The Early Effect

Current in Forward-Active a Function impacted by VCE
Main reason: Base-width modulation

Hence:

\[ i_C = I_S e^{\frac{v_{BE}}{V_{th}}} (1 + \frac{v_{CE}}{V_A}) \]
Small-Signal Models

Analogy from MOSFET s.s. model:

\[ i_D = f(v_{GS}, v_{DS}, v_{BS}) \quad i_C = f(v_{BE}, v_{CE}) \]

Transconductance \( g_m \)

- The transconductance is analogous to diode conductance
Transconductance (cont)

- Forward-active large-signal current:
  \[ i_C = I_s e^{\frac{V_{BE}}{V_{th}}} (1 + \frac{v_{CE}}{V_A}) \]

- Differentiating and evaluating at \( Q = (V_{BE}, V_{CE}) \)
  \[ \frac{di_C}{dv_{BE}} |_Q = \frac{q}{kT} I_s e^{qV_{BE}/kT} (1 + \frac{V_{CE}}{V_A}) \]
  \[ g_m = \frac{di_C}{dv_{BE}} |_Q = \frac{qI_C}{kT} \]

Comparison with MOSFET

- Typical bias point: drain/coll. current = 100 \( \mu \text{A} \); Select \((W/L) = 8/1, \mu_n C_{ox} = 100 \mu \text{A}/\text{V}^2\)
- BJT:
  \[ g_m = \frac{qI_C}{kT} = \frac{I_C}{V_{th}} \]
  \[ g_m = \frac{I_C}{V_{th}} = \frac{100\mu}{25m} = 4\text{mS} \]
- MOSFET:
  \[ g_m = \frac{2I_D}{V_{GS} - V_T} \]
  \[ g_m = \frac{2I_D}{V_{GS} - V_T} = \sqrt{2 \mu C_{ox} \frac{W}{L} I_D} = \sqrt{2 \times 100\mu \times 8 \times 100\mu} = 400\mu\text{S} \]
BJT Base Currents

Unlike MOSFET, there is a DC current into the base terminal of a bipolar transistor:

\[ I_B = I_C / \beta_F = \left( I_S / \beta_F \right) e^{q(V_{BE} + I_K)} \left( 1 + V_{CE} / V_A \right) \]

To find the change in base current due to change in base-emitter voltage:

\[ \frac{\partial i_B}{\partial V_{BE}} \bigg|_Q = \frac{\partial i_B}{\partial i_C} \bigg|_Q \frac{\partial i_C}{\partial V_{BE}} \bigg|_Q = \frac{1}{\beta} g_m \]

Small Signal Current Gain

\[ \beta = \frac{\Delta i_C}{\Delta i_B} = \beta_F \]

\[ I_C \quad \quad \quad I_B \]
Input Resistance $r_\pi$

\[ (r_\pi)^{-1} = \left. \frac{\partial i_B}{\partial v_{BE}} \right|_Q = \frac{1}{\beta} \left. \frac{\partial i_C}{\partial v_{BE}} \right|_Q = g_m \frac{\beta}{\beta} \]

In practice, the DC current gain $\beta_F$ and the small-signal current gain $\beta_o$ are both highly variable (+/- 25%) 

Typical bias point: DC collector current = 100 $\mu$A

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Output Resistance $r_o$

Why does current increase slightly with increasing $v_{CE}$?

Model: introduce the Early voltage

\[ i_C = I_S e^{v_{BE}/V_{th}} (1 + v_{CE}/V_A) \]
**Graphical Interpretation of $r_o$**

- **slope~1/ro**
- **slope**

**The Early Effect**

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Main reason: Base-width modulation

Hence:

$$i_C = I_S e^{v_{BE}/v_{th}} (1 + v_{CE}/V_A)$$

$$r_o = \frac{I_C}{V_A}$$
BJT Small-Signal Model

Bipolar transistor small-signal model: $g_m$ & $r_o$

\[ g_m = \frac{di_{CE}}{dv_{BE}} = \frac{i_{CE}}{kT/q} \]

\[ r_\pi = \frac{dv_{BE}}{di_{BE}} = \beta \frac{dv_{BE}}{di_{CE}} = \frac{\beta}{g_m} \]

\[ g_m = \frac{1}{l_{CE} / kT/q} \approx 40 \, \text{V}^{-1} \]

\[ r_o = \frac{v_E}{l_{CE}} \]

$V_{En} \approx 20 \, \text{V}$

$V_{Ep} \approx 10 \, \text{V}$
BJT Capacitances

Base-charging capacitance $C_b$: due to minority carrier charge storage (mostly electrons in the base)

$$C_b = g_m \tau_F$$

Base-emitter depletion capacitance: $C_{jE} = 1.4 \ C_{jE0}$

Total B-E capacitance: $C_\pi = C_{jE} + C_b$

Bipolar transistor capacitance $C_\pi$

$$C_\pi = C_{jBE} + C_D$$

$$C_{jBE} = \frac{C_{jBE0}}{\sqrt{1 + V_{BE}/\phi_{jE}}} \quad \phi_{jE} = 0.7 \, V$$

$C_D$ is the diffusion capacitance
Diffusion capacitance $C_D$

$$C_D = \frac{Q_B}{v_{BE}} = \tau_F \frac{d}{dv_{BE}} = \tau_F g_m = \tau_F \frac{I_{CE}}{kT/q}$$

Base transit time $\tau_F = \frac{W_B^2}{2D_n}$ or now $= \frac{W_B}{v_{sat}} \approx 10 \ldots 200$ ps

Bipolar transistor capacitances $C_\mu$ & $C_{CS}$

$$C_\mu = C_{jBC} \quad C_{jBC} = \frac{C_{jBC0}}{\sqrt{1 + V_{BC} / \phi_j}}$$

$$C_{CS} = C_{jCS} \quad C_{jCS} = \frac{C_{jCS0}}{\sqrt{1 + V_{CS} / \phi_j}} \quad \phi_j \approx \phi_s \approx 0.5 \text{ V}$$
Bipolar transistor $f_T$

\[ f_T = \frac{g_m}{2\pi C_\pi} = \frac{1}{2\pi} \frac{1}{\tau_F + \frac{C_{jBE} + C_\mu}{g_m}} \]

or \approx \frac{v_{Sat}}{2\pi W_B}

For a current drive:

\[ f_{max} \approx \sqrt{f_T / 8\pi r_B C_\mu} \]

Bipolar transistor $f_T$ versus $I_{CE}$

\[ f_T = \frac{1}{2\pi} \frac{1}{\tau_F + \frac{C_{jBE} + C_\mu}{g_m}} \]

\[ \frac{1}{2\pi f_T} = \tau_F + \frac{kT}{q} \frac{1}{I_{CE}} \]
Single-page Bipolar transistor model

\[ I_{CE} = I_S \exp \left( \frac{V_{BE}}{kT/q} \right) \]

\[ I_S \approx 10^{-15} \text{ A} \quad kT/q = 26 \text{ mV at } 300 \text{ K} \]

\[ g_m = \frac{I_{CE}}{kT/q} \quad r_o = \frac{V_E}{I_{CE}} \]

\[ V_{En} \approx 20 \text{ V} \quad V_{Ep} \approx 10 \text{ V} \]

\[ f_T = \frac{1}{2\pi} \frac{1}{\tau_F + \frac{C_{le} + C_{lc}}{g_m}} \]

or \[ \approx \frac{v_{sat}}{2\pi W_B} \]

Complete Small-Signal Model
SiGe BJT/CMOS vs. RF CMOS

IBM SiGe Heterojunction BJT

From “IBM and Cadence collaborate to accelerate silicon-accurate design of advanced RF integrated circuits,”
IBM Microelectronics Division, March 11, 2005.
Figure 2

Cutoff frequency $f_T$ vs. $I_C$ for four lithographic generations of SiGe. The InP curve shows recent production InP results.