Terminal Gain and I/O Resistances of MOS Amplifiers

<table>
<thead>
<tr>
<th>Common Source (CS)</th>
<th>Common Drain (CD)</th>
<th>Common Gate (CG)</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="CS Diagram" /></td>
<td><img src="image" alt="CD Diagram" /></td>
<td><img src="image" alt="CG Diagram" /></td>
</tr>
</tbody>
</table>

### Common Source (CS)

\[
A_{V,t} = - \frac{g_m R_L}{1 + g_m R_S} \\
R_i = \infty \\
R_o = \left[ r_o \left( 1 + g_m R_E \right) \right] \\
A_{I,t} = \infty
\]

Without degeneration:

Simply set \( R_S = 0 \)

### Common Drain (CD)

\[
A_{V,t} = \frac{R_L}{1 + g_m R_o} \\
R_i = \infty \\
R_o = \frac{1}{g_m} \\
A_{I,t} = \infty
\]

### Common Gate (CG)

\[
A_{V,t} = g_m R_L \\
R_i = \frac{1}{g_m} \\
R_o = \left[ r_o \left( 1 + g_m R_E \right) \right] \\
A_{I,t} \approx 1
\]

For the gain, \( R_i \), \( R_o \) of the whole amplifier, you need to include voltage/current dividers at input and output stages.
## Summary of MOS Single-Transistor Amplifiers

<table>
<thead>
<tr>
<th>MOS</th>
<th>Common Source</th>
<th>Common Source with Deg.</th>
<th>Common Drain</th>
<th>Common Gate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_i$</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>Small</td>
</tr>
<tr>
<td>$R_o$</td>
<td>Large</td>
<td>Very Large</td>
<td>Small</td>
<td>Large</td>
</tr>
<tr>
<td>$A_v$</td>
<td>Moderate</td>
<td>Small</td>
<td>$\sim 1$</td>
<td>Moderate</td>
</tr>
<tr>
<td>$f_H$</td>
<td>Small</td>
<td>Moderate</td>
<td>Large</td>
<td>Large</td>
</tr>
</tbody>
</table>
Single Stage Amplifier Cannot Meet All Requirements

• For example, a general purpose operational amplifier requires
  – High input resistance ~ 1MΩ
  – Low output resistance ~ 100Ω
  – High voltage gain ~ 100,000

• No single transistor amplifier can satisfy all spec’s

• Cascading multiple stages of amplifiers offers a path towards the design
Multistage Amplifiers

• Usually
  – An input stage to provide required input resistance
  – Middle stage(s) to provide gain
  – An output stage to provide required output resistance or drive external loads

• More gain!
  – Gain/stage limited, especially in nanoscale devices

• Improve Bandwidth
  – De-couple high impedance nodes from large capacitors

• DC coupling (no passive elements to block the signal)
  – Use amplifiers to naturally “level shift” signal
Impedance “Match”

- On-chip circuits often use “voltage/current” matching to minimize loading.
- Keep in mind the input resistance and output resistance of each type of stage so that the loading does not create an undesired effect.

<table>
<thead>
<tr>
<th>Amplifier Type</th>
<th>Ideal $R_{in}$</th>
<th>Ideal $R_{out}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage Amplifier</td>
<td>$\infty$</td>
<td>0</td>
</tr>
<tr>
<td>Current Amplifier</td>
<td>0</td>
<td>$\infty$</td>
</tr>
<tr>
<td>Transconductance Amplifier</td>
<td>$\infty$</td>
<td>$\infty$</td>
</tr>
<tr>
<td>Transresistance Amplifier</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Two-Stage Voltage Amplifier

- Boost gain by cascading Common-Source stages

Can combine into a single 2-port model

Results of new 2-port: $R_{in} = R_{in1}$, $R_{out} = R_{out2}$
Results of new 2-port:

\[ R_{in} = R_{in1} = \]
\[ R_{out} = R_{out2} = \]
\[ A_V = \frac{v_{out}}{v_{in}} = \]
CS Cascade Bandwidth

Two time constants:

\[ \tau_1 = \]

\[ \tau_2 = \]
Bandwidth Extension

- Common Source stage has high gain, but low bandwidth
- Note that Miller effect is the culprit
- Follower stage can buffer source resistance from Miller cap
Bandwidth Extension Using Source Follower (SF)
CS Example with Cap Load

- $C_{in}$ and $C_S$ are very large, therefore they look like short circuits to the AC signal.

- If $C_L$ is very large, its pole dominates, let’s analyze
CS with Cap Load – Small Signal

- What are the time constants associated with the capacitors in this circuit?
- What can we do if we have to drive a large \( C_L \)?
How can we reduce the impact of $C_L$?

One way is to reduce the resistance $R_d$, but this reduces our low-frequency gain.

To recover the gain we can increase $g_{m1}$. What does this cost us?
A better way to extend the bandwidth is to add a source-follower stage.

Similar to previous example.
By adding a CD (Source Follower) we can increase the bandwidth

It costs us power for the CD stage

Remember that increasing the BW by increasing $g_{m1}$ costs us much more
CS + CG

- Common source provides gain, CG acts as a buffer, but is it even helping?
- How do you bias this circuit?
Merged CS + CG = Cascode

- Let’s apply 2-port small-signal analysis

In this case, we care about the input current to the second stage

Note that the input resistance of the CG is low, therefore the majority of the CS current is fed to the CG

- $A_v =$
Cascode Bandwidth

- Draw in the $C_{gs}$ and $C_{gd}$ capacitors.
- Which ones are Miller effected?
- Is this better or worse than a CS without a CG?
Cascode Bandwidth

- Draw in the capacitors and input resistance

\[ i_{out} \quad \quad i_{in} \]
\[ r_o || r_{oc} \quad V_{int} \quad \frac{1}{g_m} \]
\[ -i_{in} \quad R_D || (r_o + g_m r_o R_s) \]
Cascode Biasing

- CG has a very large output resistance
- Loading it with $R_D$ is likely to reduce the voltage gain
- We can increase the gain by using a current source load, but $r_{oc}$ needs to be very large. Can use a cascode current mirror!
Complete Amplifier Design

Goals: $g_{m1} = 1 \text{ mS}, R_{out} = 5 \text{ M}\Omega$

For simplicity, let’s assume all $g_m$ and $r_o$ values are equal

$A_V \approx -g_{m1}R_{out} = -1\text{ mS} \times 5\text{ M}\Omega = -5,000$

$R_{out} \approx \frac{1}{2} g_m r_o^2 = 5\text{ M}\Omega$

$r_o = \sqrt{\frac{20\text{ M}\Omega}{g_m}} = \sqrt{\frac{10\text{ M}\Omega}{1\text{ mS}}} = 100\text{k}\Omega$
Bias Current & Device Sizing

Need to know process parameters to solve for W/L

\[ k' = 100 \, \mu A/V^2 \]
\[ \lambda = 0.1 \, [V^{-1}] \]

\[
\begin{align*}
  r_o &= \frac{1}{\lambda I_{DS}} = 100k\Omega \\
  I_{DS} &= \frac{1}{0.1V^{-1} \times 100k\Omega} = 100 \mu A \\
  g_m &= \sqrt{2k'(\frac{W}{L})} I_{DS} = 1mS \\
  \frac{W}{L} &= \frac{g_m^2}{2k'I_{DS}} = \frac{(1mS)^2}{2 \times 100\mu \times 100\mu A} = 50
\end{align*}
\]
Output (Voltage) Swing

Need to know $V_{GS} - V_T$ (e.g. $V_{DSAT}$, $V_{OV}$)

$$g_m = \frac{2I_{DS}}{V_{GS} - V_T} = 1mS$$

$$V_{GS} - V_T = \frac{2I_{DS}}{g_m} = \frac{2 \times 100 \mu A}{1mS} = 0.2V$$

Maximum $V_{OUT} =$
Minimum $V_{OUT} =$
Input Bias $V_{IN} =$