Overview

- Last lecture
  - Capacitance
  - pn Junction

- This lecture
  - pn Junction (cntd)
  - Diode operation and models
**Administrativia**

- Labs starting tomorrow!

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**PN Junction – Summary so far**

- The most important device is a junction between a p-type region and an n-type region.
- When the junction is first formed, due to the concentration gradient, mobile charges transfer near junction.
- Electrons leave n-type region and holes leave p-type region.
- These mobile carriers become minority carriers in new region (can’t penetrate far due to recombination).
- Due to charge transfer, a voltage difference occurs between regions.
- This creates a field at the junction that causes drift currents to oppose the diffusion current.
- In thermal equilibrium, drift current and diffusion must balance.

**Question:** width of depletion region.
Depletion Approximation

- Let’s assume that the transition region is completely depleted of free carriers (only immobile dopants exist).
- Then the charge density is given by

\[ \rho_0(x) = \begin{cases} -qN_a & -x_{po} < x < 0 \\ +qN_d & 0 < x < x_{n0} \end{cases} \]

- The solution for electric field is now easy

\[ E_0(x) = \int_{-x_{po}}^{x} \frac{\rho_0(x')}{\varepsilon_s} \, dx' = -\frac{qN_a}{\varepsilon_s} (x + x_{po}) \quad E_0(x) = -\frac{qN_d}{\varepsilon_s} (x_{n0} - x) \]

Plot of Fields In Depletion Region

- E-Field zero outside of depletion region
- Note the asymmetrical depletion widths
- Which region has higher doping?
- Slope of E-Field larger in n-region. Why?
- Peak E-Field at junction. Why continuous?
Continuity of E-Field Across Junction

- Recall that E-field diverges on charge. For a sheet charge at the interface, the E-field could be discontinuous.
- In our case, the depletion region is only populated by a background density of fixed charges so the E-Field is continuous.
- What does this imply?
  
  \[
  E_n^o(x = 0) = -\frac{qN_n}{\epsilon_s} x_{po} = -\frac{qN_d}{\epsilon_s} x_{no} = E_p^o(x = 0)
  \]
  
  \[qN_n x_{po} = qN_d x_{no}\]

- Total fixed charge in n-region equals fixed charge in p-region! Somewhat obvious result.

Potential Across Junction

- From our earlier calculation we know that the potential in the n-region is higher than p-region.
- The potential has to smoothly transition form high to low in crossing the junction.
- Physically, the potential difference is due to the charge transfer that occurs due to the concentration gradient.
- Let’s integrate the field to get the potential:

  \[
  \phi(x) = \phi(-x_{po}) + \int_{-s_{po}}^{x} \frac{qN_a}{\epsilon_s} (x' + x_{po}) dx' \\
  \phi(x) = \phi_p + \frac{qN_a}{\epsilon_s} \left( \frac{x^2}{2} + x' x_{po} \right)_{-s_{po}}^{x}
  \]
Potential Across Junction

- We arrive at potential on p-side (parabolic)
  \[ \phi^p_n(x) = \phi_p + \frac{qN_n}{2\varepsilon_s} (x + x_{p0})^2 \]

- Do integral on n-side
  \[ \phi_n(x) = \phi_n - \frac{qN_d}{2\varepsilon_s} (x - x_{n0})^2 \]

- Potential must be continuous at interface (field finite at interface)
  \[ \phi_n(0) = \phi_n - \frac{qN_d}{2\varepsilon_s} x_{n0}^2 = \phi_p + \frac{qN_n}{2\varepsilon_s} x_{p0}^2 = \phi_p(0) \]

Solve for Depletion Lengths

- We have two equations and two unknowns. We are finally in a position to solve for the depletion depths

  \[ \phi_n - \frac{qN_d}{2\varepsilon_s} x_{n0}^2 = \phi_p + \frac{qN_n}{2\varepsilon_s} x_{p0}^2 \quad (1) \]

  \[ qN_n x_{p0} = qN_d x_{n0} \quad (2) \]

\[ x_{n0} = \sqrt{\frac{2\varepsilon_s \phi_{bi}}{qN_d} \left( \frac{N_n}{N_a + N_d} \right)} \]

\[ x_{p0} = \sqrt{\frac{2\varepsilon_s \phi_{bi}}{qN_n} \left( \frac{N_d}{N_a + N_n} \right)} \]

\[ \phi_{bi} = \phi_n - \phi_p > 0 \]
Sanity Check

- Does the above equation make sense?
- Let’s say we dope one side very highly. Then physically we expect the depletion region width for the heavily doped side to approach zero:

\[ x_{n0} = \lim_{N_d \to \infty} \sqrt{\frac{2\varepsilon \phi_{bi}}{qN_d} \frac{N_d}{N_d + N_a}} = 0 \]

\[ x_{p0} = \lim_{N_a \to \infty} \sqrt{\frac{2\varepsilon \phi_{bi}}{qN_a} \left( \frac{N_d}{N_d + N_a} \right)} = \sqrt{\frac{2\varepsilon \phi_{bi}}{qN_a}} \]

- Entire depletion width dropped across p-region

Total Depletion Width

- The sum of the depletion widths is the “space charge region”

\[ X_{d0} = x_{p0} + x_{n0} = \sqrt{\frac{2\varepsilon \phi_{bi}}{q} \left( \frac{1}{N_a} + \frac{1}{N_d} \right)} \]

- This region is essentially depleted of all mobile charge
- Due to high electric field, carriers move across region at velocity saturated speed

\[ X_{d0} = \sqrt{\frac{2\varepsilon \phi_{bi}}{q} \left( \frac{1}{10^{15}} \right)} \approx 1 \mu \]

\[ E_{pm} = \frac{1V}{1\mu} = 10^4 \frac{V}{cm} \]
Have we invented a battery?

- Can we harness the PN junction and turn it into a battery?

\[ \phi_{hi} \equiv \phi_n - \phi_p = V_{th} \left( \ln \frac{N_D}{n_i} + \ln \frac{N_A}{n_i} \right) = V_{th} \ln \frac{N_D N_A}{n_i^2} \]

- Numerical example:

\[ \phi_{ui} = 26 \text{mV} \ln \frac{N_D N_A}{n_i^2} = 60 \text{mV} \times \log \frac{10^{15} \times 10^{15}}{10^{20}} = 600 \text{mV} \]

Contact Potential

- The contact between a PN junction creates a potential difference
- Likewise, the contact between two dissimilar metals creates a potential difference (proportional to the difference between the work functions)
- When a metal semiconductor junction is formed, a contact potential forms as well
- If we short a PN junction, the sum of the voltages around the loop must be zero:

\[ 0 = \phi_{hi} + \phi_{pm} + \phi_{mn} \]

\[ \phi_{hi} = - (\phi_{pm} + \phi_{mn}) \]
PN Junction Capacitor

- Under thermal equilibrium, the PN junction does not draw any (much) current
- But notice that a PN junction stores charge in the space charge region (transition region)
- Since the device is storing charge, it’s acting like a capacitor
- Positive charge is stored in the n-region, and negative charge is in the p-region:
  \[ qN_d x_{po} = qN_a x_{no} \]

Reverse Biased PN Junction

- What happens if we “reverse-bias” the PN junction?
  \[ -\phi_w^+ + V_D = V_D \]
  \[ V_D < 0 \]
- Since no current is flowing, the entire reverse biased potential is dropped across the transition region
- To accommodate the extra potential, the charge in these regions must increase
- If no current is flowing, the only way for the charge to increase is to grow (shrink) the depletion regions
Voltage Dependence of Depletion Width

- Can redo the math but in the end we realize that the equations are the same except we replace the built-in potential with the effective reverse bias:

\[ x_n(V_D) = \sqrt{\frac{2\varepsilon_s (\phi_{bi} - V_D)}{qN_d}} \left( \frac{N_a}{N_a + N_d} \right) = x_{n0} \sqrt{\frac{1 - V_D}{\phi_{bi}}} \]

\[ x_p(V_D) = \sqrt{\frac{2\varepsilon_s (\phi_{bi} - V_D)}{qN_a}} \left( \frac{N_d}{N_a + N_d} \right) = x_{p0} \sqrt{\frac{1 - V_D}{\phi_{bi}}} \]

\[ X_d(V_D) = x_p(V_D) + x_n(V_D) = x_{d0} \sqrt{\frac{2\varepsilon_s (\phi_{bi} - V_D)}{q}} \left( \frac{1}{N_a} + \frac{1}{N_d} \right) \]

\[ X_d(V_D) = X_{d0} \sqrt{1 - \frac{V_D}{\phi_{bi}}} \]

Charge Versus Bias

- As we increase the reverse bias, the depletion region grows to accommodate more charge

\[ Q_J(V_D) = -qN_a x_p(V_D) = -qN_a \sqrt{1 - \frac{V_D}{\phi_{bi}}} \]

- Charge is not a linear function of voltage
- This is a non-linear capacitor
- We can define a small signal capacitance for small signals by breaking up the charge into two terms

\[ Q_J(V_D + \nu_D) = Q_J(V_D) + q(\nu_D) \]
Derivation of Small Signal Capacitance

- From last lecture we found
  \[ Q_j(V_D + v_D) = Q_j(V_D) + \frac{dQ_j}{dV}
  \bigg|_{V_D} v_D + \cdots \]
  \[ C_j = C_j(V_D) = \left. \frac{dQ_j}{dV} \right|_{V = V_D} = \frac{d}{dV} \left( -qN_a x_{p0} \sqrt{1 - \frac{V}{\phi_{bi}}} \right) \bigg|_{V = V_D} \]
  \[ C_j = \frac{qN_a x_{p0}}{2\phi_{bi} \sqrt{1 - \frac{V_D}{\phi_{bi}}}} = C_{j0} \frac{N_a N_d}{2\phi_{bi} N_a + N_d} \]

- Notice that
  \[ C_{j0} = \frac{qN_a x_{p0}}{2\phi_{bi}} \]
  \[ = \frac{qN_a}{2\phi_{bi}} \left( \frac{2\epsilon_s \phi_{bi}}{qN_a} \right) \left( \frac{N_d}{N_a + N_d} \right) \]
  \[ = \frac{\epsilon_s N_a N_d}{2\phi_{bi} N_a + N_d} \]

Physical Interpretation of Depletion Cap

- Notice that the expression on the right-hand-side is just the depletion width in thermal equilibrium
  \[ C_{j0} = \epsilon_s \sqrt{\frac{q}{2\epsilon_s \phi_{bi}} \left( \frac{1}{N_a} + \frac{1}{N_d} \right)^{-1}} = \frac{\epsilon_s}{X_{d0}} \]

- This looks like a parallel plate capacitor!
  \[ C_j(V_D) = \frac{\epsilon_s}{X_d(V_D)} \]
A Variable Capacitor (Varactor)

- Capacitance varies versus bias:

- Application: Radio Tuner

“Diffusion” Resistor

- Resistor is capacitively isolation from substrate
  - Must Reverse Bias PN Junction!
Diode under Thermal Equilibrium

- Diffusion small since few carriers have enough energy to penetrate barrier
- Drift current is small since minority carriers are few and far between: Only minority carriers generated within a diffusion length can contribute current
- Important Point: Minority drift current independent of barrier!
- Diffusion current strong (exponential) function of barrier

Reverse Bias

- Reverse Bias causes an increased barrier to diffusion
- Diffusion current is reduced exponentially
- Drift current does not change
- Net result: Small reverse current
**Forward Bias**

- Forward bias causes an exponential increase in the number of carriers with sufficient energy to penetrate barrier
- Diffusion current *increases* exponentially

![Diode schematic](image)

- Drift current does not change
- Net result: Large forward current

**Diode I-V Curve**

- Diode IV relation is an exponential function
- This exponential is due to the Boltzmann distribution of carriers versus energy
- For reverse bias the current saturations to the drift current due to minority carriers

\[
I_d (V_d \to -\infty) = -I_S \\
I_d = I_s \left( \frac{qV_d}{kT} - 1 \right)
\]
Minority Carriers at Junction Edges

Minority carrier concentration at boundaries of depletion region increase as barrier lowers … the function is

\[
\frac{p_n(x = x_n)}{p_p(x = -x_p)} = \frac{\text{(minority) hole conc. on n-side of barrier}}{\text{(majority) hole conc. on p-side of barrier}} = e^{-(\text{Barrier Energy})/kT}
\]

\[
\frac{p_n(x = x_n)}{N_A} = e^{-q(\phi_B - V_D)/kT}
\]  
(Boltzmann’s Law)

“Law of the Junction”

Minority carrier concentrations at the edges of the depletion region are given by:

\[
p_n(x = x_n) = N_A e^{-q(\phi_B - V_D)/kT}
\]

\[
n_p(x = -x_p) = N_D e^{-q(\phi_B - V_D)/kT}
\]

Note 1: \(N_A\) and \(N_D\) are the majority carrier concentrations on the other side of the junction

Note 2: we can reduce these equations further by substituting \(V_D = 0\) V (thermal equilibrium)

Note 3: assumption that \(p_n << N_D\) and \(n_p << N_A\)
**Minority Carrier Concentration**

The minority carrier concentration in the bulk region for forward bias is a decaying exponential due to recombination.

\[
\begin{align*}
P_n(x) = P_{n0} + P_{e0} \left( e^{\frac{qV}{kT} - 1} \right) e^{\frac{qV_x}{kT}}
\end{align*}
\]

**Steady-State Concentrations**

Assume that none of the diffusing holes and electrons recombine \( \Rightarrow \) get straight lines …

This also happens if the minority carrier diffusion lengths are much larger than \( W_{n,p} \)

\[
L_{n,p} \gg W_{n,p}
\]
Diode Current Densities

\[
\frac{dn_{p}}{dx}(x) = n_{p0} e^{\frac{qV}{kT}} - n_{p0} \frac{1}{-x_{p} - (-W_{p})}
\]

\[
n_{p0} = \frac{n_{i}^{2}}{N_{c}}
\]

\[
J_{n}^{\text{diff}} = qD_{n} \frac{dn_{n}}{dx} \bigg|_{x=x_{n}} = q \frac{D_{n}}{W_{p}} n_{p0} \left( e^{\frac{qV_{p}}{kT}} - 1 \right)
\]

\[
J_{p}^{\text{diff}} = -qD_{p} \frac{dp_{p}}{dx} \bigg|_{x=x_{n}} = -q \frac{D_{p}}{W_{n}} p_{n0} \left( 1 - e^{\frac{qV_{p}}{kT}} \right)
\]

\[
J^{\text{diff}} = qn_{i}^{2} \left( \frac{D_{p}}{N_{d} W_{n}} + \frac{D_{n}}{N_{c} W_{p}} \right) \left( e^{\frac{qV_{p}}{kT}} - 1 \right)
\]

Fabrication of IC Diodes

- Start with p-type substrate
- Create n-well to house diode
- p and n+ diffusion regions are the cathode and anode
- N-well must be reverse biased from substrate
- Parasitic resistance due to well resistance
Diode Small Signal Model

- The I-V relation of a diode can be linearized

\[ I_D + i_D = I_S \left( e^{\frac{q(V_D + V_x)}{kT}} - 1 \right) \approx I_S e^{\frac{qV_x}{kT}} e^{\frac{qV_D}{kT}} \]

\[ e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \]

\[ I_D + i_D = I_D \left( 1 + \frac{q(V_D + V_x)}{kT} + \cdots \right) \]

\[ i_D = \frac{qV_D}{kT} = g_D V_D \]

Diode Capacitance

- We have already seen that a reverse biased diode acts like a capacitor since the depletion region grows and shrinks in response to the applied field. The capacitance in forward bias is given by

\[ C_j = A \frac{e_S}{X_{dep}} \approx 1.4C_{j0} \]

- But another charge storage mechanism comes into play in forward bias
- Minority carriers injected into p and n regions “stay” in each region for a while
- On average additional charge is stored in diode
Charge Storage

- Increasing forward bias increases minority charge density
- By charge neutrality, the source voltage must supply equal and opposite charge
- A detailed analysis yields:

\[ C_d = \frac{1}{2} \frac{qI}{kT} \tau \]

Time to cross junction (or minority carrier lifetime)

Diode Circuits

- Rectifier (AC to DC conversion)
- Average value circuit
- Peak detector (AM demodulator)
- DC restorer
- Voltage doubler / quadrupler /...