Differential & Common Mode Signals

- Consider positive and negative input terminal signals \( V_i^+ \) and \( V_i^- \)
- Define differential signal as: \( V_{id} = V_{in}^+ - V_{in}^- \)
- Define common mode signal as: \( V_{ic} = \frac{(V_{in}^+ + V_{in}^-)}{2} \)
- We can create arbitrary \( V_i^+ \) and \( V_i^- \) signals from differential and common mode components:
  \[
  V_{in}^+ = V_{ic} + \frac{1}{2} V_{id} \\
  V_{in}^- = V_{ic} - \frac{1}{2} V_{id}
  \]
- This also applies to differential output signals:
  \[
  V_{o}^+ = V_{oc} + \frac{1}{2} V_{od} \\
  V_{o}^- = V_{oc} - \frac{1}{2} V_{od}
  \]
Why Differential?

• Differential circuits are much less sensitive to noises and interferences

• Differential configuration enables us to bias amplifiers and connect multiple stages without using coupling or bypass capacitors

• Differential amplifiers are widely used in IC’s
  – Excellent matching of transistors, which is critical for differential circuits
  – Differential circuits require more transistors → not an issue for IC
Neural Recording

An array of electrodes is implanted in the motor cortex and senses extracellular signals that include firing from nearby neurons.

The propagation of signals from neuron to neuron is called an Action Potential, which is analogous to a digital “pulse.”
The goal of a neural recording device is to record the small-amplitude neural signals and pick out the meaningful signals from the “noise”.

These signals are then decoded to create trajectories, movements, and speeds for controlling prostheses, computers, etc.
60Hz and Other Interferers

- In reality, the tiny signals recorded from the brain can get corrupted by numerous interferers.
- Ambient 60Hz noise couples into electrical signals in and on the body.
- Motion can cause voltage artifacts from the movement of the electrodes relative to the neurons.
MOS Differential-Pair

Two matched MOS transistors
Common current bias
"Differential signals" applied to $v_{G1}$ and $v_{G2}$
(equal amplitude but opposite sign)
"Differential outputs" are produced
at $v_{D1}$ and $v_{D2}$

Note in differential configuration,
$V_{GS}$ is fixed for both Q$_1$ and Q$_2$

$I_{D1} = I_{D2} = \frac{I}{2}$

$\frac{I}{2} = \frac{k_n}{2} (V_{GS} - V_{m})^2$

$V_{GS} = V_{m} + \sqrt{\frac{I}{k_n}}$
Large Signal Behavior of Diff Mode Operation

\[ v_{id} = v_{in+} - v_{in-} = \left( V_{Tn} + \sqrt{\frac{I_1}{k_n}} \right) - \left( V_{Tn} + \sqrt{\frac{I_2}{k_n}} \right) \]

\[ v_{id} = \sqrt{\frac{1}{k_n}} - \sqrt{\frac{2}{k_n}} = \sqrt{\frac{I + \Delta i/2}{k_n}} - \sqrt{\frac{I - \Delta i/2}{k_n}} \]

\[ \Delta i = i_{D1} - i_{D2} \]

solve

\[ \Delta i = k_n v_{id} \sqrt{\frac{2I}{k_n} - v_{id}^2} \]

\[ v_{id} \leq \sqrt{\frac{I}{k_n}} \]
Common Mode Rejection

Differential Pair
Rejects Common-Mode Inputs

The common voltages applied to both Q₁ and Q₂ are referred to as common mode, \( V_{CM} \).

Common mode inputs usually come from noises or interferences.

Differential pair should reject \( V_{CM} \):

Since \( V_{GS1} = V_{GS2} = V_m + \sqrt{\frac{I}{k_n}} \)

is fixed in differential pair,

\( V_{CM} \) simply changes the voltage at Source, \( V_S \).

The drain currents remain fixed:

\[
I_1 = I_2 = \frac{I}{2}
\]

\[
v_{D1} = V_{DD} - \frac{I}{2} R_D = v_{D2}
\]

Differential output \( v_{D1} - v_{D2} = 0 \)
Common Mode Input Range

What is the maximum and minimum common-mode input voltage?
Small Signal Operation

\[ v_{G1} = V_{CM} + \frac{1}{2}v_{id}; \quad v_{G2} = V_{CM} - \frac{1}{2}v_{id} \]

AC equivalent circuit

For differential AC small signal, the differential pair is “anti-symmetric”. The potential at the mid point (Source) is zero. This is called “Virtual Ground”.

This virtual ground is obtained without using a large bypass capacitor → much smaller area and better frequency response.
Because the two halves of the circuits are anti-symmetric, and Source is at virtual ground, we can simply simplify and just analyze the "half circuit"
AC Equivalent Circuit for Common Mode Input

Non-ideal current source consists of an ideal current source shunted by a large resistance, \( R_{SS} \)
Common Mode “Half Circuit”

- For common-mode inputs, the two half circuits are symmetric. The Source is not virtual ground any more.

- $R_{SS}$ can be considered as two parallel combination of $2R_{SS}$.
- Each CM half circuit has $2R_{SS}$ connected to the source.
Ideal CM Output Voltage

The common-mode half-circuit is basically a common-source amplifier with source degeneration.

The gain is

\[ \frac{v_{o1}}{v_{icm}} = \frac{v_{o2}}{v_{icm}} = \frac{-R_D}{1/g_m + 2R_{SS}} \]

Since \( 2R_{SS} >> 1/g_m \),

\[ \frac{v_{o1}}{v_{icm}} = \frac{v_{o2}}{v_{icm}} \approx \frac{-R_D}{2R_{SS}} \]

\[ v_{od} = v_{o2} - v_{o1} = 0 \]

Output voltage is zero for ideal differential pair with perfectly matched transistors and resistors, and the CM voltage is small enough that \( Q_1 \) and \( Q_2 \) remains in Saturation.

Common-Source with degeneration
Useful Metric for Diff Amps: CMRR

- **Common Mode Rejection Ratio (CMRR)**
  - Define: $a_{vd}$: differential gain, $a_{vc}$: common mode gain
    \[
    \text{CMRR} = \left( \frac{a_{vd}}{a_{vc}} \right)
    \]
  - CMRR corresponds to ratio of differential to common mode gain and is related to received signal-to-noise ratio
    \[
    V_{od} = a_{vd} V_{sig} + a_{vc} V_{noise}
    \]
    \[
    \Rightarrow \frac{\text{Signal}}{\text{Noise}} = \left( \frac{a_{vd}}{a_{vc}} \right) \left( \frac{V_{sig}}{V_{noise}} \right) = \text{CMRR} \left( \frac{V_{sig}}{V_{noise}} \right)
    \]

Courtesy: M.H. Perrott
Differential Amplifier with Current-Source Loads

\[ Q_3 \text{ and } Q_4 \text{ are PMOS current sources (active loads)} \]

From half-circuit

\[ A_d = \frac{V_{od}}{V_{id}} = g_{m1} \left( r_{o1} \parallel r_{o3} \right) \]
Cascode Differential Amplifier

Cascode configurations for both amplifying transistors and current source loads.

From half-circuit

\[ A_d = \frac{v_{od}}{v_{id}} = g_m \left( R_{on} \parallel R_{op} \right) \]

\[ R_{on} = (g_m r_o) r_o \]

\[ R_{op} = (g_m r_o) r_o \]

If all transistors are identical,

\[ R_{on} = R_{op} = g_m r_o^2 \]

\[ A_d = \frac{1}{2} g_m r_o^2 \]
Differential Input, Single-End Output

Differential-in, Differential-out

Differential input, Single-ended Output
MOS Differential Pair with Current Mirror Load

AC equivalent circuit for differential input

Current mirror forces small-signal currents through Q₃ and Q₄ to be the same → output currents = 2x that of half circuit