Lecture 15

• Last time:
  – Square-law MOSFET model
  – Linear MOSFET model

• Today:
  – MOSFET small-signal model: three (four) terminal device → complicated!

Why Find an Incremental Model?

• Signals of interest in analog ICs are often of the form:
  \[ v_{GS}(t) = V_{GS} + v_{gs}(t) \]

  Direct substitution into \( i_D = f(v_{GS}, v_{DS}) \) is tedious AND doesn’t include charge-storage effects … pretty rough approximation
Which Operating Region?

Changing One Variable at a Time

Assumption: $V_{DS} > V_{DS,SAT} = V_{GS} - V_{Th}$ (square law)
The Transconductance $g_m$

Defined as the change in drain current due to a change in the gate-source voltage, with everything else constant

$$g_m = \frac{\Delta i_D}{\Delta v_{GS}} = \left. \frac{\partial i_D}{\partial v_{GS}} \right|_{V_{GS}, V_{DS}}$$

Square-law: MOSFET saturation region

$$i_D = \left( \frac{W}{2L} \right) \mu_n C_{ox} (V_{GS} - V_{Th})^2 (1 + \lambda_n V_{DS})$$

Another Way to Find $g_m$
Evaluating $g_m$

Square-law characteristic: H&S 1st Edition

$$g_m =$$

Linear characteristic: better for submicron CMOS

$$i_{D,SAT} = v_{sat} W C_{ox} (v_{GS} - V_{Th}) (1 + \lambda_n v_{DS})$$

Output Resistance $r_o$

Defined as the inverse of the change in drain current due to a change in the drain-source voltage, with everything else constant.
Evaluating $r_o$

$$r_o = \left( \frac{\partial i_D}{\partial V_{DS}} \right)_{V_{GS}, V_{DS}}^{-1}$$

Typical value:

Putting Together a Circuit Model

```
gate          drain
     +-----+      +-----+
        |    |      |    |
        vgs   |      vds
     -    -      -    -
source
```
Role of the Substrate Potential

Need not be the source potential, but \( V_B < V_S \)

Effect: changes threshold voltage, which changes the drain current ... substrate acts like a “backgate”

\[
g_{mb} = \frac{\Delta i_D}{\Delta V_{BS}} \bigg|_Q = \frac{\partial i_D}{\partial V_{BS}} \bigg|_Q
\]

\[Q = (V_{GS}, V_{DS}, V_{BS})\]