Overview

- Last lecture
  - Second Order
- This lecture
  - Frequency Response Common Source
Common Source Amplifier: $A_i(j\omega)$

Unity Gain Frequency of Amplifier:
Frequency where current gain drops to 1.

MOS Unity Gain Frequency

- Since the zero occurs at a higher frequency than pole, assume it has negligible effect:

$$A_i = \frac{g_m}{j\omega(C_{gs} + C_{gd})} = 1 \quad \Rightarrow \quad \omega_T = \frac{g_m}{(C_{gs} + C_{gd})}$$

$$\omega_T \approx \frac{g_m}{C_{gs}} = \frac{\mu C_{ox} W}{L} \frac{V_{GS} - V_T}{2 \frac{W}{L} C_{ox}} = 2 \frac{3}{2} \frac{\mu (V_{GS} - V_T)}{L^2}$$

Performance improves with $L^2$ for long channel devices!
For short channel devices the dependence is $\sim L^1$

$$\omega_T \approx \frac{3}{2} \frac{\mu (V_{GS} - V_T)}{L} \sim \frac{\mu V_{GS} - V_T}{L} \approx \frac{\mu E_{off}}{L} = \frac{v}{L} = \frac{1}{\tau_L}$$

Time to cross channel
Common-Source Voltage Amplifier

Small-signal model: omit $C_s$ due to avoid complicated analysis

CS Voltage Amp Small-Signal Model
Frequency Response

KCL at input and output nodes; analysis is made complicated due to $Z_{gd}$ branch → see H&S pp. 639-640 (for common emitter)

$$\frac{V_{out}}{V_{in}} = -g_m \left( \frac{r_o \parallel r_{oc} \parallel R_L \left( 1 - j\omega/\omega_z \right)}{(1 + j\omega/\omega_p1)(1 + j\omega/\omega_p2)} \right)$$

Low-frequency gain:

Zero: $\omega_z = \frac{g_m}{C_{gd}} > \omega_T = \frac{g_m}{C_{gs} + C_{gd}}$

---

Poles

$$\omega_{p1} \approx \frac{1}{R_S \left( C_{gs} + \left( 1 + g_m R'_{out} \right) C_{gd} \right) + R'_{out} C_{gd}}$$

$$\omega_{p2} \approx \frac{R'_{out} / R_S}{R_S \left( C_{gs} + \left( 1 + g_m R'_{out} \right) C_{gd} \right) + R'_{out} C_{gd}}$$
**Poles**

\[
\omega_{p1} = \frac{1}{R_S \left[ C_{gs} + \left( 1 + g_m R'_{out} \right) C_{gd} \right] + R'_{out} C_{gd}}
\]

\[
\omega_{p2} = \frac{R'_{out} / R_S}{R_S \left[ C_{gs} + \left( 1 + g_m R'_{out} \right) C_{gd} \right] + R'_{out} C_{gd}}
\]

- Low frequency voltage gain!

**Miller Impedance**

- Consider the current flowing through an impedance \( Z \) hooked up to a “black-box” where the voltage gain from one terminal to the other is fixed.

\[
A_v = \frac{v_2}{v_1}
\]

\[
I = \frac{v_1 - v_2}{Z} = \frac{v_1 - A_v v_1}{Z} = v_1 \frac{1 - A_v}{Z}
\]
**Miller Impedance**

- Notice that the current flowing into $Z$ from terminal 1 looks like an equivalent current to ground where $Z$ is transformed down by the Miller factor:

\[ I = v_i \frac{1 - A_v}{Z} \rightarrow Z_{M,1} = \frac{Z}{1 - A_v} \]

- From terminal 2, the situation is reciprocal

\[ -I = \frac{v_2 - v_1}{Z} = v_2 - A_v^{-1}v_2 = v_2 \frac{1 - A_v^{-1}}{Z} \]

\[ Z_{M,2} = \frac{Z}{1 - A_v^{-1}} \]

---

**Miller Equivalent Circuit**

**Note:** $Z_{M,1} + Z_{M,2} = Z$

- We can decouple these terminals if we can calculate the gain $A_v$ across the impedance $Z$
- Often the gain $A_v$ is weakly dependent on $Z$
- The approximation is to ignore $Z$, calculate $A_v$, and then use the decoupled Miller impedances
CS Amplifier using Miller Approx.

Use Miller to transform $C_{gd}$

Analysis using Miller
Comparison with “Exact Analysis”

Miller result:

\[ \omega_{p1}^{-1} = \]

Exact result:

\[ \omega_{p1}^{-1} = R_S \{C_{gs} + (1 + g_m R'_{out}) C_{gd}\} + R'_{out} C_{gd} \]

Miller Effect Examples

Common source amplifier:

\[ A_v C_{gd} = \text{negative, large number (-100)} \]

Miller multiplied cap has detrimental impact on bandwidth

Common drain amplifier:

\[ A_v C_{gd} = \text{slightly less than 1} \]

“Bootstrapped” cap does not hurt bandwidth!
## Dominant Pole and Its Impact

- For each capacitor in the circuit you calculate an equivalent resistor “seen” by capacitor and form the time constant $\tau_i = R_i C_i$.
- The dominant pole then is the sum of these time constants in the circuit $\omega_{p,\text{dom}} = \frac{1}{\tau_1 + \tau_2 + \cdots}$.

## Method of Open Circuit Time Constants

- This is a technique to find the dominant pole of a circuit (only valid if there really is a dominant pole!).
- For each capacitor in the circuit you calculate an equivalent resistor “seen” by capacitor and form the time constant $\tau_i = R_i C_i$.
- The dominant pole then is the sum of these time constants in the circuit $\omega_{p,\text{dom}} = \frac{1}{\tau_1 + \tau_2 + \cdots}$. 

Method of Open Circuit Time Constants

- General two-pole transfer function:

\[
A(j\omega) = A_0 \frac{(1 + j \frac{\omega}{\omega_p})}{(1 + j \frac{\omega}{\omega_{p1}})(1 + j \frac{\omega}{\omega_{p2}})}
\]

\[
A(j\omega) = A_0 \frac{N(j\omega)}{1 + a_1 j\omega + a_2 (j\omega)^2}
\]

\[
a_1 = \frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}
\]

\[
a_2 = \frac{1}{\omega_{p1}\omega_{p2}}
\]

Sum of time constants
Equivalent Resistance “Seen” by Capacitor

- For each “small” capacitor in the circuit:
  - Open-circuit all other “small” capacitors
  - Short circuit all “big” capacitors
  - Turn off all independent sources
  - Replace cap under question with current or voltage source
  - Find equivalent input impedance seen by cap
  - Form RC time constant

- This procedure is best illustrated with an example...

Example Calculation

\[
\begin{align*}
\text{Rs} + \text{RL} + v_{\text{out}} = v_{\text{in}} + \frac{v_{\text{gs}}}{C_{\text{gs}}} + \frac{v_{\text{gd}}}{C_{\text{gd}}} + g_{\text{m}}v_{\text{gs}} + \frac{g_{\text{d}}|v_{\text{oc}}}{R_{\text{L}}}
\end{align*}
\]
Example Calculation (C_{GS})

\[ v_{out} = \frac{v_{gs}}{R_s + C_{gs} + C_{gd} + g_m V_{gs} + \frac{r_d}{R_L}} \]

Example Calculation (C_{gd})
Higher-Order Time Constants

- General two-pole transfer function:

\[
A(j\omega) = A_0 \frac{(1 + j \frac{\omega}{\omega_{p1}})(1 + j \frac{\omega}{\omega_{p2}})}{(1 + j \frac{\omega}{\omega_{p1}})(1 + j \frac{\omega}{\omega_{p2}})}
\]

\[
A(j\omega) = A_0 \frac{N(j\omega)}{1 + a_1 j\omega + a_2 (j\omega)^2}
\]

\[
a_1 = \frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}} \quad a_2 = \frac{1}{\omega_{p1}\omega_{p2}}
\]
Higher-Order Time Constants

- Coefficient $a_1$
  
  \[ a_1 = R_{011}C_1 + R_{022}C_2 \]

- Coefficient $a_2$:
  
  \[ a_2 = R_{011}C_1R_{022}C_2 = R_{011}C_1R_{122}C_2 \]

- This is exact!

- If $\omega_{p1} << \omega_{p2}$:  
  
  \[ a_1 \approx \frac{1}{\omega_{p1}} \quad a_2 \approx \frac{a_1}{\omega_{p2}} \]
Gain-Bandwidth Product

Result from Miller:

\[ \omega_p^{-1} \approx (R_S) \{ C_{gs} + (1 + g_m R'_{out}) C_{gd} \} \]

Low-frequency gain:

\[ A_{vo} = \left. \frac{v_{out}}{v_s} \right|_{R_S, R_L} = -g_m R'_{out} \]
**Gain-Bandwidth Product**

Considering only the first pole (assuming zero and 2\textsuperscript{nd} pole are at much higher frequencies):

\[ |A_v(j\omega)|_{\text{dB}} \]

\[ |A_v(j\omega)| \approx \frac{A_{v0}}{1 + j\omega/\omega_p} \approx \frac{A_{v0}}{\omega_p} = \frac{A_{v0}\omega_p}{\omega} \]

\[ |A_v(j\omega^*)| = 1 \Rightarrow \omega^* = A_{v0}\omega_p \]

---

**Gain-Bandwidth Product**

For common-source amplifier:

\[ |A_{v0}|_{\omega_p} = \frac{g_mR_{\text{out}}'}{R_S C_{gs} + R_S(1 + g_m R_{\text{out}}') C_{gd}} \]

Special case: \( R_S \approx R_L < r_o, r_{oc} \)

\[ |A_{v0}|_{\omega_p} \approx \frac{g_mR_L}{R_S(C_{gs} + g_mR_L C_{gd})} \ll \omega_T \text{ not that great!} \]