EE 105 Announcement

* Midterm Next Thu 9/28/17
  in Class [3:40 → 5:00]

* Closed Book

* One page allowed

* Bring you calculator

* Practice Midterm will be posted today

* Discussion of solution in Discussion Session next week

* No HW, but one problem, for practice

* No Lab next week
\[ \sigma = \oint \mathbf{E} \cdot d\mathbf{A} = \frac{\epsilon_0}{\epsilon_s} \]

\[ \epsilon_s = \epsilon_{r,sc} \epsilon_0 \]

\[ \epsilon_{r,sc} \approx 13 \quad 8.854 \times 10^{-14} \, \text{F/um} \]

Depletion region
fully depleted, electrons, hole
\[ E \leftrightarrow \rightarrow (\neg) \]

\[ E_{\text{max}} = \frac{8 N_A x_p}{\epsilon_s} \]

\[ V_1 = \frac{1}{2} \frac{8 N_A x_p^2}{\epsilon_s} \quad V_2 = \frac{8 N_D x_n^2}{2 \epsilon_s} \]

\[ V_0 = V_1 + V_2 = \frac{8}{2 \epsilon_s} \left( N_A x_p^2 + N_D x_n^2 \right) \]

\[ x_n = \frac{N_A}{N_A + N_D} \cdot W \]

\[ x_p = \frac{N_D}{N_A + N_D} \cdot W \]

Total depletion width
\[ V_0 = \frac{q}{2\varepsilon S} \cdot \frac{N_A N_D^2 + N_A^2 N_D}{(N_A + N_D)^2} \cdot W^2 = \frac{q}{2\varepsilon S} \cdot \frac{N_A N_D}{N_A + N_D} \cdot W^2 \]

Hole conc follows Boltzman distribution
\[ p(x) = p_0 e^{\frac{-\phi(x)}{kT}} = p_0 e^{\frac{-\phi}{kT}} \]

Just solve \( \phi(x) = -\int_{-\infty}^{x} E \cdot dx \)
\[ \phi(x) = \begin{cases} 0 & x < -x_p \text{ (p side)} \\ V_0 & x > x_n \text{ (n side)} \end{cases} \]

On p-side:
\[ E \bigg|_p \bigg| \approx p(x) = p_0 e^{\frac{x}{kT}} = p_0 = N_A = \text{doping conc.} \]

On n-side:
\[ E \bigg|_n \bigg| \approx n(x) = \frac{N_c^2}{N} = \frac{N_c^2}{N_D} \]
\[ n \cdot p = n_c^2 \quad \text{Mass Action Law} \]

\[ \frac{1}{2} : \quad e^{\frac{V_0}{kT}} = \frac{N_A N_D}{n_c^2} \quad \uparrow \]

\[ \ln : \quad V_0 = V_T \cdot \ln \left( \frac{N_A N_D}{n_c^2} \right) \quad \uparrow \]
\[ \text{Built-in Potential of pn junction} \]
\[ \Rightarrow \text{only dep. on doping conc.} \]

\[ W = \frac{2\varepsilon S}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) \cdot V_0 \quad \text{solved W} \quad \uparrow \Rightarrow \text{solve } x_p, x_n \]
Depletion Width Under Bias

\[ W = \sqrt{\frac{2e_S}{q}} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) (V_0 - V) + |V_R| \]

\( V \) is the applied voltage to the pn junction, it's positive for forward bias and negative for reverse bias. Depletion width is widened in reverse bias.
\[
I_D = I_S \left( e^{\frac{V_F}{V_T}} - 1 \right)
\]

\[I_0 = e^{\frac{V_F}{V_T}} = e^{28} \approx 10^{14}\]

\[V_F = 0.7V, \quad V_T = 25mV \Rightarrow \frac{1}{V_T} = 40 \quad [V]\]

\[I_S \sim 10^{-16} \text{ Amp (typical)} \quad I_0 = 10mA\]
Under forward bias, minority carriers at the edge of depletion region is boosted up by \( e^{V/V_T} - 1 \):

\[
p_n(x) = p_{n0} + p_{n0} \left( e^{V/V_T} - 1 \right) e^{x-x_p} \]

\( L_p \): hole diffusion length in n-type

Hole diffusion current density on n-side

\[
J_p = -q D_p \frac{dp_n(x)}{dx} \bigg|_{x=x_p} = q \frac{D_p}{L_p} p_{n0} \left( e^{V/V_T} - 1 \right)
\]

Similarly, electron diffusion current density in p-side

\[
J_n = q D_n \frac{dn_p(x)}{dx} \bigg|_{x=-x_p} = q \frac{D_n}{L_n} n_{p0} \left( e^{V/V_T} - 1 \right)
\]

Total current:

\[
I = (J_p + J_n) A = A \left( q \frac{D_p}{L_p} p_{n0} + q \frac{D_n}{L_n} n_{p0} \right) \left( e^{V/V_T} - 1 \right)
\]

\[
I = I_S \left( e^{V/V_T} - 1 \right) \]

where

\[
I_S = A q n_i^2 \left( \frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A} \right)
\]
I-V Curve

\[ I = I_S \left( e^{V/V_T} - 1 \right) \]

where

\[ I_S = A q n_i^2 \left( \frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A} \right) \]
Capacitance in pn Junction (1): Depletion Capacitance (Mainly Reverse Bias)

Parallel plate capacitance:
\[ C_j = \frac{\varepsilon_s A}{W} \]

Plate separation, \( W \), is voltage dependent:
\[ W = \sqrt{\frac{2\varepsilon_s}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) (V_0 + V_R)} \]

Variable capacitance:
\[ C_j(V) = \frac{C_{j0}}{\sqrt{1 + \frac{V_R}{V_0}}} \]

\[ C = \frac{\varepsilon A}{d} = \frac{\varepsilon_s A}{W(V_R)} \left\langle \text{Variable Capacitor} \right\rangle \]
\[ \omega_0 = \frac{1}{\sqrt{LC}} \]
Capacitance in pn Junction (2): Diffusion Capacitance (Forward Bias)

Extra minority carriers stored outside junction under forward bias

\[ Q_p = Aq \times \text{shaded area under } p_n(x) \]

\[ Q_p = Aq \int_{x_n}^{\infty} p_{n0} \left( e^{V/V_T} - 1 \right) \cdot e^{-\frac{x-x_n}{L_p}} \, dx \]

\[ Q_p = AqL_p p_{n0} \left( e^{V/V_T} - 1 \right) = \frac{L_p^2}{D_p} I_p = \tau_p I_p \]

\[ \frac{L_p^2}{D_p} \] has unit of time, its physical meaning is minority carrier lifetime: \[ \tau_p = \frac{L_p^2}{D_p} \]

Similarly, \[ Q_n = \tau_n I_n \]

Total charge stored: \[ Q = \tau_p I_p + \tau_n I_n = \tau_T I \]

\[ \tau_T \] is mean transit time

These stored charges correspond to another nonlinear capacitor call "diffusion capacitance":

\[ C_d = \frac{dQ}{dV} = \frac{d(\tau_T I)}{dV} = \tau_T \frac{dI}{dV} \quad \Rightarrow \quad C_d = \left( \frac{\tau_T}{V_T} \right) I \]
Summary of pn Junction

Built-in potential: \( V_0 = V_T \ln \left( \frac{N_A N_D}{n_i^2} \right) \)

Under forward bias:

I-V curve: \( I = I_S \left( e^{V/V_T} - 1 \right) \)

Diffusion capacitance: \( C_d = \left( \frac{\tau_T}{V_T} \right) I \) (x)

Under reverse bias:

Negligible current, \( I = -I_S \)

Depletion capacitance: \( C_j = \frac{C_{j_0}}{\sqrt{1 + \frac{V_R}{V_0}}} \)

Other important parameter:

Depletion Width: \( W = \sqrt{\frac{2e_S}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) (V_0 - V)} \)
Appendix

• Rigorous derivation of pn junction potential
• Rigorous derivation of junction capacitance
Rigorous Derivation of pn Junction Potential

\[ E(x) = \begin{cases} \frac{-qN_A(x+x_p)}{\varepsilon_s}, & -x_p < x < 0 \\ \frac{qN_D(x-x_n)}{\varepsilon_s}, & 0 < x < x_n \end{cases} \]

\[ V(x) = -\int_{-x_p}^{x} E(x')dx' \]

(1) for \(-x_p < x < 0\) : \(V(x) = -\int_{-x_p}^{0} E(x')dx' = \int_{-x_p}^{0} \frac{qN_A(x'+x_p)}{\varepsilon_s} dx' = \frac{qN_A}{2\varepsilon_s} (x'+x_p)^2 \]

(2) for \(0 < x < x_n\) : Because \(E(x)\) has different expression for \(x < 0\) and \(x > 0\), the integration should be performed in two separate ranges, first from \(-x_p\) to 0, and then from 0 to \(x\). We can use \(V(x=0)\) from the above equation for the first integration. Therefore,

\[ V(x) = \frac{qN_A}{2\varepsilon_s} x_p^2 - \int_{0}^{x} \frac{qN_D(x'-x_n)}{\varepsilon_s} dx' = \frac{qN_A}{2\varepsilon_s} x_p^2 - \frac{qN_D(x'-x_n)^2}{2\varepsilon_s} \]

\[ = \frac{qN_A}{2\varepsilon_s} x_p^2 - \left( \frac{qN_D(x-x_n)^2}{2\varepsilon_s} - \frac{qN_Dx_n^2}{2\varepsilon_s} \right) = \frac{qN_A}{2\varepsilon_s} x_p^2 + \frac{qN_Dx_n^2}{2\varepsilon_s} - \frac{qN_D(x-x_n)^2}{2\varepsilon_s} \]

Built-in potential : \(V_0 = V(x_n) = \frac{q}{2\varepsilon_s} \left( N_Ax_p^2 + N_Dx_n^2 \right)\)
Rigorous Derivation of Junction Capacitance

Total charge $Q_j$ in depletion width at $V = -V_R$

$$Q_j = AqN_D x_n = AqN_D \frac{N_A}{N_A + N_D} W$$

$$W = \sqrt{\frac{2\varepsilon_s}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right)(V_0 + V_R)}$$

As bias voltage change, the amount of charge in the junction change. This is a "nonlinear" capacitor. The capacitance value is

$$C_j = \frac{dQ_j}{dV_R} = Aq \frac{N_D N_A}{N_A + N_D} \sqrt{\frac{2\varepsilon_s}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right)} \frac{d}{dV} \sqrt{(V_0 + V_R)}$$

$$C_j = A \sqrt{\frac{\varepsilon_s q}{2} \left( \frac{N_A N_D}{N_A + N_D} \right)} \frac{1}{\sqrt{(V_0 + V_R)}} \quad \text{Note:} \quad C_j = \frac{\varepsilon_s A}{W}$$

At zero bias, $V_R = 0$

$$C_{j_0} = A \sqrt{\frac{\varepsilon_s q}{2} \left( \frac{N_A N_D}{N_A + N_D} \right)} \frac{1}{\sqrt{V_0}}$$

Therefore at $V = -V_R$, \[ C_j = \frac{C_{j_0}}{\sqrt{1 + \frac{V_R}{V_0}}} \]

This is a variable capacitor, controllable by voltage.