Lecture 22: Small-Signal Modeling

- Announcements:
  - HW#7 online
  - Lab#5 online (this is your first project)
    - Due Tuesday, Oct. 30, 5 p.m.
  - Z scores today (at end of class)

- Lecture Topics:
  - Hybrid-π Model for the npn BJT
  - T Model
  - Pnp Transistor Hybrid-π Model
  - Saturated NMOS Hybrid-π Model

- Last Time:
  - Introduced the BJT Hybrid-π Model
  - Now, continue by deriving its elements…
Determine the small-signal elements:

\[ g_m = \frac{I_c}{V_{be}} \]

\[ I_f = I_c \exp \left( \frac{V_{be}}{V_t} \right) \]

\[ I_c + I_f = I_c \left( 1 + \frac{V_{be}}{V_t} \right) = I_c + \frac{I_c}{V_t} V_{be} \]

This is the condition that should be satisfied to have a "small-signal" condition.

Typical: \( I_c = 1 \text{mA} \)

\[ g_m = \frac{1 \text{mA}}{25 \text{mV}} = 0.04 \text{mS} \]

Typical: \( \beta = 100 \)

\[ r_n = \frac{r_{ie}}{\beta} = \frac{100}{0.04} = 2500 \text{ohm} \]
Remarks:
- \( g_m \) is independent of device specifics, i.e., \( \beta, I_s \).
- Depends only on temperature (via \( V_T \)) and biasing (\( I_c \)).
- Small-signal model valid for \( v_{be} \ll V_T \).

What about emitter resistance?

Why is this not included in the hybrid-\( T \) model?
- Well... it is!
To explicitly show the emitter resistance in the small-signal model, use the T-model:

\[ i_c = \frac{N_{ie}}{r_t} - g_m N_{be} \]

\[ (V_N = N_{be}) \Rightarrow i_c = V_N \left( \frac{1}{r_t} + g_m \right) \]

\[ r_e = \frac{N_{ic}}{i_c} = \frac{1}{r_t + g_m} \leq \frac{r_t}{1 + g_m r_t} = \frac{\beta}{g_m (1 + \beta)} \]

\[ r_e = \frac{\alpha}{g_m} \]

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Where (as before):

\[ g_m = \frac{I_e}{V_T} \]

\[ r_e = \frac{V_T}{I_e} = \frac{\alpha}{g_m} \]