D/V_{th}} - 1)$$



- Diode current: multiply by area A and (for simplicity) assume that x_n, x_p << W_n, W_p
-

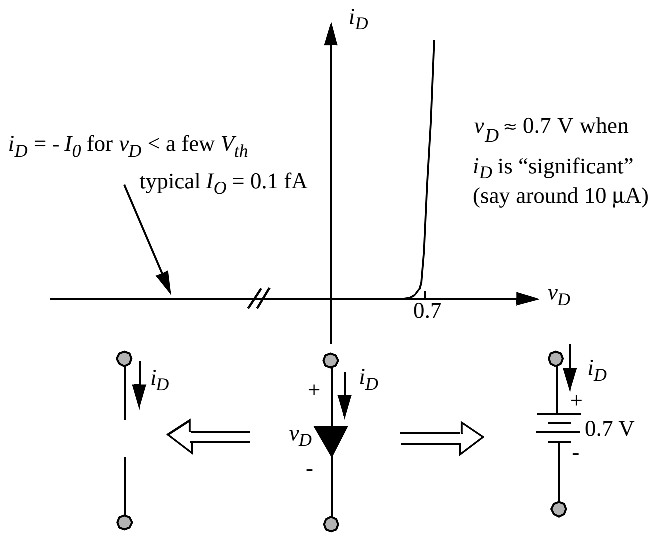
$$\boxed{I_D \cong q n_i^2\left(\frac{D_n}{N_a W_p} + \frac{D_p}{N_d W_n}\right)(e^{V_D/V_{th}} - 1) = I_o(e^{V_D/V_{th}} - 1)}$$

Circuit Models for the Junction Diode

Large-signal model:

plug into diode equation the symbols for total voltage and current (assumption is that time variation is “slow” since we derived the equation for DC)

$$i_D = I_O(e^{v_D/V_{th}} - 1)$$



Small-Signal Model: Diode Resistance r_d

Substitute (DC + small-signal) for current and voltage

$$i_D = I_D + i_d = I_O(e^{(V_D + v_d)/V_{th}} - 1)$$

Goal: find relationship between $v_D = V_D + v_d$ and $i_D = I_D + i_d$ for forward-bias

$$I_D + i_d \cong I_O e^{(V_D + v_d)/V_{th}} = I_O e^{V_D/V_{th}} e^{v_d/V_{th}} \cong I_D e^{v_d/V_{th}}$$

Taylor’s expansion for the exponential

$$I_D + i_d \cong I_D e^{v_d/V_{th}} = I_D\left(1 + \left(\frac{v_d}{V_{th}}\right) + \frac{1}{2}\left(\frac{v_d}{V_{th}}\right) + \ldots\right)$$

keeping only the linear term (meaning the small-signal voltage $v_d \ll V_{th}$)

$$I_D + i_d = I_D + I_D\left(\frac{v_d}{V_{th}}\right)$$

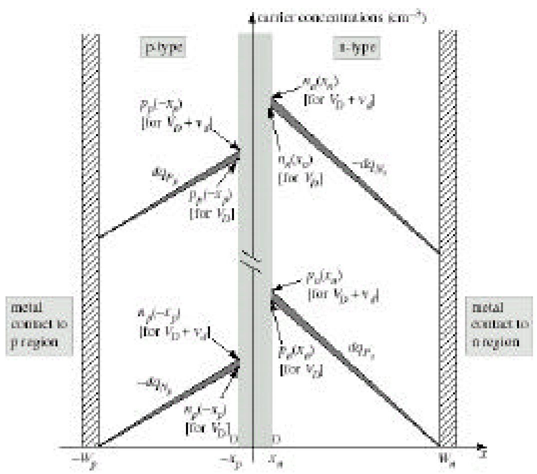
$$\boxed{i_d = \left(\frac{I_D}{V_{th}}\right)v_d = \left(\frac{1}{r_o}\right)v_d}$$

Small-Signal Diode Capacitances

Depletion region width varies with the diode voltage, but the reverse bias model is not quite right with so many electrons and holes in the depletion region.

$$C_j \approx \sqrt{2} C_{jo} \text{ (approximation)}$$

Minority carrier concentrations change with changing diode voltage, which means that there is a charge under control of the diode voltage ... *another capacitor*



Diffusion Capacitance C_d

See section 6.4 for derivation of C_d

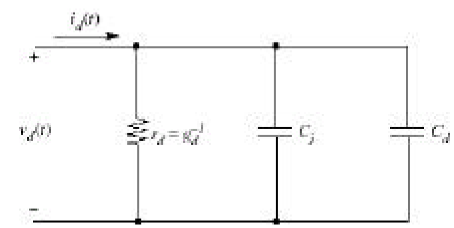
For a one-sided (asymmetric) diode with $N_d >> N_a$ most of the injected charge storage is on the p-side

$$C_d \cong \left(\frac{I_D}{V_{th}}\right)\left(\frac{W_p^2}{2D_n}\right) = g_d\left(\frac{W_p^2}{2D_n}\right)$$

The units of the second term are [seconds] and it is called the *transit time* τ_T

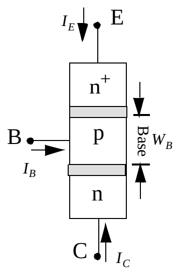
$$\boxed{C_d = g_o \tau_T}$$

Small-signal model:



The Bipolar Junction Transistor Concept

Reverse-biased junction ("collector") close to forward-biased junction ("emitter")



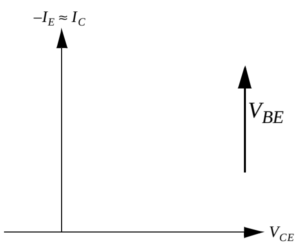
$V_{BE} \sim 0.6$ to 0.7 (large injection of electrons into base)

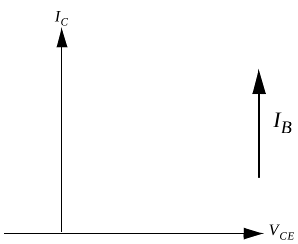
narrow base width (e.g., $< 0.2\mu m$)

$V_{BC} \leq 0$ (injected electrons collected)

$I_C \approx |I_E| \gg I_B$

I_B is also an exponential function of the base-emitter voltage





Current Relationships in BJT's

A. **Forward-active Mode** (NPN V_{BE} , V_{BC})

Since most of the emitter current is electrons injected into base and most are collected

$I_C \equiv -\alpha_F I_E$ where α_F is less than, but close to 1

KCL: $I_C + I_B + I_E = 0$

so $$I_B = -I_C - I_E = -I_C + I_C/\alpha_F = I_C \frac{(1-\alpha_F)}{\alpha_F}$$

$$\beta_F \equiv \frac{I_C}{I_B} = \text{"Current Gain"} = \frac{\alpha_F}{1-\alpha_F} \quad \text{typically} \sim 100$$

B. **Cutoff** (NPN V_{BE} , V_{BC})

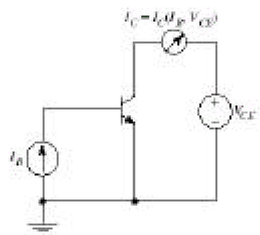
C. **Saturation** (NBN V_{BE} , V_{BC})

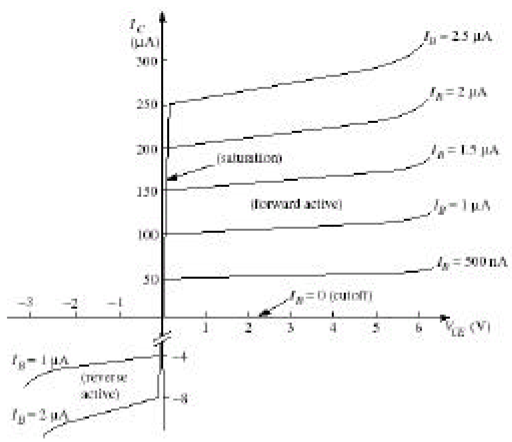
E-B Junction

B-C Junction

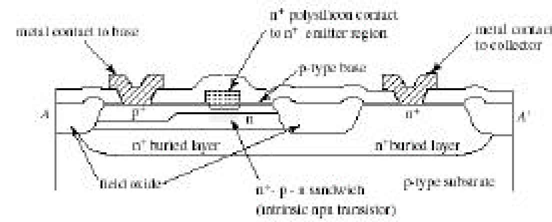
Is $I_C = \beta_F I_B$?

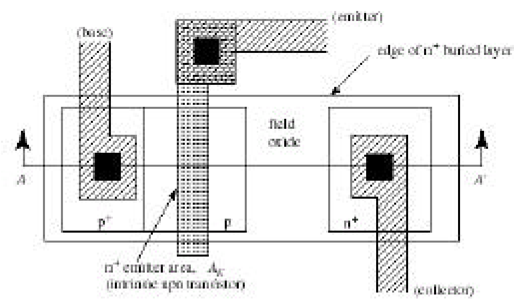
npn Collector Characteristics





Integrated Bipolar Transistor



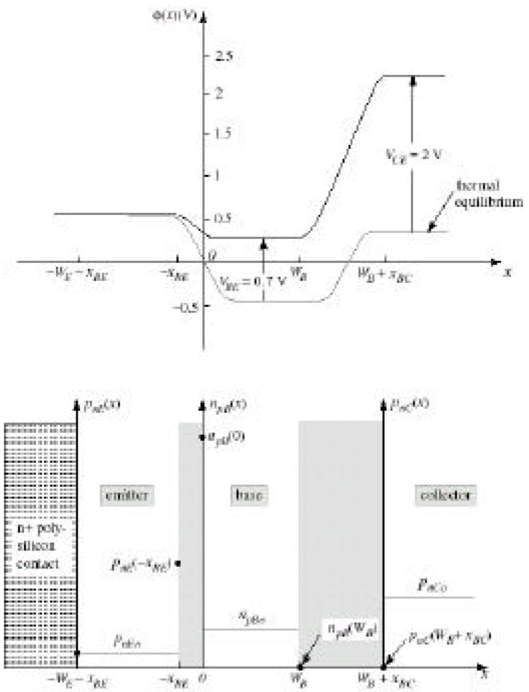


Note the depletion layers between:

1. E-B (small area)
2. B-C (bigger area)
3. C-S (substrate) (biggest area)

Forward-Active Region

Use the Law of the Junction at both the E-B and the B-C junctions



The transistor is designed with a *very* short base ...

Forward-Active Region (cont.)

Diffusion currents:

$$I_{E_n} = -A_E \frac{qD_n n_{pBo}}{W_B} e^{V_{BE}/V_{th}}$$ (electrons injected from emitter into base)

$$I_{E_p} = -A_E \frac{qD_p p_{nEo}}{W_E} e^{V_{BE}/V_{th}}$$ (holes injected from base into emitter)

Collector current I_C =

Emitter current $I_E = I_{En} + I_{Ep}$

The ratio of collector current to the magnitude of the emitter current is defined as "alpha-F"

$$\frac{I_C}{-I_E} = \frac{\left(\frac{qD_n n_{pBo} A_E}{W_B}\right)}{\left(\frac{qD_p p_{nEo} A_E}{W_E}\right) + \left(\frac{qD_n n_{pBo} A_E}{W_B}\right)} = \alpha_F$$

Forward-Active Current Gain

Base current $I_B = -I_C - I_E = -I_E - I_C$

Substitute for the emitter current:

$$I_C = -I_E\alpha_F \text{ which implies that } -I_E = \frac{I_C}{\alpha_F}$$

$$I_B = \left(\frac{1}{\alpha_F} - 1\right)I_C = \left(\frac{1-\alpha_F}{\alpha_F}\right)I_C$$

$$I_B = \left(\frac{1}{\beta_F}\right)I_C$$

The forward-active current gain is

$$\boxed{I_C = \beta_F I_B}$$

Don't forget that this result is ONLY true for the case where:

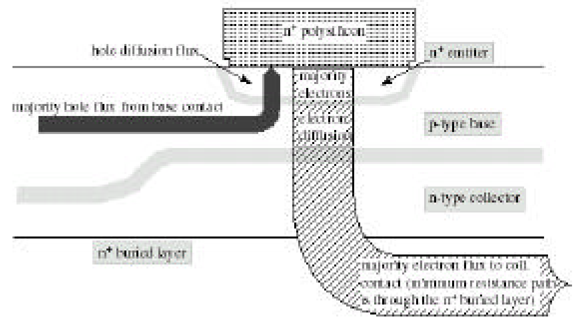
B-E junction *is* forward-biased

B-C junction *is not* forward-biased

Carrier Fluxes in Forward Active Bias

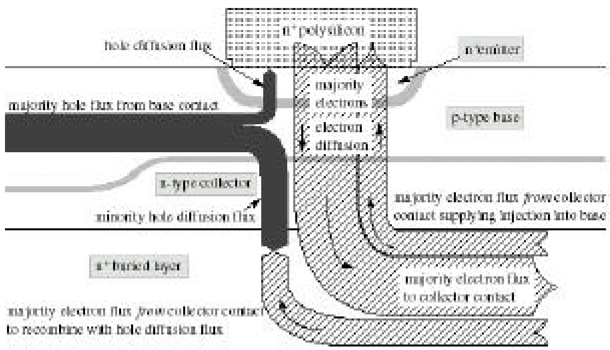
After collection by the electric field in the base-collector junction, the electrons are majority carriers in the n-type collector region

The heavily doped buried layer minimized the resistance between the collector metal interconnect and the base-collector junction



The Saturation Region

- $V_{CE(sat)}$ = 0.1 V (approx) from the characteristics --> both the emitter-base and the base-collector junctions are forward-biased
- Law of the Junction --> find minority carrier concentrations in the emitter, base, and the collector



- Both junctions are injecting and both are also collecting ... since the electric field in the depletion region remains in the same direction under forward bias.
- Separate the electron diffusion current in the base into two components: one due to the emitter-base junction (with zero bias on the base-collector junction) and the other due to the base-collector junction:

$$J_{nB}^{diff} = -qD_n \frac{n_{pBo}(e^{V_{BE}/V_{th}} - 1)}{W_B} + qD_n \frac{n_{pBo}(e^{V_{BC}/V_{th}} - 1)}{W_B}$$

Ebers-Moll Model

- Electron diffusion current in the base: multiply by the emitter area

$$I_{diff} = \boxed{-I_S(e^{V_{BE}/V_{th}} - 1)} + \boxed{I_S(e^{V_{BC}/V_{th}} - 1)} = -I_1 + I_2$$

- Emitter current I_E: three components

4. - I_1 due to injection of electrons from the emitter-base junction,
5. - I_1 / β_F due to reverse injection of holes into the emitter, and
6. I_2 due to collection of electrons from the base-collector junction.

$$I_E = -I_1 + (-I_1/\beta_F) + I_2 = -\left(1 + \frac{1}{\beta_F}\right)I_1 + I_2 = -\left(\frac{1}{\alpha_F}\right)I_1 + I_2$$

- Collector current I_C: three components (by symmetry)

1. - I_2 due to injection of electrons from the base-collector junction,
2. - I_2 / β_R due to reverse injection of holes into the collector, and
3. I_1 due to collection of electrons from the emitter-base junction

$$I_C = I_1 - I_2 - \frac{I_2}{\beta_R} = I_1 - \left(1 + \frac{1}{\beta_R}\right)I_2 = I_1 - \left(\frac{1}{\alpha_R}\right)I_2$$

$\beta_R = \alpha_R$ / (1 - α_R) is the reverse current gain

Ebers-Moll Model (cont.)

- "Standard form" for Ebers-Moll equations: define two new constants

$$I_{ES} = I_S / \alpha_F \text{ and } I_{CS} = I_S / \alpha_R,$$

- Emitter current:

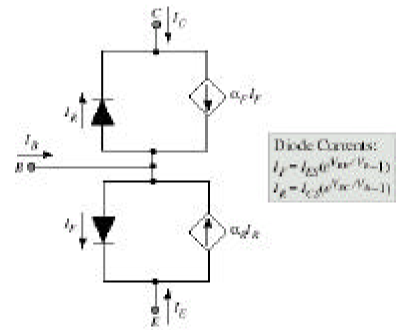
$$\boxed{I_E = -I_{ES}(e^{V_{BE}/V_{th}} - 1) + \alpha_R I_{CS}(e^{V_{BC}/V_{th}} - 1)}$$

- Collector current:

$$\boxed{I_C = \alpha_F I_{ES}(e^{V_{BE}/V_{th}} - 1) - I_{CS}(e^{V_{BC}/V_{th}} - 1)}$$

- The collector current and the emitter current represent two diodes with current-controlled current sources coupling the emitter and the collector branches

Ebers-Moll Equivalent Circuit



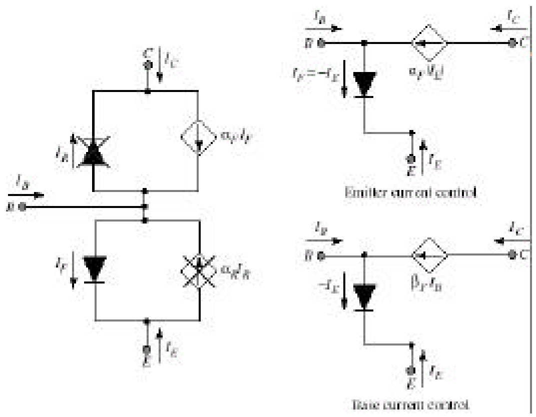
This model for the BJT applies to general device structures, with the four parameters I_{ES}, I_{CS}, α_F, and α_R being linked by "reciprocity"

$$\alpha_F I_{ES} = \alpha_R I_{CS} = I_S$$

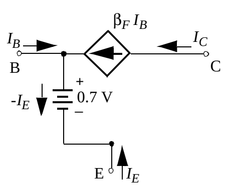
- Ebers-Moll *must* be simplified for hand calculation of DC bias currents

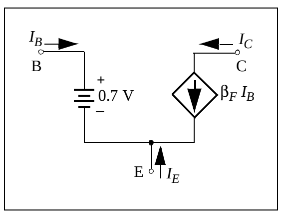
Forward Active Model

- I_R is negligible --> can neglect current through B-C diode



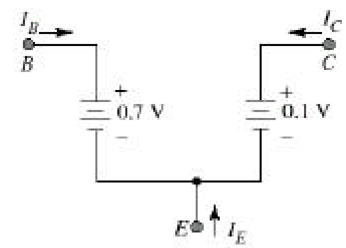
- Eliminate forward-biased diode by replacing with a 0.7 V battery:





Saturation

- Include both diodes in the circuit ... both as batteries



note that the batteries make the controlled current sources irrelevent to the circuit.

How to figure out the DC currents in a BJT?

1. Assume that it's forward-active (that's the goal for nearly all BJTs anyway)

2. If the base current is determined by other circuit elements, then find the collector current from $I_C = \beta_F I_B$.

3. If the base-emitter voltage is set by other circuit elements, then find the collector current from $I_C = I_S \exp[V_{BE} / V_{th}]$

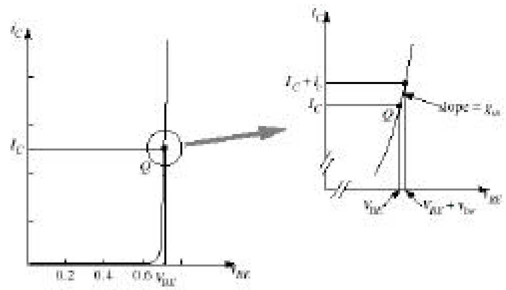
4. Verify that the BJT is saturated by finding the resulting V_{CE}.

Small-Signal Model of the Forward-Active npn BJT

- Transconductance (same concept as for MOSFET):

$$g_m = \left.\frac{\partial i_C}{\partial v_{BE}}\right|_Q$$

Ebers-Moll (forward-active): $\quad i_C = I_S e^{v_{BE}/V_{th}}$



- Evaluating the derivative, we find that

$$g_m = \left(\frac{I_S}{V_{th}}\right) e^{V_{BE}/V_{th}} = \frac{I_C}{V_{th}}$$

Input Resistance

- The collector current is a function of the base current in the forward-active region (recall $I_C = \beta_F I_B$). At the operating point Q, we define

$$\beta_o = \left.\frac{\partial i_C}{\partial i_B}\right|_Q$$

and so $i_c = \beta_o\, i_b$. (Note that the "DC beta" β_F and the small-signal β_o are both highly variable from device to device)

- Since the base current is therefore a function of the base-emitter voltage, we define the input resistance r_π as:

$$r_\pi^{-1} = \left.\frac{\partial i_B}{\partial v_{BE}}\right|_Q = \left.\frac{\partial i_B}{\partial i_C}\right|_Q \left.\frac{\partial i_C}{\partial v_{BE}}\right|_Q = \left(\frac{1}{\beta_o}\right) g_m$$

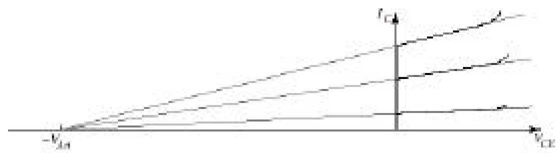
- Solving for the input resistance

$$\boxed{r_\pi = \frac{\beta_o}{g_m} = \frac{\beta_o V_{th}}{I_C} = \frac{kT\beta_o}{qI_C}}$$

- For a high input resistance (often desirable), we need a high current gain or a low DC bias current.

Output Resistance

- The Ebers-Moll model has perfect current source behavior in the forward-active region -- actual characteristics show some increase:



- Why? Base width shrinks due to encroachment by base-collector depletion region

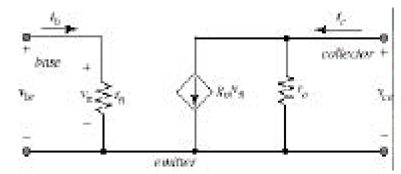
  Approximate model: introduce Early voltage V_{An} to model increase in i_C

  Model: $$i_C = I_S e^{v_{BE}/V_{th}}\left(1 + \frac{v_{CE}}{V_{An}}\right)$$

- Output resistance:

$$r_o^{-1} = \left.\frac{\partial i_C}{\partial v_{CE}}\right|_Q \cong \frac{I_C}{V_{An}}$$

Numerical Values of Small-Signal Elements



- Transconductance:

I_C = 100 µA, V_{th} = 25 mV --> $\boxed{g_m = 4 \text{ mS} = 4 \times 10^{-3} \text{ S}}$

Note: g_m varies *linearly* with collector current and is independent of device geometry, in contrast to the MOSFET

- Input resistance:

β_o = 100, I_C = 100 µA, V_{th} = 25 mV --> $\boxed{r_\pi = 25 \text{ k}\Omega}$

- Output resistance:

I_C = 100 µA, V_{An} = 35 V --> $\boxed{r_o = 350 \text{ k}\Omega}$

V_{An} = Early voltage increases with increasing base width and decreases with decreasing base doping.