Lecture 23: MOS Model and Common Emitter Amp

- Announcements:
  - HW#7 online and due Friday via Gradescope
  - Lab#5 online (this is your first project)
    - Due Tuesday, Oct. 30, 5 p.m.

- Lecture Topics:
  - T Model
  - Pnp Transistor Hybrid-π Model
  - Saturated NMOS Hybrid-π Model
  - Example: Common Emitter Amplifier

- Last Time:
  - Derived the BJT Hybrid-π Model
  - Now, continue with models …

**Hybrid-π Model Summary** (for npn BJT)

\[
\begin{align*}
V_{be} & \rightarrow i_e \rightarrow \alpha \rightarrow r_e = \frac{N_{be}}{r_{ce}} \\
i_e & \rightarrow \frac{V_{be}}{r_{be}} \rightarrow \frac{I_e}{r_{be}} = \frac{N_{be}}{r_{be}} \rightarrow \frac{I_e}{r_{be}} = \frac{V_{be}}{r_{be}} \\
\frac{r_{ce}}{r_{be}} & \rightarrow \frac{r_{ce}}{r_{be}} \rightarrow r_e = \frac{N_{be}}{r_{ce}} \rightarrow r_e = \frac{V_{be}}{r_{ce}} \\
\end{align*}
\]

**Remarks:**
- \( g_m \) is independent of device specifics, i.e., \( \beta, I_s \)
- Depends only on temperature (via \( V_T \)) and biasing (\( I_C \))
- Small-signal model valid for \( v_{be} \ll V_T \)

**What about emitter resistance?**

\[
\begin{align*}
\frac{r_{ce}}{r_{be}} & \rightarrow \frac{r_{ce}}{r_{be}} \rightarrow r_e = \frac{N_{be}}{r_{ce}} \\
r_e & = \frac{V_{be}}{r_{ce}} \\
r_e & = \frac{V_{be}}{r_{ce}} \\
\end{align*}
\]

Why is this not included in the hybrid-π model? So well… it is!
To explicitly show the emitter resistance in the small-signal model, use the T-model:

\[ i_X = -\frac{V_{be}}{r_T} + g_m V_{be} \]

\[ (V_X = V_{be}) \Rightarrow i_X = r_T (\frac{1}{r_T} + g_m) \]

\[ R_e = \frac{V_X}{i_X} = \frac{1}{r_T + g_m} = \frac{r_T}{1 + g_m r_T} = \frac{r_T}{1 + \beta} = \frac{B}{g_m(1 + \beta)} \]

\[ R_e = \frac{a}{g_m} \]

Small-Signal Models for Forward-Accen* pnp Transistors

- For pnp transistors, use the same small-signal models as npn with NO change in polarities

Hybrid-TT Model:

(same as npn model...just upside down)

(again, same as npn model, but upside down)
Note that in the above small-signal models, the current directions are the same as used for npn, i.e., no change in polarities.

Large signal directions, however, are as before.

Need Proof?

\[ i_e = I_f \exp \left( \frac{V_{EE}}{V_T} \right) \]

\[ I_t = i_c = I_f \exp \left( \frac{V_{ee} - V_{be}}{V_T} \right) = I_f \exp \left( -\frac{V_{be}}{V_T} \right) \]

2 Terms of Taylor Expansion

\[ i_C = \frac{I_f}{V_T} V_{be} \]

\[ \text{as with npn, and with the same directions as npn} \]

\[ g_m = \frac{I_d}{V_{GS}} \]

\[ g_m = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TN}) \]
Procedure for Small-Signal Analysis

- (Common Emitter Example)

\[ V_{CC} = +12V \]

\[ V_o = \frac{1}{\lambda I_D} \]

Threshold Voltage \( V_e = f(N_{BS}) \):

\[ g_m = \frac{i_D}{2V_{GS}} = \frac{g_m}{2\sqrt{2Q_0 + V_{SB}}} = g_m \]

Source [Input]

Amplifier

Output Load

[Thévenin Equivalent]

For \( Q \): \( \beta = 100 \), \( V_B = 100V \)

Find the voltage gain, \( \frac{V_o}{V_i} \).

Draw the collector voltage waveform for

\[ V_B = (0.014) \cos 60t + 1V \]

Ac small-signal component

Dc component
Determine the DC operating point
- i.e., find the relevant DC voltages at all nodes and DC currents through all branches
- Draw the DC circuit
  - Eliminate independent AC small-signal sources
    - Short AC voltage sources
    - Open AC current sources
  - Open all capacitors (in particular, open the bypass/coupling capacitors)
  - Use DC transistor models
    - this might entail nonlinearity in some cases, but approximations can alleviate

\[ V_{cc} = +12V \]
\[ R_C = 3k \]
\[ R_B = 2.3k \]
\[ R_E = 2.3k \]
\[ R_1 = 30k \]
\[ R_2 = 10k \]
\[ R_3 = 100 \]
\[ V_C = 0 \]
\[ V_E = 0 \]
\[ I_{C} = \frac{V_{BB} - V_{BE}}{R_E + \frac{R_B}{\beta}} = \frac{3 - 0.7}{2.3k + \frac{7.5k}{100}} = \frac{0.97mA}{1mA} \]
\[ I_{R} = \frac{I_C}{\beta} = 0.01mA \]
\[ V_{B} = V_{BB} - R_B I_{R} = 3 - (7.5k)(0.01m) = 2.92V \]
\[ I_{E} = 0.97mA \approx 1mA \]
\[ V_{E} = 2.92 - 0.7 = 2.22V \]
\[ V_{o} = V_{cc} - I_{C}R_{C} = 12 - 3 = 9V \]

Faster Way:
- Ignore \( I_{EE} \):
  \[ V_{B} = V_{cc} \left( \frac{R_2}{R_1 + R_2} \right) = 3V \]
  \[ V_{E} = V_{B} - V_{BE} = 3 - 0.7 = 2.3V \]
  \[ I_{E} = \frac{V_{E}}{R_E} = \frac{2.3}{2.3k} = 1mA = I_{C} \]
  \[ I_{R} = \frac{I_{C}}{\beta} = \frac{1mA}{100} = 0.01mA \]
  \[ V_{o} = V_{cc} - I_{C}R_{C} = 9V \]

**For stable bias pt.**