

LTI Systems

Linear Time Invariant System

$$x_1(t) \rightarrow y_1(t)$$

$$x_1(t+\tau) \rightarrow y_1(t+\tau)$$

$$x_1(t) \rightarrow [H(s)] \rightarrow y_1(t)$$

$$a_1 x_1(t) + a_2 x_2(t) \rightarrow [H(s)] \rightarrow a_1 y_1(t) + a_2 y_2(t)$$

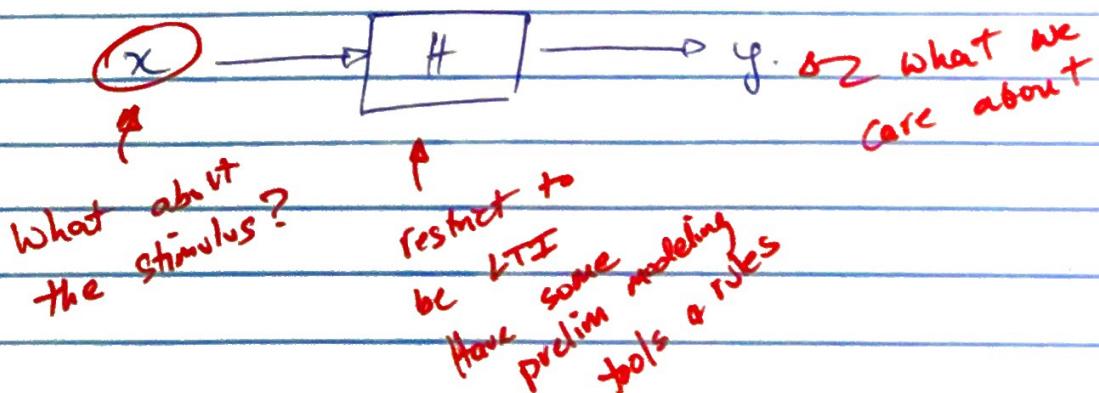
Linear: Can scale & sum

Time-Invariant: Output does not depend on time of a particular signal.

What's nice about LTI Systems? Time \leftrightarrow frequency

- # Can use superposition
- # Convenient conversion between functional & vector representations
- # Most systems in real life are LTI Systems.
 - Focus of this class.

How do we study & model LTI systems?



What is the stimulus?

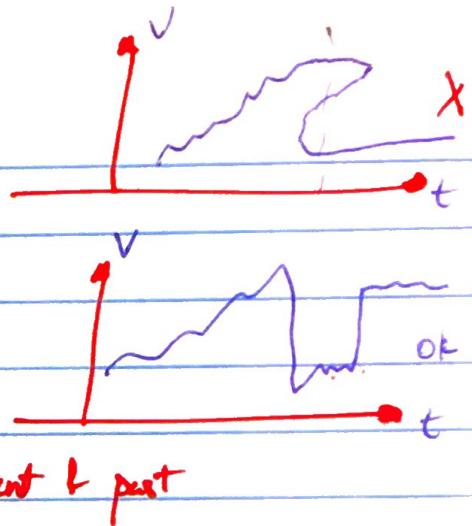
Can be anything

* Swift kick (ie unit step)

* A sharp jab (ie impulse)

* Sinusoidal/exponential

* Enforce causality ↗



depends on present & past

Useful tool to "convert" or represent differently the
Stimulus: **Fourier Series** / **Fourier Transform**

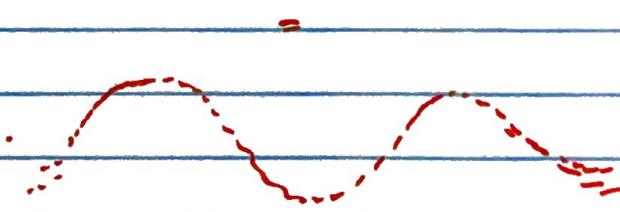
Can represent most signals as sum or
integral of sinusoids.



Some LTI

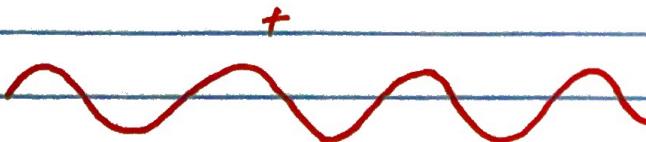
System
by \downarrow

y_1



LTI

Phase shift
Amp shift
attenuation



LTI

attenuation

LTI System alters phase & freq amplitude



What have we accomplished so far?

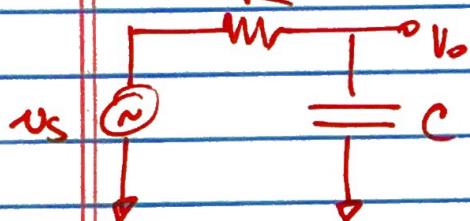
* Can model the system as LTI

* Have some model for stimulus as superposition of sinusoids

* Can change from 1 domain to another freely.

Example:

Use your favorite method to analyze:



$$v_o(t) = v_s(t) - i(t) \cdot R$$

$$i(t) = C \frac{dv_o}{dt}$$

$$v_o(t) = v_s(t) - R \cdot C \frac{dv_o}{dt}$$

differential
eqn.

$$v_s(t) = v_o(t) + \omega \frac{dv_o}{dt}$$

Represent sinusoid as exponentials

Euler's formula

$$e^{j\theta} = \cos(\theta) + j \sin(\theta).$$

$$\cos(\theta) = \frac{1}{2} (e^{j\theta} + e^{-j\theta})$$

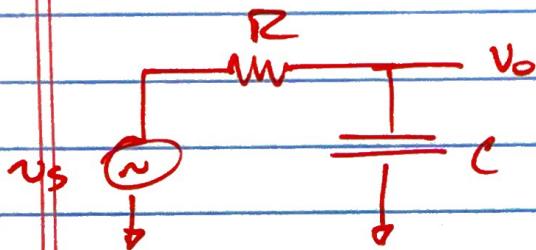
$$\sin(\theta) = \frac{1}{2j} (e^{j\theta} - e^{-j\theta}).$$

Use conversion: Turn ODE into algebra!

$$\frac{d}{dt} e^{j\omega t} = j\omega e^{j\omega t}$$

$$\int e^{j\omega t} dt = \frac{1}{j\omega} e^{j\omega t}$$

Return to R-C Example:



$$V_S(t) = V_0(t) + j \frac{dV_0}{dt}$$

Represent $V_S(t)$ as exponential

$$V_S(t) = V_s e^{j\omega t}$$

Capacitor Impedance :

$$\frac{1}{j\omega C} \rightarrow Z_C$$

$$V_o = \frac{Z_C}{R + Z_C} \cdot V_S = \frac{1/j\omega C}{R + 1/j\omega C} \cdot V_S = \frac{1}{1 + j\omega RC} \cdot V_S$$

Can go back & forth between time & frequency domains using Fourier

$$i = C \frac{dV_o}{dt}, \quad V_o = V_o e^{j\omega t} \quad i = I_o e^{j\omega t}$$

$$\Rightarrow I_o e^{j\omega t} = C j\omega V_o e^{j\omega t}$$

$$Z_C = \frac{V_o}{I_o} = \frac{1}{j\omega C}$$

Similarly, Inductor:

$$\boxed{Z_L = j\omega L}$$

from $\frac{L di}{dt} = V$

$$\text{Transfer Function: } \frac{V_o}{V_i} = \frac{1}{1 + j\omega RC}$$

Convenient and intuitive metrics: Magnitude \rightarrow to Sinusoidal wave

Phase \rightarrow Going back

If transfer function: $A + B\omega$

Mag: $|H(s)| = \frac{C + D\omega}{\sqrt{R^2 E_N \omega^2 + I_m^2 E_N \omega^2}}$

Phase: $\Rightarrow H(s) = \tan^{-1}\left(\frac{I_m E_N \omega}{R E_N \omega}\right) - \tan^{-1}\left(\frac{I_m D\omega}{R E_N \omega}\right)$.

Going back: $H(\omega) = \frac{V_o}{V_s} = \frac{1}{1 + j\omega RC}$.

$$|H(\omega)| = \left| \frac{V_o}{V_s} \right| = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

$$\Rightarrow H(\omega) = -\tan^{-1}(\omega RC)$$

What does this mean?

V plane domain
↔ $j\omega$
interchangeable

$$x \rightarrow [H] \rightarrow y$$

Ex: $e^{j\omega t} \rightarrow [H] \rightarrow |H(\omega)| \cdot e^{j(\omega t + \phi)}$.

LTI property $e^{-j\omega t} \rightarrow [H] \rightarrow |H(-\omega)| \cdot e^{j(-\omega t + \phi)}$.

$$\frac{e^{j\omega t} + e^{-j\omega t}}{2} \rightarrow [H] \rightarrow \frac{|H(\omega)| e^{j(\omega t + \phi)} + H(-\omega) e^{j(-\omega t + \phi)}}{2}$$

Works for all harmonics! AGAIN, becomes algebra.

So far, transfer function working w/ $V_{in} \rightarrow V_o$

Sometimes care about $I \rightarrow V / V \rightarrow I$

Trans Impedance: $\frac{V_o}{I_{in}} = |V_o| e^{j(\phi_v - \phi_i)}$.

Trans Admittance: $\frac{I_o}{V_{in}} = |I_o| e^{j(\phi_i - \phi_v)}$.

I/V Gain: $\frac{I_o}{I_{in}}$ or $\frac{V_o}{V_{in}}$.

↳ unitless

Summary to now:

- # Convert between time & freq domains
- # One usually easier
- # In freq domain, caps/ind become impedances
 - use node analysis etc. to analyze