Lecture 25: High Frequency Small-Signal Models

Announcements:
- HW#8 online and due Friday via Gradescope
- Lab#5 due Tuesday, Oct. 30, 5 p.m.
- Monday's lecture as recorded by ETS is online in your CalCentral EE105 course page
  - Good news!
  - Remember, this is the lecture that I lost due to a hard drive failure
  - You'll need to turn your sound volume up
- Next Monday: No lecture
  - I will be on travel (as indicated in the schedule shown on the first day)
  - Lecture will be by video

Lecture Topics:
- Finish Common Emitter Amplifier
- Frequency Response
- High Frequency Model for BJT

Last Time:
- Going through a Common Emitter Amplifier small-signal analysis example
- Now, continue with this …

Procedure for Small-Signal Analysis

(Common Emitter Example)

\[ V_{cc} = +12V \]
\[ V_{be} = 0.7V \]
\[ R_s = 100 \Omega \]
\[ R_1 = 30k \Omega \]
\[ R_c = 3k \Omega \]
\[ C_1 = 10\mu F \]
\[ R_e = 5k \Omega \]
\[ C_e = 100\mu F \]
\[ R_f = 5k \Omega \]

Source (Input) \hspace{1cm} \text{Amplifier} \hspace{1cm} \text{Output Load}

[Thevenin Equivalent]

For Q: \( \beta = 100, V_A = 100V \)

Find the voltage gain, \( \frac{V_o}{V_i} \).

Draw the collector voltage waveform for

\[ V_c = (0.01 \mu V) \cos 1000t + 1V \]

\[ AC \text{ small-signal component} \hspace{1cm} DC \text{ component} \]
② Determine the elements in the small-signal transistor model(s)
   - If more than one transistor, might need to determine SS element values for several of them
     \[
     g_m = \frac{I_c}{V_T} = \frac{1}{25} = 0.04 \text{ mS} \quad r_o = \frac{V_h}{I_c} = \frac{100}{1} = 100 \Omega
     \]
     \[
     r_T = \frac{100}{0.04} = 2500 \Omega \quad r_e = \frac{r_o + r_T}{g_m} = \frac{2500}{25} = 100 \Omega
     \]
     It gives all of there!

③ Obtain the small-signal circuit
   - Eliminate independent DC sources
     - Short DC voltage sources
     - Open DC current sources
   - Short large coupling capacities (c's > 10 nF)
   - Use small-signal Xstain models

\[V_{in} = \text{short} \quad \text{midband} \quad V_{out} \]

\[\text{L.F.} \quad \text{H.F.} \]

\[\text{Wanted There} \]

\[\text{Specifically, want values in the amplifier in the box, i.e., with } R_S = 0 \text{ and } R_L = \infty.\]
4. Use standard circuit analysis (i.e., KCL or KVL with superposition) to determine the parameters of interest.
   - Usually, the parameters of interest include:
     - Gain, $A_v$
     - Input Resistance, $R_i$
     - Output Resistance, $R_o$
     - Low Frequency Cut-off, $\omega_b$
     - High Frequency Cut-off, $\omega_h$
   - Determine all of these during small-signal analysis.
   - The total gain of the simplified amplifier circuit takes the form:

   $\frac{V_o}{V_i} = \frac{R_l}{R_x + R_s} \frac{A_v}{R_l + R_o}$

   For ideal worst: $R_i = \infty$, $R_o = 0$
   $(R_i > R_s)$ $(R_o < R_l)$

   **Amplifier Gain** (for the circuit in the box)

   $A_v = \left. \frac{V_o}{V_i} \right|_{r_e = \infty}$
   $(i_o = 0)$

   $V_o = (g_m V_i)(r_o || R_c) \Rightarrow A_v = \left. \frac{V_o}{V_i} \right|_{r_e = \infty} = -g_m (r_o || R_c) = -A_m$

   $A_N \approx -g_{m} r_c = -120$
   $(r_o > R_c)$

\[ \text{Max Gain occurs when } R_c = \infty : \]

\[ \text{Max Gain} = -g_m r_c = -\frac{i_o V_i}{V_{T}} = \frac{V_A}{V_T} = \text{Max Gain} \]

\[ R_i = \frac{V_A}{A_m} = r_o || R_c \approx R_c = 1875 \Omega \]

\[ R_o = \frac{V_A}{A_m} = r_o || R_c \approx R_c = 3 \text{ k}\Omega \]
5. Use the gain and resistances determined above to obtain the small-signal voltages and currents at each node/branch of the circuit.

- To obtain the actual node & branch signals, superpose the DC and small-signal AC solutions.
Select amplifier properties, e.g., $R_i, R_o, A_i, f_t$

$V_C (DC)$

$N_{CE} = N_S \times (\frac{V_C}{V_S}) = (0.01V)(-71.2) \cos \phi = -(IV) \cos \phi$

$V_C (Total Signal)$

Contains the information

$N_C (AC S.S.)$

1V

What happens from a device perspective?

$N_C, N_B E$

$R_C, R_L$

$V_C, V_{BE}$

$A_i, f_t$

$V_{CC}, R_C$

$10V$

$V_{BE}$

$V_{BE}, V_{BE0}$

$V_{BE0}, N_B E$

$N_{CE}, N_{BE}$

$+V_{CC}, -V_{BE}$

$I_C, I_E$

$V_{CC}(R_C + R_L)$

$N_{CE}(0.01V)(-71.2)(0.004A)$

$N_{BE}$

$V_{BE}(N_B E)$

$N_{CE}$

An Inking of Inspection Analysis

$= \text{for fast SSckt analysis.}$

$R_o = \text{fall/}R_c = R_c$

$R_{f1} + R_1 + R_2$

$N_C = \frac{N_{OE}}{N_S}$

$N_{OE} = \frac{r_i R_1 R_2}{r_i R_1 R_2 + r_o}$

$\text{...but more later...}$
Frequency Response of Amplifier

\[ A(s) = A_m F_L(s) F_H(s) \]

**General Form:**

- \( A(s) = A_m F_L(s) F_H(s) \)
- \( A_m \) = midband gain (constant w freq)
- \( F_L(s) \) models low frequency behavior generally
  - \( \omega \rightarrow 0 : F_L(s) \rightarrow 0 \) It's a HPF
  - \( \omega \rightarrow \infty : F_L(s) \rightarrow 1 \)
- \( F_H(s) \) models high frequency behavior
  - governed by parasitic capacitors (often inside the transistor)
  - \( \omega \rightarrow 0 : F_H(s) \rightarrow 1 \)
  - \( \omega \rightarrow \infty : F_H(s) \rightarrow 0 \) It's a LPF

**High Frequency Hybrid-\pi Model**

- \( R_b \) = base resistance
- \( C_b \) = small-signal capacitance
- \( C_m \) = action happens here

**Linear Capacitor**

\[ q = CV \]

**Nonlinear Capacitor**

\[ q = f(v) \]
From before:
- The depletion region width $x_d$ is a function of the applied reverse-bias voltage $v_{CB}$: $x_d = f(v_{CB})$
- $Q = qN_0 x_d A$, where $A =$ cross sectional area
- Since $Q$ is a function of $x_d$, it is also a function of $v_{CB}$: $Q = g(x_d) = g[f(v_{CB})]$

In general, for a $p$-$n$ junction:
$$C_j = \frac{dQ}{dV_R} = \frac{C_{j0}}{\sqrt{1 + \frac{V_R}{\phi_j}}}; \quad C_{j0} = \frac{eA}{x_d}; \quad \phi_j = \text{built-in potential between } p-n$$

$C_j$ is a junction capacitance:

More generally:
$$C_j = \frac{C_{j0}}{\left(1 + \frac{V_{CB}}{\phi_j}\right)^m}; \quad m = \text{fun of junction interface}$$

We assume this ($= \frac{1}{2}$ for abrupt)

$C_{m}$ - Base-to-Collector Capacitance

$C_{m} = \frac{C_{m0}}{\sqrt{1 + \frac{V_{CB}}{\phi_j}}}$ (for an abrupt junction)

where $C_{m0}$ = capacitance for $V_{CB} =$ 0V
$\phi_j =$ built-in potential between $p$-$n$ type semiconductor
$$= \frac{kT}{q} \ln\left(\frac{N_{A0}N_{D0}}{n_0^2}\right); \quad n_0 = 1.45 \times 10^{10} \ \text{cm}^{-3}$$

$C_{m}$ - Base-to-Collector Capacitance

$\Rightarrow$ two components comprise $C_{m}$:
1. Junction capacitance, $C_{je}$
2. Diffusion Capacitance, $C_{bd}$

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Can experimentally determine $C_\mu$ by measuring the small-signal capacitance between the base and collector terminals with the emitter incrementally open, i.e., use a current source to set $I_E$.

- Determine $C_{tr}$ and $C_{m}$

- $\Rightarrow$ Switch to small-signal parameter:

$$q = I_E/c$$

$$C_b = \frac{q}{N_{be}} = \frac{I_E}{N_{be}} = \frac{I_F g_m}{I_F \frac{I_E}{V_T}} = C_b$$

Putting it all together:

$$C_T = C_b + C_{je} = \frac{C_{je}}{I_E + V_{BE}}$$

- Forward Bias: $C_{je}$ is significant

- Junction Capacitance:

$$C_{je} = \frac{C_{vio}}{V_{BE}}$$

- Diffusion Capacitance:

  - Define base transit time: average time a carrier takes to cross the base

  $$\tau_c = \frac{Q}{I_c} \text{ think of } I_d \text{ as the rate of transfer of charge through the base}$$

  $$Q = I_c \tau_c$$

  $$\Delta Q = I_c \Delta I_c$$

- Can measure $C_{m}$ vs. $V_{CB}$
To find $C_n$, find an expression for $C_n$ in terms of $C_\mu$ and known measurable parameters.

One parameter we can conveniently measure is the short-circuit current gain:

$$h_{fe} = \left. \frac{i_c}{i_b} \right|_{V_e=0}$$

Find $\frac{i_c}{i_b} |_{V_e=0}$:

$$V_{be} = i_b \left( r_{\pi} || \frac{1}{sC_{\pi}} || \frac{1}{sC_\mu} \right)$$

$$i_c = g_m V_{be} - sC_m V_{be} = (g_m - sC_m) V_{be}$$