

Lecture 22: Small-Signal Modeling

- **Announcements:**
- HW#7 online
- Lab#5 online (this is your first project)
  - Due Tuesday, Oct. 30, 5 p.m.
- Z scores today (at end of class)
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- Lecture Topics:
  - Hybrid-π Model for the npn BJT
  - T Model
  - Pnp Transistor Hybrid-π Model
  - Saturated NMOS Hybrid-π Model
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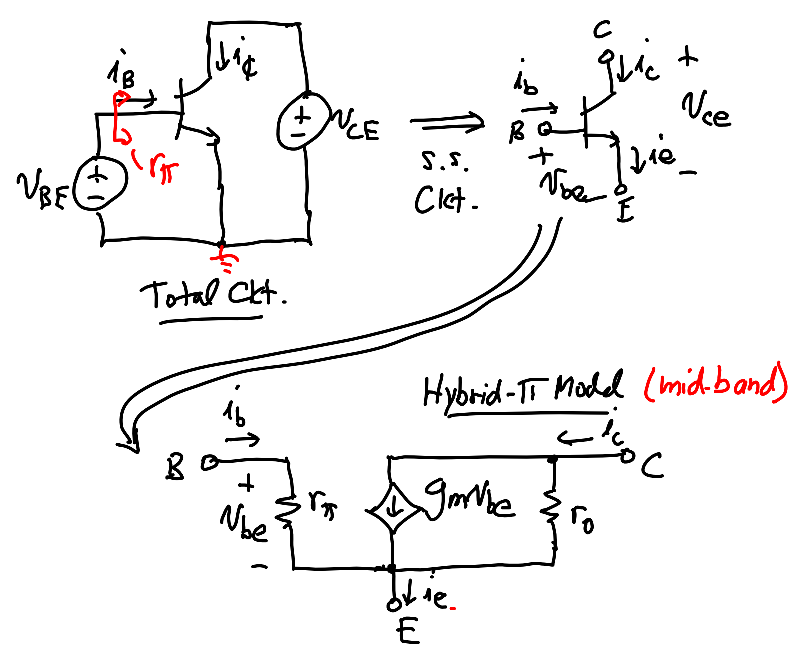
- **Last Time:**
- Introduced the BJT Hybrid-π Model
- Now, continue by deriving its elements …

Small-Signal Models for Forward-Active BJTs



$$\left.\begin{aligned} v_{be}\uparrow \to i_b\uparrow &\Rightarrow r_\pi = \frac{v_{be}}{i_b} \\ v_{be}\uparrow \to i_c\uparrow &\Rightarrow g_m = \frac{i_c}{v_{be}} \\ v_{ce}\uparrow \to i_c\uparrow &\Rightarrow r_o = \frac{v_{ce}}{i_c} \end{aligned}\right\} \text{Each of these determined by the bias pt.}$$

Determine the Small-Signal Elements

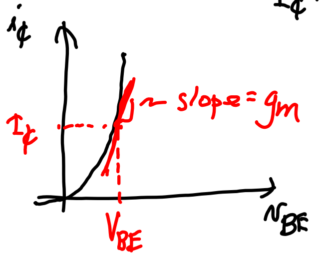
$g_m = \frac{i_c}{v_{be}}$:

$i_C = I_S \exp\left(\frac{v_{BE}}{V_T}\right)$ ← $= V_{BE} + v_{be}$

$I_C + i_c = \underbrace{I_S \exp\left(\frac{V_{BE}}{V_T}\right)}_{I_C} \exp\left(\frac{v_{be}}{V_T}\right)$

$= I_C \exp\left(\frac{v_{be}}{V_T}\right)$

↑ large signal DC collector current



For $v_{be} \ll V_T$ (25mV), can use Taylor series approximation:

$e^x \simeq 1 + x$ for small x

$\cancel{I_C} + i_c = I_C\left(1 + \frac{v_{be}}{V_T}\right) = \cancel{I_C} + \frac{I_C}{V_T} v_{be}$

This is the condition that should be satisfied to have a "small-signal"

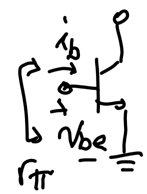
$\frac{i_c}{v_{be}} = \boxed{g_m = \frac{I_C}{V_T}}$ ← fcn. of I_C

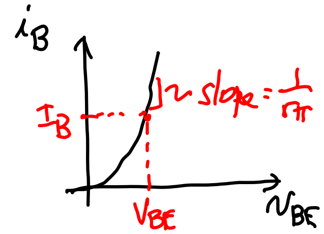
↑ comes from the DC operating pt. (Q-pt.) ← quiescent

Typical: $I_C = 1mA$

$g_m = \frac{1m}{25m} = 0.04$ ℧

$r_\pi = \frac{v_{be}}{i_b}$:





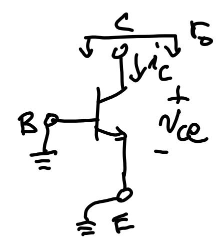
$r_\pi = \frac{v_{be}}{i_b} = \frac{v_{be}}{\frac{i_c}{\beta}} = \frac{\beta}{\frac{i_c}{v_{be}}} = \boxed{\frac{\beta}{g_m} = \frac{V_T}{I_B} = r_\pi}$

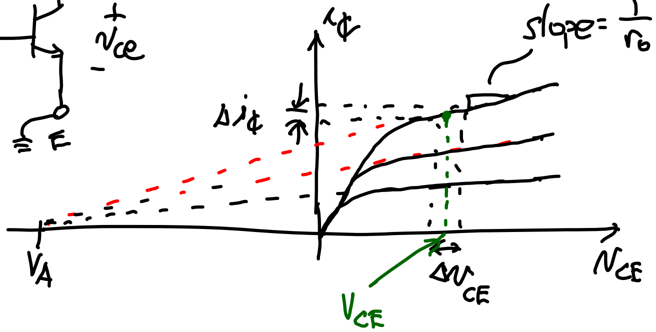
↑ again, fcn. of DC operating pt.

Typical: $\beta = 100$

$r_\pi = \frac{100}{0.04} = 2.5k\Omega$

$r_o = \frac{v_{ce}}{i_c}$:





$$i_C = I_s \exp\left(\frac{v_{BE}}{V_T}\right)\left(1 + \frac{v_{CE}}{V_A}\right)$$

$$r_o = \left[\frac{\partial i_C}{\partial v_{CE}}\bigg|_{Q\text{-pt.}}\right]^{-1} = \left[\underbrace{I_s \exp\left(\frac{v_{BE}}{V_T}\right)}_{= I_C}\frac{1}{V_A}\bigg|_{v_{BE}=V_{BE}}\right]^{-1}$$

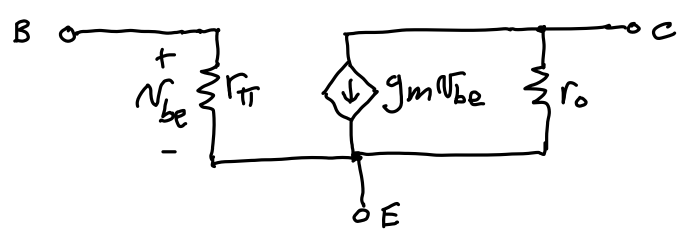
$$\boxed{r_o = \frac{V_A}{I_C}}$$

Typical: $V_A = 100V$

$$r_o = \frac{100}{1m} = 100k\Omega$$

Large, so often we neglect!

Hybrid-π Model Summary (for npn BJT)



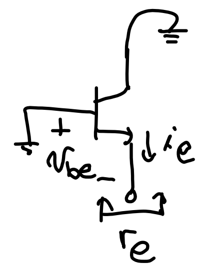
$$r_\pi = \frac{\beta}{g_m} = \frac{V_T}{I_B}$$

$$g_m = \frac{I_C}{V_T}$$

$$r_o = \frac{V_A}{I_C}$$

- **Remarks**:
- g_m is independent of device specifics, i.e., β, I_s
- Depends only on temperature (via V_T) and biasing (I_C)
- Small-signal model valid for $v_{be} << V_T$

What about emitter resistance?

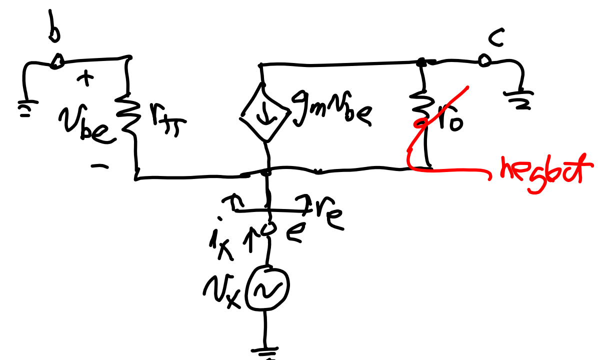


$$v_{be}\uparrow \rightarrow i_e\uparrow \rightarrow r_e = \frac{v_{be}}{i_e}$$

$$r_e = \frac{v_{be}}{i_e} = \frac{v_{be}}{\frac{i_c}{\alpha}} = \frac{\alpha}{g_m} = \frac{\alpha V_T}{I_C} = \frac{V_T}{I_E}$$

$$\boxed{r_e = \frac{\alpha}{g_m} \approx \frac{1}{g_m} = \frac{V_T}{I_E}}$$

Why is this not included in the hybrid-π model?
↳ well... it is!



$$i_x = -\frac{v_{be}}{r_\pi} - g_m v_{be}$$

$$(v_x = -v_{be}) \Rightarrow i_x = v_x\left(\frac{1}{r_\pi} + g_m\right)$$

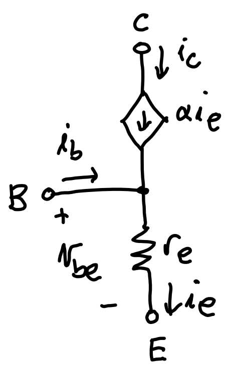
$$r_e = \frac{v_x}{i_x} = \frac{1}{\frac{1}{r_\pi} + g_m} = \frac{r_\pi}{1 + g_m r_\pi} = \frac{r_\pi}{1+\beta} = \frac{\beta}{g_m(1+\beta)}$$

$$\downarrow$$

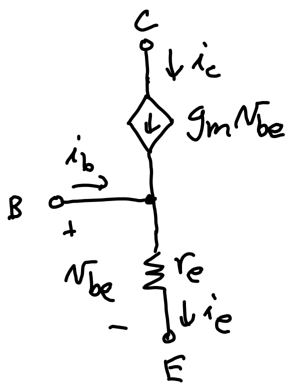
$$r_e = \frac{\alpha}{g_m} \checkmark$$

- **To explicitly show the emitter resistance in the small-signal model, use the T-model:**

$\boxed{\text{T-Model}}$ (Common Base Model)



or



where (as before): $g_m = \frac{I_C}{V_T}$

$$r_e = \frac{V_T}{I_E} = \frac{\alpha}{g_m}$$