Lecture 20: Small-Signal Analysis

- Announcements:
- HW#6 online and due Friday Oct. 12 via Gradescope
- Lab#4 (the experimental part) online very soon
- I hear that those who did not have access to 125 Cory now have access
- Midterm 1 Exams coming back today
  - I will hand them out with solutions in the last 15 minutes
  - Will show you Z-scores on Wednesday
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- Lecture Topics:
  - Small Signal Analysis
    - Linearizing Non-Linear Elements
    - DC and Small-Signal AC Components
    - Taylor Series Approximation
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- Last Time:
- Going through biasing for discrete MOS transistors
- Now, continue with this …

For practical amplifier requirements: compromise:

① $V_{BB} \approx \frac{1}{3} V_{CC}$

② $V_{CE} \approx \frac{1}{3} V_{CC}$

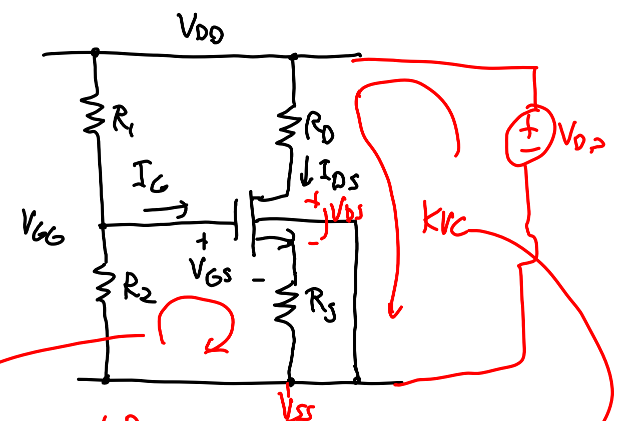
③ $V_{R_C} = I_C R_C \approx \frac{1}{3} V_{CC}$

④ $0.1 I_E < I_{BIAS} < I_E$

Good starting pt. … but not rules! → must adjust for a given design

MOS Biasing

⇒ can use a similar biasing strategy for discrete MOS ckts.



$V_{GG} = V_{DD}\left(\frac{R_2}{R_1+R_2}\right)$, $I_G = 0 \rightarrow V_G = V_{GG}$

KVL: $V_{GG} = V_{GS} + I_{DS} R_S$ (2)

$V_{DD} = I_{DS} R_D + V_{DS} + I_{DS} R_S + V_{SS}$ (1)

To find the DC operating point: (by hand)

① Assume saturation:

can often neglect

$$I_{DS} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_t)^2(1+\cancel{\lambda V_{DS}})$$

w/ $V_t = f(V_{SB}) = V_{to} + \gamma\left(\sqrt{2\phi_f - V_{SB}} - \sqrt{2\phi_f}\right)$

⇒ using (2): $V_{GG} = V_{GS} + \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_t)^2 R_S$ (3)

② Solve for V_{GS} assuming $V_t = V_{to}$.

③ $V_S = V_{GG} - V_{GS} \to V_{SB} = V_S - V_{SS} \to$ find $V_t(V_{SB}) = V_t'$

④ Plug $V_t' = V_t(V_{SB})$ into (3) → Get V_{GS}'

⑤ Back to ③ → iterate to convergence

⋮

⑥ Check operating pt. → saturated?

if yes → done

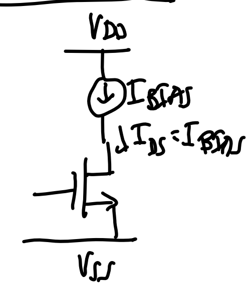
if no → assume linear & start over

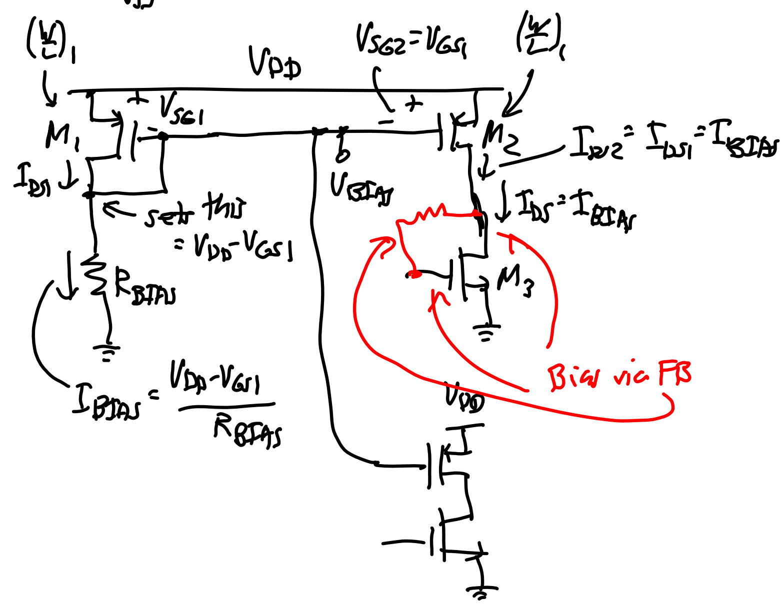
⇒ tedious, but effective for discrete (i.e., off-chip) MOS ckts.

⇒ on-chip, we generally use current mirrors...

- For discrete (off-chip) circuits, avoid transistors
  - ↳ Discrete transistors are more expensive than discrete resistors
- For integrated (on-chip) circuits, avoid resistors
  - ↳ Resistors take up much more space than transistors, and space is money
  - ↳ Use lots of transistors and few if any resistors

$\boxed{\text{Current Mirror}}$ ← for on-chip

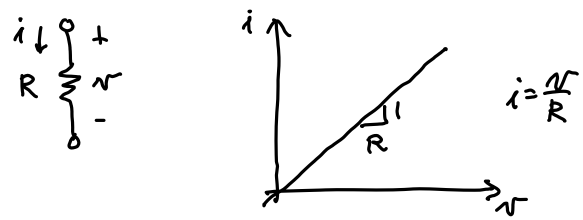




Small-Signal Analysis

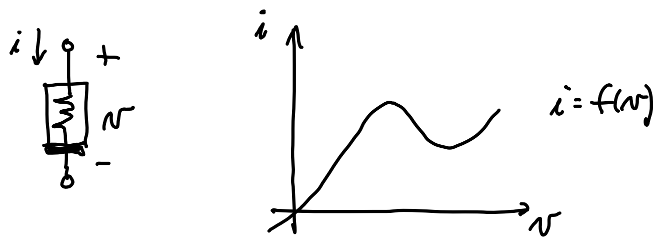
- A method to solve nonlinear problems by linearizing them around a specific coordinate → Bias Pt.

Linear Resistor



$i = \frac{v}{R}$

Non-linear Resistor

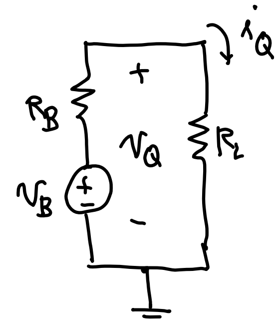


$i = f(v)$

- As we've seen, transistors are nonlinear devices
- We've already experienced difficulty solving for DC operating points for nonlinear transistors
- So solving circuits with more complex inputs, e.g., sinusoids or sums of them, will become even more difficulty
- Need some way to simplify these problems → Small-Signal Analysis
- Take a two-terminal nonlinear resistor circuit as an example:

Example: Two-Terminal Non-Linear Ckt.

⇒ First, look at a linear ckt:

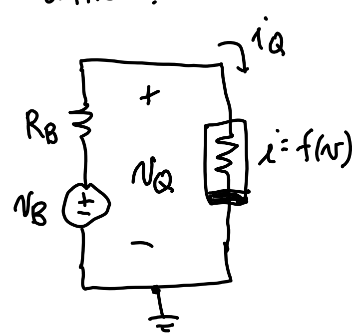


$$\underbrace{\begin{aligned} v_Q &= v_B - i_Q R_B \\ v_Q &= i_Q R_L \end{aligned}}$$

Solve two simultaneous linear equations:

$$v_Q = \left(\frac{R_L}{R_L + R_B}\right) v_B$$

⇒ With non-linear ckts., things become more difficult:



$$\underbrace{\begin{aligned} v_Q &= v_B - i_Q R_B \\ i_Q &= f(v_Q) \end{aligned}}$$

Must solve two simultaneous equations

$$v_Q = v_B - f(v_Q) R_B$$

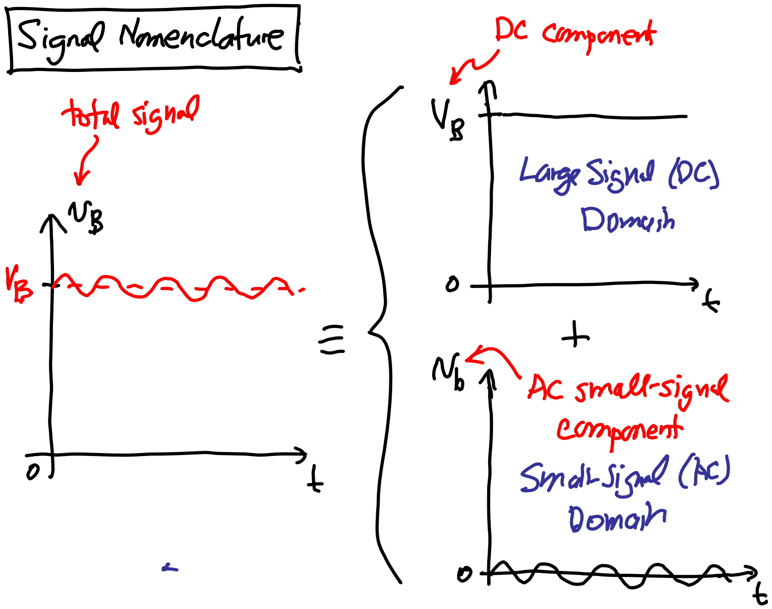
Generally, can't get into closed form.

Not easy to solve!

[e.g., $i = f(v) = 2v^2 + v^3 \rightarrow v_Q = v_B - (2v_Q^2 + v_Q^3) R_B$]

- Need a convenient method to solve this nonlinear circuit, i.e., need some way to linearize it
- Such linearization is possible for analog circuits when the signals can break down into DC and small-signal AC components

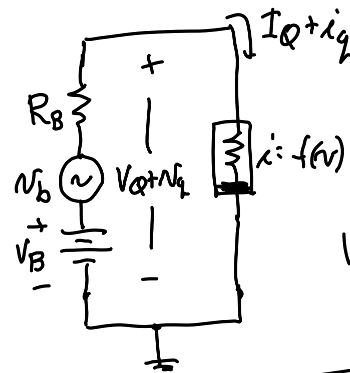
$\boxed{\text{Signal Nomenclature}}$



$$v_B \triangleq V_B + v_b$$

Total Signal	DC Component
lower case variable	upper case variable
upper case subscript	upper case subscript
or	
upper case variable	**AC Small-Signal**
lower case subscript	lower case variable
	lower case subscript

Using this notation:



$$V_Q + v_q = V_B + v_b - \underbrace{f[V_Q + v_q]}R_B$$

For this: Use Taylor Series approx. for $f(v)$ around $v = V_Q$.

Review Taylor Series:

⇒ function $f(x)$ can be approximated for points near $x = a$ by:

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n + \cdots$$

1st derivative: $\left.\frac{df}{dx}\right|_{x=a}$

$$\approx f(a) + f'(a)(x-a) \quad \text{for } (x-a) = \text{small}$$

small-signal

linear! → easy to solve!

total signal

bias point

$$V_Q + v_q = V_B + v_b - f[V_Q]R_B - f'(V_Q)v_q R_B$$

$$V_Q + v_q = V_B + v_b - I_Q R_B - \frac{R_B}{R_{s.s.}} v_q$$

Evaluate nonlinear fcn

$I_Q = f(V_Q)$

$$R_{s.s.} = \frac{1}{\left.\frac{\partial f}{\partial v}\right|_{V_Q}} \triangleq \text{Small-signal resistance}$$