Overview

- Last lecture
  - Diffusion currents
  - Overview of IC fabrication process
  - Review of electrostatics

- This lecture
  - Capacitances
  - pn Junctions
Administrativia

- Another Make-up Lecture Monday at 4pm (streamed)
  NO LECTURE ON TUESDAY

IC MIM Capacitor

- By forming a thin oxide and metal (or polysilicon) plates, a capacitor is formed
- Contacts are made to top and bottom plate
- Parasitic capacitance exists between bottom plate and substrate

\[ Q = CV \]
Review of Capacitors

For an ideal metal, all charge must be at surface
Gauss’ law: Surface integral of electric field over closed surface equals charge inside volume

Capacitor Q-V Relation

Total charge is linearly related to voltage
Charge density is a delta function at surface (for perfect metals)
A Non-Linear Capacitor

- We’ll soon meet capacitors that have a non-linear Q-V relationship.
- If plates are not ideal metal, the charge density can penetrate into surface.

What’s the Capacitance?

- For a non-linear capacitor, we have
  \[ Q = f(V_s) \neq CV_s \]
- We can’t identify a capacitance.
- Imagine we apply a small signal on top of a bias voltage:
  \[ Q = f(V_s + V_x) \approx f(V_s) + \left. \frac{d f(V)}{dV} \right|_{V=V_s} V_x \]
  \[ \text{Constant charge} \]
- The incremental charge is therefore:
  \[ Q = Q_0 + q = f(V_s) + \left. \frac{d f(V)}{dV} \right|_{V=V_s} V_x \]
Small Signal Capacitance

- Break the equation for total charge into two terms:
  
  \[ Q = Q_s + q \approx f(V_s) + \frac{df(V)}{dV} \bigg|_{V=V_s} v_s \]

  Incremental Charge

  \[ q = \frac{df(V)}{dV} \bigg|_{V=V_s} v_s = C v_s \]

  Constant Charge

  \[ C \equiv \frac{df(V)}{dV} \bigg|_{V=V_s} \]

Example of Non-Linear Capacitor

- Next lecture we’ll see that for a PN junction, the charge is a function of the reverse bias:
  
  \[ Q_j(V) = -qN_a x_p \sqrt{1 - \frac{V}{\phi_b}} \]

  Charge At N Side of Junction

  Voltage Across NP Junction

- Small signal capacitance:
  
  \[ C_j(V) = \frac{dQ_j}{dV} = \frac{qN_a x_p}{2\phi_b} \frac{1}{\sqrt{1 - \frac{V}{\phi_b}}} = \frac{C_{j0}}{\sqrt{1 - \frac{V}{\phi_b}}} \]
PN Junctions (Diodes)

Carrier Concentration and Potential

- In thermal equilibrium, there are no external fields and we thus expect the electron and hole current densities to be zero:

\[ J_n = 0 = qn_0 \mu_n E_0 + qD_n \frac{dn_o}{dx} \]

\[ \frac{dn_n}{dx} = -\left(\frac{\mu_n}{D_n}\right)n_n E_0 = \left(\frac{q}{kT}\right)n_n \frac{d\phi_0}{dx} \]

\[ d\phi_0 = \left(\frac{kT}{q}\right)\frac{dn_o}{n_0} = V_{th} \frac{dn_o}{n_0} \]
Carrier Concentration and Potential (2)

- We have an equation relating the potential to the carrier concentration

\[ d\phi_0 = \left( \frac{kT}{q} \right) \frac{dn}{n_0} = V_{th} \frac{dn}{n_0} \]

- If we integrate the above equation we have

\[ \phi_0(x) - \phi_0(x_0) = V_{th} \ln \frac{n_i(x)}{n_0(x_0)} \]

- We define the potential reference to be intrinsic Si:

\[ \phi_0(x_0) = 0 \quad n_0(x_0) = n_i \]

Carrier Concentration Versus Potential

- The carrier concentration is thus a function of potential

\[ n_0(x) = n_i e^{\phi_0(x)/V_{th}} \]

- Check that for zero potential, we have intrinsic carrier concentration (reference).

- If we do a similar calculation for holes, we arrive at a similar equation

\[ p_0(x) = n_i e^{-\phi_0(x)/V_{th}} \]

- Note that the law of mass action is upheld

\[ n_0(x)p_0(x) = n_i^2 e^{-\phi_0(x)/V_{th}} e^{\phi_0(x)/V_{th}} = n_i^2 \]
The Doping Changes Potential

- Due to the log nature of the potential, the potential changes linearly for exponential increase in doping:

\[ \phi_b(x) = V_{th} \ln \frac{n_d(x)}{n_i(x_0)} = 26 \text{mV} \ln \frac{n_d(x)}{n_i(x_0)} \approx 26 \text{mV} \cdot (\ln 10) \cdot \log 10^{n_d(x)} \]

\[ \phi_b(x) \approx 60 \text{mV} \log 10^{n_d(x)} \]

\[ \phi_b(x) \approx -60 \text{mV} \log 10^{p_d(x)} \]

- Quick calculation aid: For a p-type concentration of $10^{16}$ cm$^{-3}$, the potential is $-360$ mV

- N-type materials have a positive potential with respect to intrinsic Si

PN Junction: Overview

- p-type silicon
- ionized acceptor (fixed)
- hole (mobile)

- n-type silicon
- ionized donor (fixed)
- electron (mobile)
PN Junction: Overview

- Present in most IC structures
PN Junctions: Overview

- The most important device is a junction between a p-type region and an n-type region.
- When the junction is first formed, due to the concentration gradient, mobile charges transfer near junction.
- Electrons leave n-type region and holes leave p-type region.
- These mobile carriers become minority carriers in new region (can’t penetrate far due to recombination).
- Due to charge transfer, a voltage difference occurs between regions.
- This creates a field at the junction that causes drift currents to oppose the diffusion current.
- In thermal equilibrium, drift current and diffusion must balance.

PN Junction Currents

- Consider the PN junction in thermal equilibrium.
- Again, the currents have to be zero, so we have

\[ J_n = 0 = qn_0 \mu_n E_0 + qD_n \frac{dn_n}{dx} \]

\[ qn_0 \mu_n E_0 = -qD_n \frac{dn_n}{dx} \]

\[ E_0 = -\frac{D_n}{n_0 \mu_n} \frac{dn_n}{dx} = -\frac{kT}{q} \frac{1}{n_0} \frac{dn_0}{dx} \]

\[ E_0 = \frac{D_p}{n_0 \mu_p} \frac{dp_0}{dx} = -\frac{kT}{q} \frac{1}{p_0} \frac{dp_0}{dx} \]
PN Junction Fields

To solve for the electric fields, we need to write down the charge density in the transition region:

\[ \rho_o(x) = q(p_o - n_o + N_d - N_a) \]

In the p-side of the junction, there are very few electrons and only acceptors:

\[ \rho_o(x) \approx q(p_o - N_a) \quad -x_p < x < 0 \]

Since the hole concentration is decreasing on the p-side, the net charge is negative:

\[ N_a > p_o \quad \rho_o(x) < 0 \]
Charge on N-Side

- Analogous to the p-side, the charge on the n-side is given by:
  \[ \rho_0(x) = q(-n_0 + N_d) \quad 0 < x < x_{n0} \]

- The net charge here is positive since:
  \[ N_d > n_0 \quad \rho_0(x) > 0 \]

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\begin{align*}
\rho_0 &= \frac{n_0^2}{N_n} \\
n_0 &= N_d
\end{align*}
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Transition Region


“Exact” Solution for Fields

- Given the above approximations, we now have an expression for the charge density
  \[ \rho_0(x) = \begin{cases} 
  q(n_i e^{-\varphi(x)/V_n} - N_a) & -x_{po} < x < 0 \\
  q(N_d - n_i e^{\varphi(x)/V_n}) & 0 < x < x_{n0}
\end{cases} \]

- We also have the following result from electrostatics
  \[ \frac{dE_0}{dx} = -\frac{d^2\phi}{dx^2} = \frac{\rho_0(x)}{\epsilon_e} \]

- Notice that the potential appears on both sides of the equation... difficult problem to solve

- A much simpler way to solve the problem...
Depletion Approximation

- Let’s assume that the transition region is completely depleted of free carriers (only immobile dopants exist)
- Then the charge density is given by

\[ \rho_0(x) \equiv \begin{cases} 
-qN_a & -x_{p0} < x < 0 \\
+qN_d & 0 < x < x_{n0} 
\end{cases} \]

- The solution for electric field is now easy

\[ E_0(x) = \int_{-x_{p0}}^{x} \frac{\rho_0(x')}{\varepsilon_x} dx' + E_0(-x_{p0}) \]

Field zero outside transition region

Depletion Approximation (2)

- Since charge density is a constant

\[ E_0(x) = \int_{-x_{p0}}^{x} \frac{\rho_0(x')}{\varepsilon_x} dx' = -\frac{qN_a}{\varepsilon_x} (x + x_{p0}) \]

- If we start from the n-side we get the following result

\[ E_0(x_{n0}) = \int_{x}^{x_{n0}} \frac{\rho_0(x')}{\varepsilon_x} dx' + E_0(x) = \frac{qN_d}{\varepsilon_x} (x_{n0} - x) + E_0(x) \]

Field zero outside transition region

\[ E_0(x) = -\frac{qN_d}{\varepsilon_x} (x_{n0} - x) \]
**Plot of Fields In Depletion Region**

- E-Field zero outside of depletion region
- Note the asymmetrical depletion widths
- Which region has higher doping?
- Slope of E-Field larger in n-region. Why?
- Peak E-Field at junction. Why continuous?

**Continuity of E-Field Across Junction**

- Recall that E-field diverges on charge. For a sheet charge at the interface, the E-field could be discontinuous
- In our case, the depletion region is only populated by a background density of fixed charges so the E-Field is continuous
- What does this imply?

\[
E^n_0 (x = 0) = -\frac{qN_d}{\varepsilon_s} x_{po} = -\frac{qN_d}{\varepsilon_s} x_{no} = E^p_0 (x = 0)
\]

\[
qN_d x_{po} = qN_d x_{no}
\]

- Total fixed charge in n-region equals fixed charge in p-region! Somewhat obvious result.
Potential Across Junction

- From our earlier calculation we know that the potential in the n-region is higher than p-region.
- The potential has to smoothly transition from high to low in crossing the junction.
- Physically, the potential difference is due to the charge transfer that occurs due to the concentration gradient.
- Let’s integrate the field to get the potential:

\[
\phi(x) = \phi(-x_{po}) + \int_{-x_{pi}}^{x} \frac{qN_a}{\varepsilon_s} (x' + x_{po}) dx'
\]

\[
\phi(x) = \phi_p + \frac{qN_a}{\varepsilon_s} \left( \frac{x'^2}{2} + x'x_{po} \right)_{-x_{pi}}^{x}
\]

Potential Across Junction

- We arrive at potential on p-side (parabolic)

\[
\phi_p(x) = \phi_p + \frac{qN_a}{2\varepsilon_s} (x + x_{po})^2
\]

- Do integral on n-side

\[
\phi_n(x) = \phi_n - \frac{qN_d}{2\varepsilon_s} (x - x_{n0})^2
\]

- Potential must be continuous at interface (field finite at interface)

\[
\phi_n(0) = \phi_n - \frac{qN_d}{2\varepsilon_s} x_{n0}^2 = \phi_p + \frac{qN_a}{2\varepsilon_s} x_{po}^2 = \phi_p(0)
\]
We have two equations and two unknowns. We are finally in a position to solve for the depletion depths

\[ \phi_n - \frac{qN_d}{2\varepsilon_s} x_{n0}^2 = \phi_p + \frac{qN_a}{2\varepsilon_s} x_{p0}^2 \]  
\[ qN_ax_{p0} = qN_ax_{no} \]

\[ x_{no} = \sqrt{\frac{2\varepsilon_r\phi_{bi}}{qN_a} \left( \frac{N_a}{N_a + N_d} \right)} \]
\[ x_{p0} = \sqrt{\frac{2\varepsilon_r\phi_{bi}}{qN_a} \left( \frac{N_d}{N_d + N_a} \right)} \]

\[ \phi_{bi} \equiv \phi_n - \phi_p > 0 \]

**Sanity Check**

- Does the above equation make sense?
- Let’s say we dope one side very highly. Then physically we expect the depletion region width for the heavily doped side to approach zero:

\[ x_{n0} = \lim_{N_d \to \infty} \sqrt{\frac{2\varepsilon_r\phi_{bi}}{qN_a} \left( \frac{N_d}{N_d + N_a} \right)} = 0 \]

\[ x_{p0} = \lim_{N_d \to \infty} \sqrt{\frac{2\varepsilon_r\phi_{bi}}{qN_a} \left( \frac{N_d}{N_d + N_a} \right)} = \sqrt{\frac{2\varepsilon_r\phi_{bi}}{qN_a}} \]

- Entire depletion width dropped across p-region
Total Depletion Width

- The sum of the depletion widths is the “space charge region”
  \[ X_{d0} = x_{p0} + x_{n0} = \sqrt{\frac{2e \phi_{bi} \left( \frac{1}{N_a} + \frac{1}{N_d} \right)}{q}} \]

- This region is essentially depleted of all mobile charge
- Due to high electric field, carriers move across region at velocity saturated speed

\[ X_{d0} = \sqrt{\frac{2e \phi_{bi} \left( \frac{1}{10^{15}} \right)}{q}} \approx 1\mu \quad E_{pn} = \frac{1V}{1\mu} = 10^4 \frac{V}{cm} \]

Have we invented a battery?

- Can we harness the PN junction and turn it into a battery?
  \[ \phi_{bi} \equiv \phi_a - \phi_p = V_{th} \left( \ln \frac{N_D}{n_i} + \ln \frac{N_A}{n_i} \right) = V_{th} \ln \frac{N_D N_A}{n_i^2} \]

- Numerical example:
  \[ \phi_{bi} = 26mV \ln \frac{N_D N_A}{n_i^2} = 60mV \times \log \frac{10^{15} \times 10^{15}}{10^{20}} = 600mV \]
Contact Potential

- The contact between a PN junction creates a potential difference.
- Likewise, the contact between two dissimilar metals creates a potential difference (proportional to the difference between the work functions).
- When a metal semiconductor junction is formed, a contact potential forms as well.
- If we short a PN junction, the sum of the voltages around the loop must be zero:

\[ 0 = \phi_{in} + \phi_{pn} + \phi_{nn} \]

PN Junction Capacitor

- Under thermal equilibrium, the PN junction does not draw any (much) current.
- But notice that a PN junction stores charge in the space charge region (transition region).
- Since the device is storing charge, it’s acting like a capacitor.
- Positive charge is stored in the n-region, and negative charge is in the p-region:

\[ qN_d x_{po} = qN_a x_{no} \]
Reverse Biased PN Junction

- What happens if we “reverse-bias” the PN junction?

\[ -\phi_{bi} + V_D^+ \quad \text{to} \quad V_D^- \quad V_D < 0 \]

- Since no current is flowing, the entire reverse biased potential is dropped across the transition region
- To accommodate the extra potential, the charge in these regions must increase
- If no current is flowing, the only way for the charge to increase is to grow (shrink) the depletion regions

Voltage Dependence of Depletion Width

- Can redo the math but in the end we realize that the equations are the same except we replace the built-in potential with the effective reverse bias:

\[
\begin{align*}
x_n(V_D) &= \sqrt{\frac{2\varepsilon_s (\phi_{bi} - V_D)}{q N_d}} \left( \frac{N_a}{N_a + N_d} \right) = x_{n0} \sqrt{1 - \frac{V_D}{\phi_{bi}}} \\
x_p(V_D) &= \sqrt{\frac{2\varepsilon_s (\phi_{bi} - V_D)}{q N_a}} \left( \frac{N_d}{N_a + N_d} \right) = x_{p0} \sqrt{1 - \frac{V_D}{\phi_{bi}}} \\
X_d(V_D) &= x_p(V_D) + x_n(V_D) = \sqrt{\frac{2\varepsilon_s (\phi_{bi} - V_D)}{q}} \left( \frac{1}{N_a} + \frac{1}{N_d} \right) \\
X_d(V_D) &= x_{d0} \sqrt{1 - \frac{V_D}{\phi_{bi}}}
\end{align*}
\]
Charge Versus Bias

- As we increase the reverse bias, the depletion region grows to accommodate more charge

\[ Q_J(V_D) = -qN_a x_p(V_D) = -qN_a \sqrt{1 - \frac{V_D}{\phi_{bi}}} \]

- Charge is not a linear function of voltage
- This is a non-linear capacitor
- We can define a small signal capacitance for small signals by breaking up the charge into two terms

\[ Q_J(V_D + v_D) = Q_J(V_D) + q(v_D) \]

Derivation of Small Signal Capacitance

- From last lecture we found

\[ Q_J(V_D + v_D) = Q_J(V_D) + \left. \frac{dQ_D}{dV} \right|_{V_D} v_D + \cdots \]

\[ C_j = C_j(V_D) = \left. \frac{dQ_j}{dV} \right|_{V = V_D} = \left. \frac{d}{dV} \left( -qN_a x_{p0} \sqrt{1 - \frac{V}{\phi_{bi}}} \right) \right|_{V = V_D} \]

\[ C_j = \frac{qN_a x_{p0}}{2\phi_{bi} \sqrt{1 - \frac{V_D}{\phi_{bi}}} \sqrt{1 - \frac{V_D}{\phi_{bi}}}} = \frac{C_{j0}}{2\phi_{bi} \sqrt{1 - \frac{V_D}{\phi_{bi}}} \sqrt{1 - \frac{V_D}{\phi_{bi}}}} \]

- Notice that

\[ C_{j0} = \frac{qN_a x_{p0}}{2\phi_{bi}} = \frac{qN_a}{2\phi_{bi} \sqrt{\frac{2\epsilon_s \phi_{bi}}{qN_a} \left( \frac{N_d}{N_a + N_d} \right)}} = \frac{q\epsilon_s N_a N_d}{2\phi_{bi} \sqrt{N_a + N_d}} \]
Physical Interpretation of Depletion Cap

\[ C_{j0} = \sqrt{\frac{q\varepsilon_s N_a N_d}{2\phi_{bi} N_a + N_d}} \]

- Notice that the expression on the right-hand-side is just the depletion width in thermal equilibrium

\[ C_{j0} = \varepsilon_s \sqrt{\frac{q}{2\varepsilon_s \phi_{bi} \left( \frac{1}{N_a} + \frac{1}{N_d} \right)^{-1}}} = \frac{\varepsilon_s}{X_{d0}} \]

- This looks like a parallel plate capacitor!

\[ C_j(V_D) = \frac{\varepsilon_s}{X_d(V_D)} \]

A Variable Capacitor (Varactor)

- Capacitance varies versus bias:

\[ \frac{C_j}{C_{j0}} \]

- Application: Radio Tuner
“Diffusion” Resistor

- Resistor is capacitively isolation from substrate
  - Must Reverse Bias PN Junction!