Lecture 23: MOS Model and Common Emitter Amp

- **Announcements**:
- HW#7 online and due Friday via Gradescope
- Lab#5 online (this is your first project)
  - Due Tuesday, Oct. 30, 5 p.m.
- ------------------------------------
- Lecture Topics:
  - T Model
  - Pnp Transistor Hybrid-π Model
  - Saturated NMOS Hybrid-π Model
  - Example: Common Emitter Amplifier
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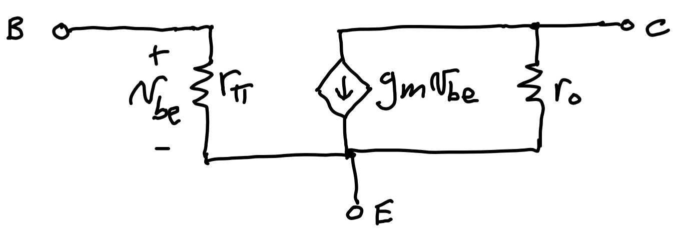
- **Last Time**:
- Derived the BJT Hybrid-π Model
- Now, continue with models …

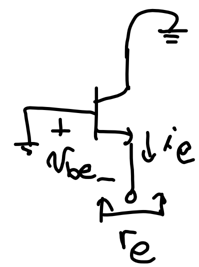
Hybrid-π Model Summary (for npn BJT)



$$\boxed{\begin{aligned} r_\pi &= \frac{\beta}{g_m} = \frac{V_T}{I_B} \\ g_m &= \frac{I_C}{V_T} \\ r_o &= \frac{V_A}{I_C} \end{aligned}}$$

- **Remarks**:
- g_m is independent of device specifics, i.e., β, I_s
- Depends only on temperature (via V_T) and biasing (I_C)
- Small-signal model valid for $v_{be} << V_T$

What about emitter resistance?

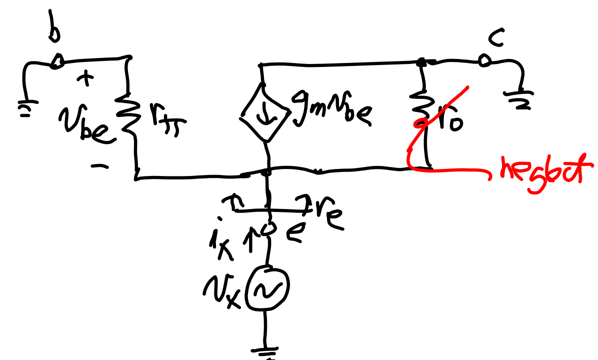


$v_{be}\uparrow \to i_e\uparrow \to r_e = \frac{v_{be}}{i_e}$

$$r_e = \frac{v_{be}}{i_e} = \frac{v_{be}}{\frac{i_c}{\alpha}} = \frac{\alpha}{g_m} = \frac{\alpha V_T}{I_C} = \frac{V_T}{I_E}$$

$$\boxed{r_e = \frac{\alpha}{g_m} \approx \frac{1}{g_m} = \frac{V_T}{I_E}}$$

Why is this not included in the hybrid-π model?
↳ well… it is!



$$i_x = -\frac{v_{be}}{r_\pi} - g_m v_{be}$$

$$(v_x = -v_{be}) \Rightarrow i_x = v_x\left(\frac{1}{r_\pi} + g_m\right)$$

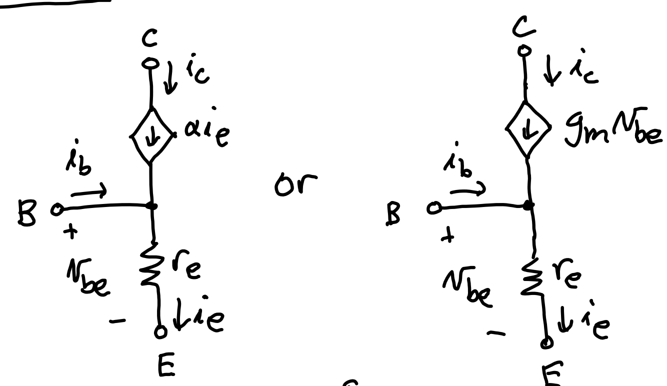
$$r_e = \frac{v_x}{i_x} = \frac{1}{\frac{1}{r_\pi} + g_m} = \frac{r_\pi}{1 + g_m r_\pi} = \frac{r_\pi}{1+\beta} = \frac{\beta}{g_m(1+\beta)}$$

$$\downarrow$$

$$r_e = \frac{\alpha}{g_m} \quad ✓$$

- To explicitly show the emitter resistance in the small-signal model, use the T-model:

$\boxed{\text{T-Model}}$ (Common Base Model)

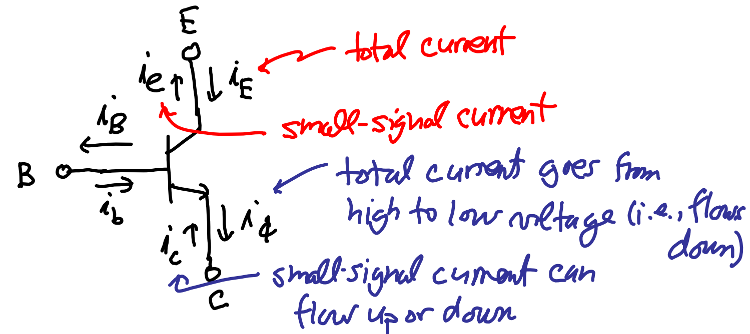


where (as before): $g_m = \frac{I_C}{V_T}$

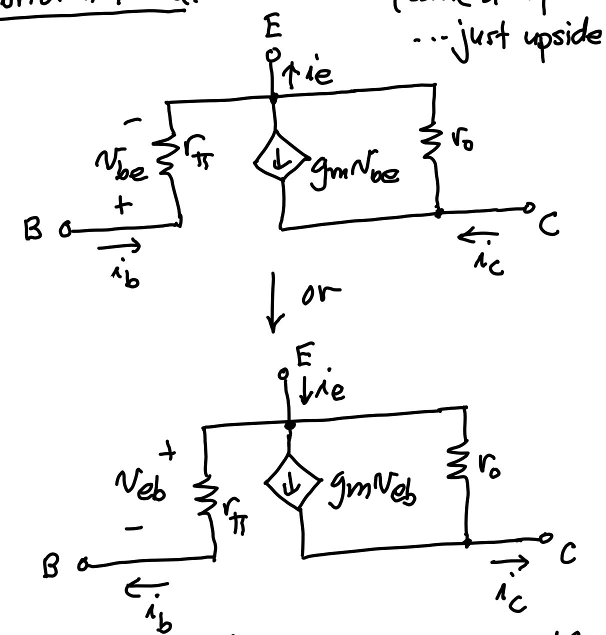
$$r_e = \frac{V_T}{I_E} = \frac{\alpha}{g_m}$$

Small-Signal Models for Forward-Active pnp Xsistors

- For pnp transistors, use the same small-signal models as npn with NO change in polarities

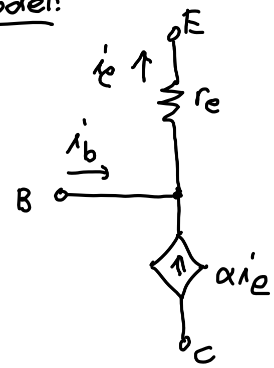


Hybrid-π Model: (same as npn model ...just upside down)



(again, same as npn model, but upside down)

T-Model:



- Note that in the above small-signal models, the current directions are the same as used for npn, i.e., no change in polarities
- Large signal directions, however, are as before

Need Proof?

$$i_C = I_S \exp\left(\frac{v_{EB}}{V_T}\right)$$

$$I_C - i_c = I_S \exp\left(\frac{V_{EB} - v_{be}}{V_T}\right)$$

$$= I_S \exp\left(\frac{V_{EB}}{V_T}\right)\exp\left(-\frac{v_{be}}{V_T}\right) = I_C \exp\left(-\frac{v_{be}}{V_T}\right)$$

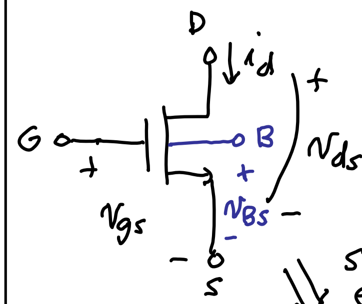
[2 terms of Taylor expansion] $\Rightarrow$ $= I_C\left(1 - \frac{v_{be}}{V_T}\right) = I_C - \frac{I_C}{V_T}v_{be}$

$$i_c = \frac{I_C}{V_T}v_{be} \quad \therefore \quad g_m = \frac{i_c}{v_{be}} = \frac{I_C}{V_T}$$

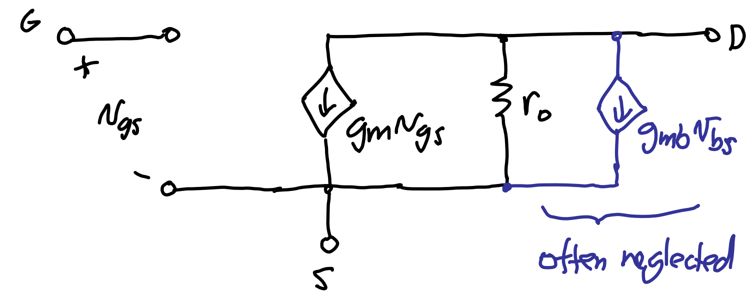
(as with npn, and with the same directions as npn)

$\boxed{\text{Small-Signal Model for Saturated NMOS Transistor}}$

for analog applications



$\Downarrow$ small-signal equivalent ckt.



where:

$$g_m = \frac{i_d}{v_{gs}} = \frac{\partial i_D}{\partial v_{GS}}\bigg|_{Q\text{-}pt} = \frac{\partial}{\partial v_{GS}}\left(\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(v_{GS} - V_{TN})^2\right)\bigg|_{Q\text{-}pt}$$

$$= \mu_n C_{ox}\frac{W}{L}(v_{GS} - V_{TN})\Big|_{v_{GS} = V_{GS}}$$

$$\therefore\ g_m = \mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TN}) = \sqrt{2\mu_n C_{ox}\frac{W}{L}I_D}$$

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TN})^2$$

$$(V_{GS} - V_{TN}) = \sqrt{\frac{2I_D}{\mu_n C_{ox}\frac{W}{L}}} = V_{ov} \triangleq \text{Overdrive Voltage}$$

$$r_o = \frac{v_{ds}}{i_d} = \left[\left.\frac{\partial i_D}{\partial v_{DS}}\right|_{Q\text{-pt}}\right]^{-1}$$

$$= \left[\frac{\partial}{\partial v_{DS}}\left(\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(v_{GS}-V_{TN})^2(1+\lambda v_{DS})\right)\Big|_{Q\text{-pt}}\right]^{-1}$$

$$= \left[i_D\lambda\Big|_{i_D=I_D}\right]^{-1} = [I_D\lambda]^{-1}$$

$$\Rightarrow \boxed{r_o = \frac{1}{\lambda I_D}}$$

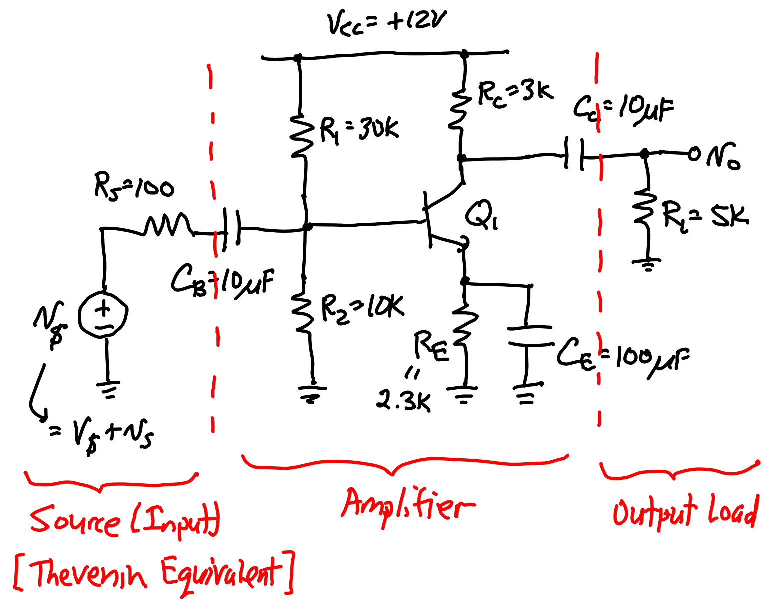
Threshold Voltage $V_t = f(v_{BS})$:

$\Rightarrow$ gives rise to a Body effect transconductance:

$$g_{mb} = \left.\frac{\partial i_D}{\partial v_{BS}}\right|_{Q\text{-pt.}} = \boxed{\frac{g_m \gamma}{2\sqrt{2\phi_f + V_{SB}}} = g_{mb}}$$

Procedure for Small-Signal Analysis

- (Common Emitter Example)



For Q: $\beta = 100$, $V_A = 100V$

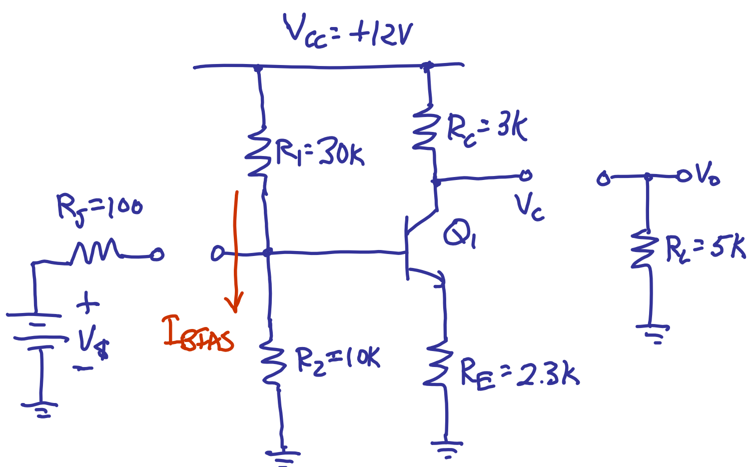
Find the voltage gain, $\frac{v_o}{v_s}$.

Draw the collector voltage waveform for

$$v_s = \underbrace{(0.014)\cos\omega_o t}_{\text{AC small-signal component}} + \underbrace{1V}_{\text{DC component}}$$

1. Determine the DC operating point

- i.e., find the relevant DC voltages at all nodes and DC currents through all branches
- Draw the DC circuit
  - Eliminate independent AC small-signal sources
    - Short AC voltage sources
    - Open AC current sources
  - Open all capacitors (in particular, open the bypass/coupling capacitors)
  - Use DC transistor models
    - this might entail nonlinearity in some cases, but approximations can alleviate



Accurate (but slow):

⇒ Get Thevenin equivalent:

$$V_{BB} = V_{cc}\left(\frac{R_2}{R_1+R_2}\right) = (12)\left(\frac{30K}{40K}\right) = 3V$$

$R_{BB} = 10k \| 30k = \underline{7.5K}$

$$I_C = \frac{V_{BB} - V_{BE}}{\frac{R_E}{\alpha} + \frac{R_B}{\beta}} = \frac{3 - 0.7}{2.3k + \frac{7.5K}{100}} = 0.97mA \approx \underline{1mA}$$

$$I_B = \frac{I_C}{\beta} = \underline{0.01mA}$$

$\therefore I_E \approx 0.97mA \approx 1mA$

$V_B = V_{BB} - R_B I_B = 3 - (7.5k)(0.01m) = \underline{2.92V}$

$\therefore V_E = 2.92 - 0.7 = \underline{2.22V}$

$V_c = V_{cc} - I_C R_C = 12 - 3 = \underline{9V}$

≈ ✓

Faster Way:

Ignore I_B. → $V_B = V_{cc}\left(\frac{R_2}{R_1+R_2}\right) = 3V$

$V_E = V_B - V_{BE(on)} = 3 - 0.7 = 2.3V$

$\therefore I_E = \frac{V_E}{R_E} = \frac{2.3}{2.3k} = \underline{1mA} = I_C$

$I_B = \frac{I_C}{\beta} = \frac{1m}{100} = \underline{0.01mA}$

$V_c = V_{cc} - I_C R_c = \underline{9V}$

$$I_{BIAS} = \frac{V_{cc}}{R_1+R_2} = \frac{12}{40k} = \underbrace{0.3mA > 10 I_B}_{\text{For stable bias pt.}} ✓$$