(a) The small-signal equivalent circuit is

\[ i_o = \frac{v_o}{r_{in}} + \frac{v_i}{r_{in}} \]

From KCL at the output,
\[ i_o = i_i \Rightarrow v_o = v_i \left( \frac{1}{r_{in}} \right) = v_i \frac{1}{r_{in}} \]  

From KCL at the node X,
\[ v_x + \frac{v_x}{r_{in}} + \frac{v_o}{r_{o1}} + \frac{v_i}{r_{o2}} = 0 \]
\[ v_x = -\frac{v_i}{r_{o1}} \left( \frac{r_{o2}}{r_{o1} + r_{o2} + r_{o3}} \right) = -\frac{v_i}{r_{o1}} \left( \frac{r_{o2}}{r_{o1} + r_{o2}} \right) \]
\[ v_x = -\frac{v_i}{r_{o1}} \left( \frac{1}{r_{o2}} \right) \]  

Using (1) and (2) gives
\[ i_o = -g_m v_i \left( \frac{1}{r_{o1}} \right) \]
\[ G_m = -g_m \]

(b) For small-signal output resistance purposes, the cascade configuration is simply a special case of a CS stage with source degeneration. More specifically, the source degeneration resistor is \( r_{o1} \)
\[ R_o = r_{o1} + r_{o2} + \Delta r_{o1} \]
\[ G_m = -\frac{i_o}{v_i} = \frac{r_{o1}}{r_{o1} + \Delta r_{o1}} \]

(c) \[ A_v = \frac{G_m R_o}{1 + \frac{r_{in}}{r_{o1}}} \]

Recall that for a simple CS amp, \( A_v = -g_m \frac{r_{o1}}{r_{o2}} \)
\[ \left| \frac{r_{in}}{r_{o1}} \right| = \frac{g_m}{r_{o2}} \] (this is typically around 100 - 1000)

Thus, one has attained a factor of 100 - 1000 increase in gain. This, of course, has come at the expense of output voltage swing for now \( r_{o2} \) transistors need to be assured to be operating in the saturation regime.

(2) The small-signal equivalent circuit is

\[ -g_m v_i \]

\[ i_o = \frac{v_o}{r_{in}} + \frac{v_i}{r_{in}} \]

Note that \( v_x \) is a floating node.

\( i_o \) current must flow in the loop \( L \) as indicated.

Since \( v_y = 0 \), \( v_x = -r_{o1} \Delta g_m v_i \) in order to have the current of the dependent source go through \( r_{o1} \).
\[ i_o = r_{o1} \Delta g_m \frac{v_i}{r_{o1}} \]
\[ G_m = -\frac{i_o}{v_i} = \frac{r_{o1}}{r_{o1} + \Delta r_{o1}} \]
(b) The small-signal equivalent circuit is

\[ \begin{align*}
\delta v_n & \quad \delta v_{\text{out}} \\
\delta i_n & \quad \delta i_{\text{out}} \\
\delta v_n & \quad \delta v_{\text{out}} \\
\delta i_n & \quad \delta i_{\text{out}} \\
\delta v_n & \quad \delta v_{\text{out}}
\end{align*} \]

As before, we have the current loop L

\[ \frac{v_n - v_{\text{test}}}{r_{\text{in}}} = g_m, \quad v_{\text{out}} \Rightarrow v_n = v_{\text{test}} + g_m, r_{\text{in}} v_{\text{test}} \]

\[ i_{\text{test}} = g_m (1 + g_m, r_{\text{in}}) v_{\text{test}} + \frac{v_{\text{test}}}{r_{\text{in}}} \]

\[ R_0 = \frac{v_{\text{test}}}{i_{\text{test}}} = \frac{1}{g_m, r_{\text{in}}, g_m + g_m, r_{\text{in}}} \]

\[ \approx \frac{1}{g_m, g_m, r_{\text{in}}} \]

(c) \[ A_v = G_m R_0 \approx 1 \]

Thus, the voltage-gain is ideally the same as that for a SF. The advantage of this new circuit is its reduced \( R_0 \)
by a factor of \( g_m, r_{\text{in}} \)
(remember that an ideal op-amp has \( R_0 = 0 \))