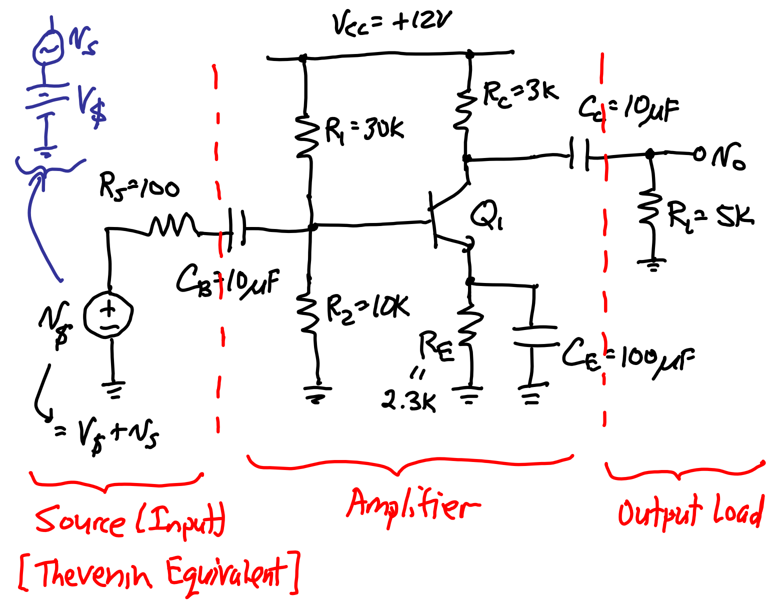
Lecture 25: High Frequency Small-Signal Models

- Announcements:
- HW#8 online and due Friday via Gradescope
- Lab#5 due Tuesday, Oct. 30, 5 p.m.
- Monday's lecture as recorded by ETS is online in your CalCentral EE105 course page
  - Good news!
  - Remember, this is the lecture that I lost due to a hard drive failure
  - You'll need to turn your sound volume up
- Next Monday: No lecture
  - I will be on travel (as indicated in the schedule shown on the first day)
  - Lecture will be by video
- ------------------------------------
- Lecture Topics:
  - Finish Common Emitter Amplifier
  - Frequency Response
  - High Frequency Model for BJT
- ------------------------------------
- Last Time:
- Going through a Common Emitter Amplifier small-signal analysis example
- Now, continue with this …

Procedure for Small-Signal Analysis

- (Common Emitter Example)



For Q: $\beta = 100$, $V_A = 100V$

Find the voltage gain, $\frac{v_o}{v_s}$.

Draw the collector voltage waveform for

$$v_S = \underbrace{(0.014)\cos\omega_o t}_{\text{AC small-signal component}} + \underbrace{1V}_{\text{DC component}}$$

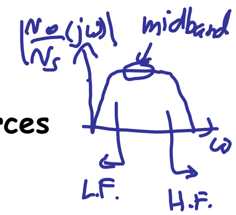
② Determine the elements in the small-signal transistor model(s)

- If more than one transistor, might need to determine SS element values for several of them

$g_m = \frac{I_C}{V_T} = \frac{1m}{25m} = \underline{0.04\,\Omega}$ $r_o = \frac{V_A}{I_C} = \frac{100}{1m} = \underline{100k\Omega}$

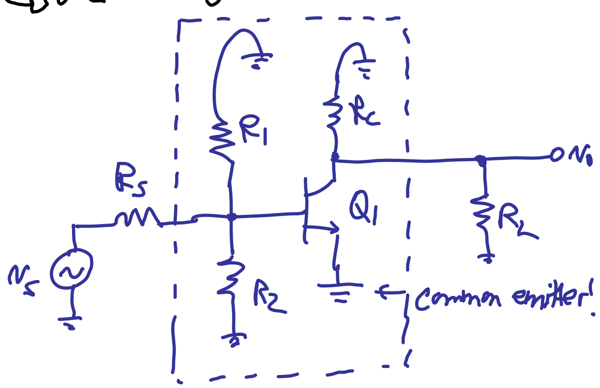
$r_\pi = \frac{\beta}{g_m} = \frac{100}{0.04} = \underline{2.5k\Omega}$ $r_e = \frac{\alpha}{g_m} \simeq \frac{1}{g_m} = \underline{25\,\Omega}$

I_C gives all of these!

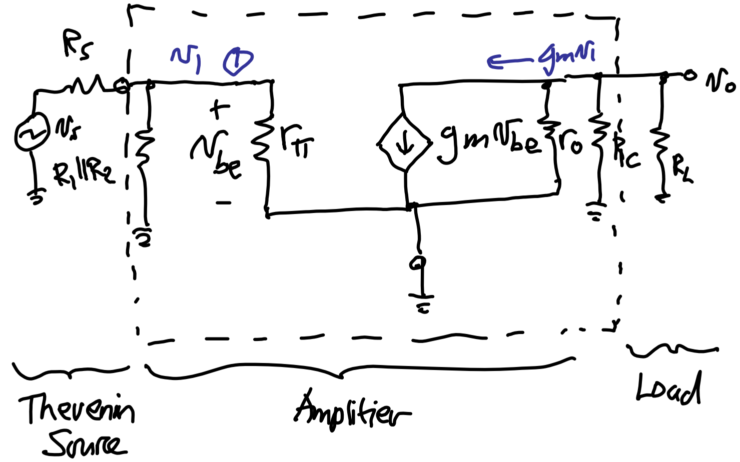


③ Obtain the small-signal circuit

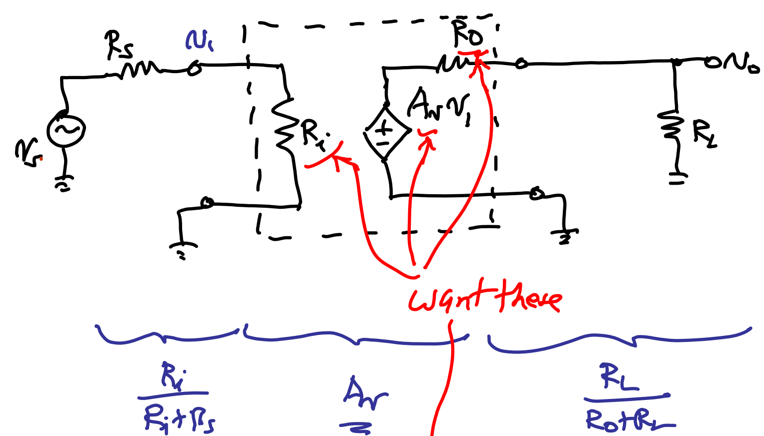
- Eliminate independent DC sources
  - Short DC voltage sources
  - Open DC current sources
- Short large coupling capacitors (C's > 10 nF)
- Use small-signal Xsistor models



⇓ replace the Xsistor w/ S.S. model



⇒ would like to get this in the form of our simple amplifier model



Specifically, want values for the amplifier in the box, i.e., w/ $R_s = 0$ & $R_L = \infty$.

④ Use standard circuit analysis (i.e., KCL or KVL with superposition) to determine the parameters of interest

- Usually, the parameters of interest include
  - Gain, A_v
  - Input Resistance, R_i
  - Output Resistance, R_o
  - Low Frequency Cut-off, ω_b
  - High Frequency Cut-off, ω_h
- Determine all of these during small-signal analysis
- The total gain of the simplified amplifier circuit takes the form

$$\boxed{\frac{v_o}{v_s} = \frac{R_i}{R_i+R_s} A_v \frac{R_L}{R_L+R_o}}$$

gain of ckt. in the box (i.e., not including R_S or R_L)

For ideal want: $R_i = \infty$ ($R_i \gg R_S$) $\quad R_o = 0$ ($R_o \ll R_L$)

Amplifier Gain – (for the ckt. in the box)

$$A_v = \left.\frac{v_o}{v_i}\right|_{R_L=\infty} \quad (\text{or } i_o = 0)$$

$$v_o = -(g_m v_i)(r_o \| R_c) \Rightarrow A_v = \left.\frac{v_o}{v_i}\right|_{R_L=\infty} = \boxed{-g_m(r_o\|R_c) = A_v}$$

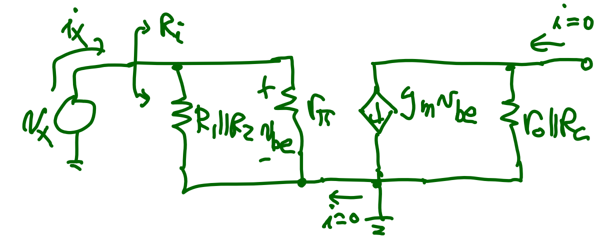
$$A_v \approx -g_m R_c = -120$$

$r_o \gg R_c$

$\Rightarrow$ max gain occurs when $R_c = \infty$:

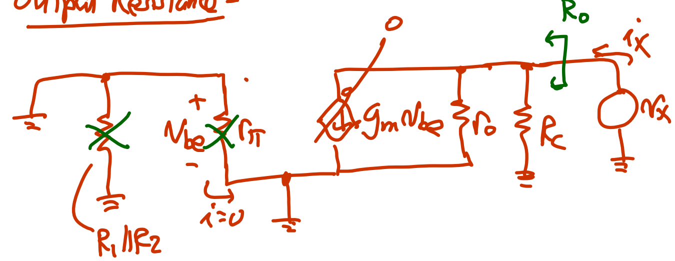
$$\text{Max Gain} = -g_m r_o = -\frac{I_c}{V_T}\frac{V_A}{I_q} = \boxed{-\frac{V_A}{V_T} = \text{Max Gain}}$$

Input Resistance –



$$\boxed{R_i = \frac{v_x}{i_x} = r_\pi \| R_1 \| R_2} = 1875\,\Omega$$

Output Resistance –



$$\boxed{R_o = \frac{v_x}{i_x} = r_o \| R_c \cong R_c} = 3\,k\Omega$$

$[r_o \gg R_c]$

For the Total Ckt:

$$\text{Gain} = \frac{v_o}{v_s} = \frac{R_i}{R_i+R_s} A_v \frac{R_L}{R_L+R_o}$$

$$= \frac{r_\pi\|R_1\|R_2}{r_\pi\|R_1\|R_2+R_s}(-g_m R_c)\left(\frac{R_L}{R_L+R_c}\right) \qquad = R_c\|R_L$$

$$= -\frac{r_\pi\|R_1\|R_2}{r_\pi\|R_1\|R_2+R_s} g_m(R_c\|R_L)$$

$$= -\frac{1875}{1875+100}(0.04)(3k\|5k)$$

$$= \boxed{-71.2 = \frac{v_o}{v_s}}$$

⇒ just for kicks...

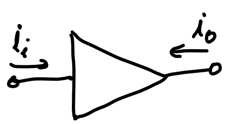
Short Ckt. Current Gain for the C.E. Amp – (ckt. in the box)

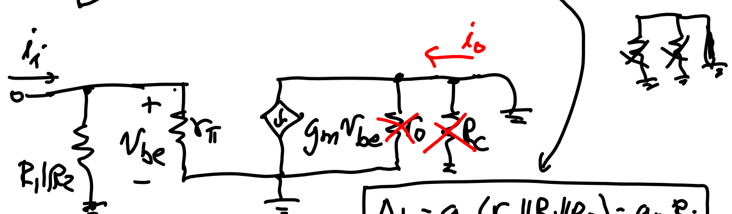
$$A_{is} = \frac{i_o}{i_i}\bigg|_{R_L=0} \quad (@\, v_o = 0)$$

$$i_o|_{R_L=0} = g_m v_{be}$$

$$v_{be} = i_i(r_\pi\|R_1\|R_2)$$

$$i_o|_{R_L=0} = g_m(r_\pi\|R_1\|R_2)\,i_i$$



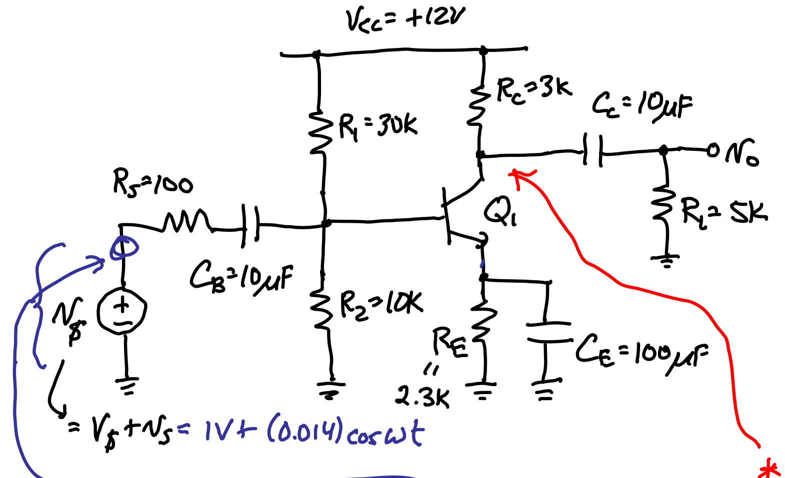


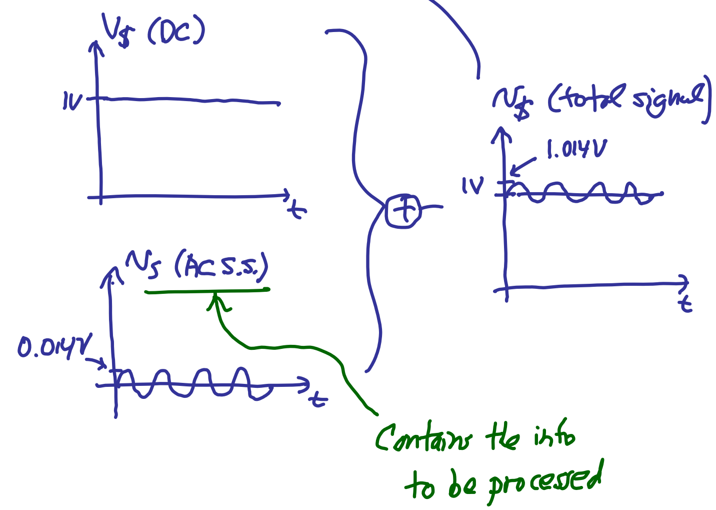
$$\boxed{A_{is} = g_m(r_\pi\|R_1\|R_2) = g_m R_i}$$

$$[R_1 = R_2 = \infty] \Rightarrow \boxed{A_{is} = g_m r_\pi = \beta}$$

⑤ Use the gain and resistances determined above to obtain the small-signal voltages and currents at each node/branch of the circuit

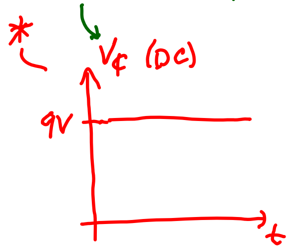
- To obtain the actual node & branch signals, superpose the DC and small-signal AC solutions



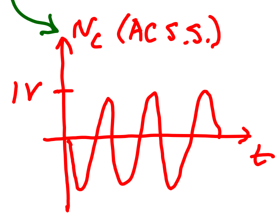


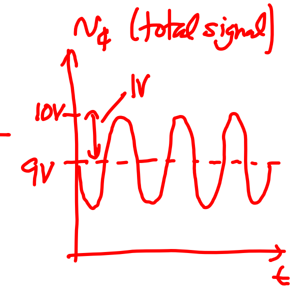
Sets amplifier properties, e.g., R_i, R_o, A_v, f_H

$v_c = v_s\left(\frac{v_o}{v_s}\right) = (0.014)(-71.2)\cos\omega t$
$= -(1V)\cos\omega t$

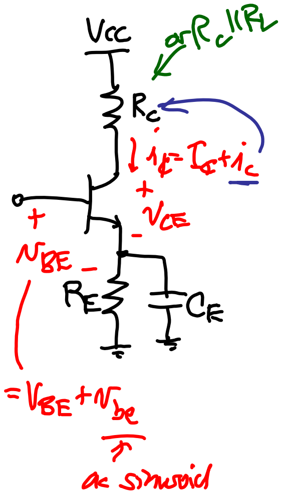


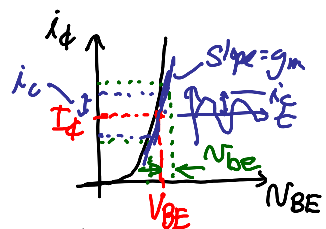
Contains the information

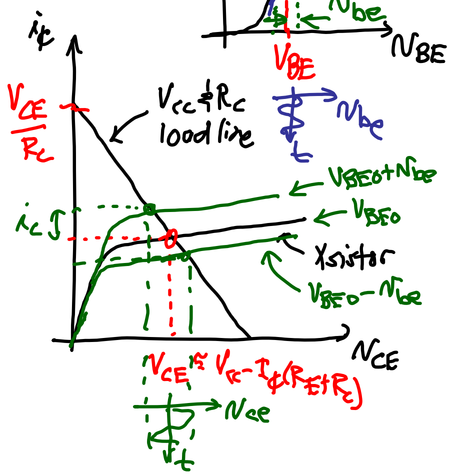




What happens from a device perspective?

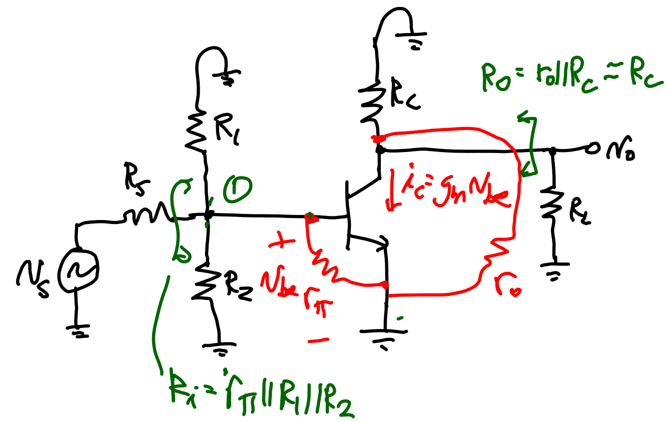






An Inkling of Inspection Analysis

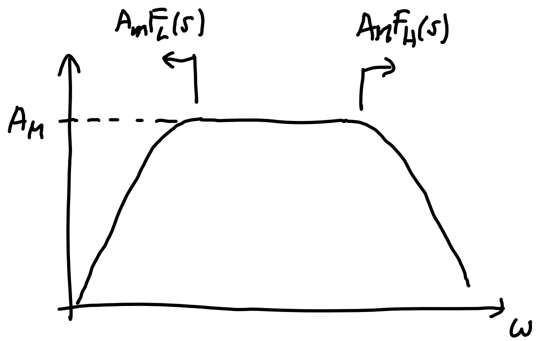
⇒ for fast s.s. ckt analysis:



$$\frac{v_o}{v_s} = \frac{v_①}{v_s}\cdot\frac{v_o}{v_①} = \frac{r_\pi \| R_1 \| R_2}{r_\pi \| R_1 \| R_2 + R_S}\left(-g_m(r_o \| R_C)\right)$$

... but more later...

Frequency Response of Amplifiers



General Form: $A(s) = A_M F_L(s) F_H(s)$

where A_M = midband gain (constant w/ freq.)

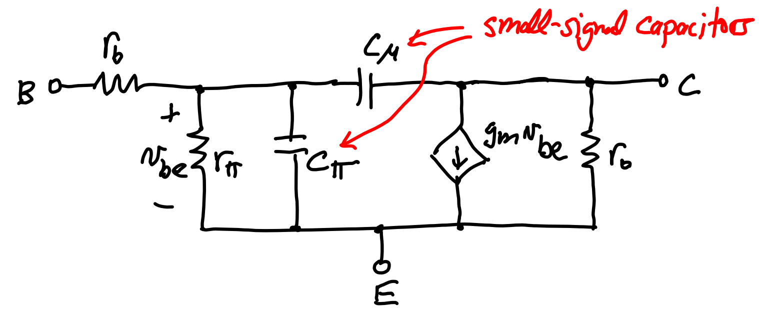
$F_L(s)$ models the low frequency behavior generally governed by coupling capacitors:

For $\omega \to 0$: $F_L(s) \to 0$
$\omega \to \infty$: $F_L(s) \to 1$
} It's a HPF

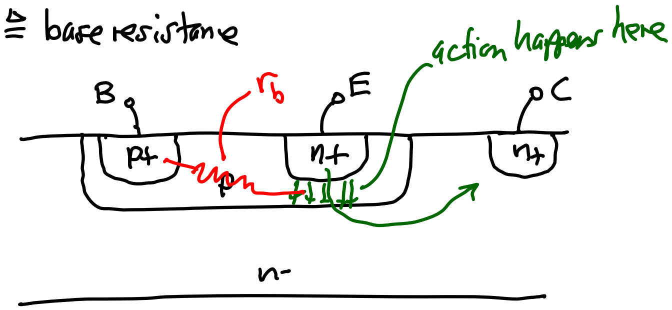
$F_H(s)$ models the high frequency behavior governed by parasitic capacitors (often inside the transistor)

For $\omega \to 0$: $F_H(s) \to 1$
$\omega \to \infty$: $F_H(s) \to 0$
} It's a LPF

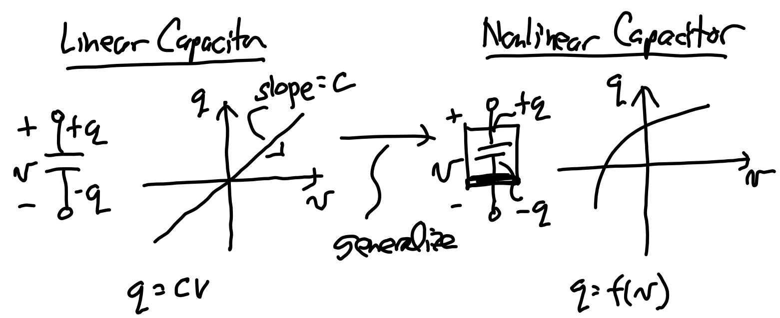
High Frequency Hybrid-π Model



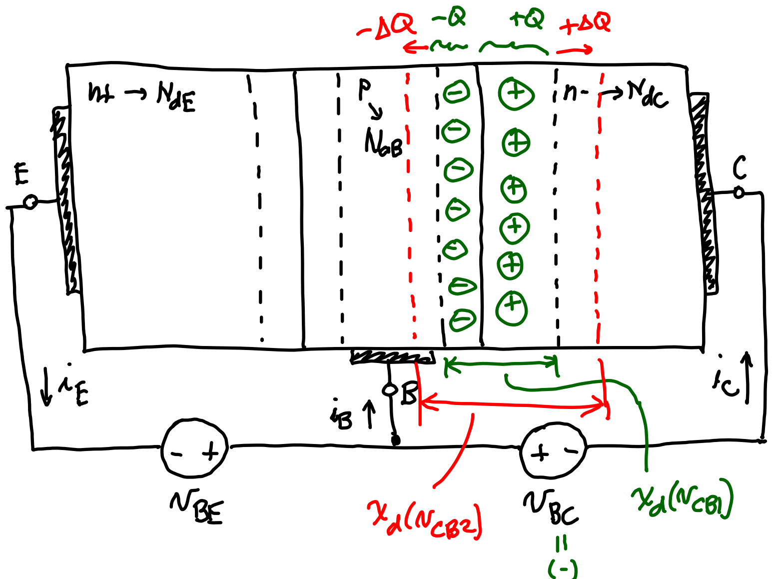
$r_b \triangleq$ base resistance



Capacitor



C_μ – Base-to-Collector Capacitance



- From before:
- The depletion region width x_d is a function of the applied reverse-bias voltage v_{CB}: $x_d=f(v_{CB})$
- $Q=qN_ax_dA$, where A=cross sectional area
- Since Q is a function of x_d, it is also a function of v_{CB}: $Q=g(x_d)=g[f(v_{CB})]$

In general, for a pn junction:

$$C_j = \frac{dQ}{dv_R} = \frac{C_{jo}}{\sqrt{1+\frac{v_R}{\phi_j}}} \quad ; \quad C_{jo} = \frac{\epsilon_s A}{x_{do}} \leftarrow @\ v_R = 0V$$

C_μ is a junction capacitance:

$$C_\mu = \frac{C_{\mu o}}{\sqrt{1+\frac{V_{CB}}{\phi_j}}} \quad \text{(for an abrupt junction)}$$

where $C_{\mu o}$ = capacitance for $V_{CB}=0V$

ϕ_j = built-in potential between p & n type semiconductors

$$= \frac{kT}{q}\ln\left(\frac{N_{aB}N_{dC}}{n_i^2}\right) \quad ; \quad n_i = 1.45\times10^{10}\,cm^{-3}$$

More generally: $C_\mu = \dfrac{C_{\mu o}}{\left(1+\frac{V_{CB}}{\phi_j}\right)^m}$; m = fcn of junction interface

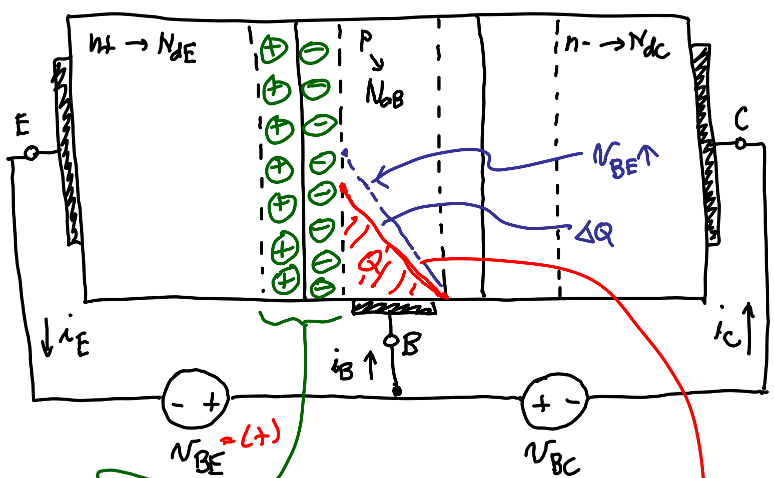
$C_{\mu o}$ ← CJC in SPICE; m ← MJC in SPICE; ϕ_j ← VJC in SPICE

We assume this → ($=\frac{1}{2}$ for abrupt)

C_π – Base-to-Emitter Capacitance

⇒ two components comprise C_π:

1. Junction capacitance, C_{je}
2. Diffusion capacitance, C_b

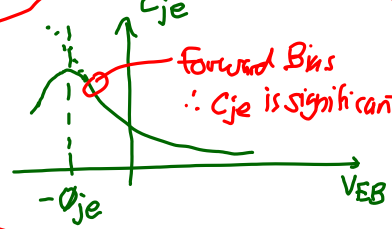


Junction Capacitance:

CJE in SPICE

$$C_{je} = \frac{C_{jeo}}{\sqrt{1 + \frac{V_{EB}}{\phi_{je}}}}$$

VJE in SPICE



Diffusion Capacitance:

⇒ define base transit time: average time a carrier takes to cross the base

$$\tau_F = \frac{Q}{I_C}$$ ← think of I_C as the rate of transfer of charge through the base

$$Q = \tau_F I_C$$
$$\Delta Q = \tau_F \Delta I_C$$

⇒ switch to small-signal parameters:

$$q = \tau_F i_c$$

$$C_b = \frac{q}{v_{be}} = \tau_F \frac{i_c}{v_{be}} = \boxed{\tau_F g_m = \tau_F \frac{I_C}{V_T} = C_b}$$

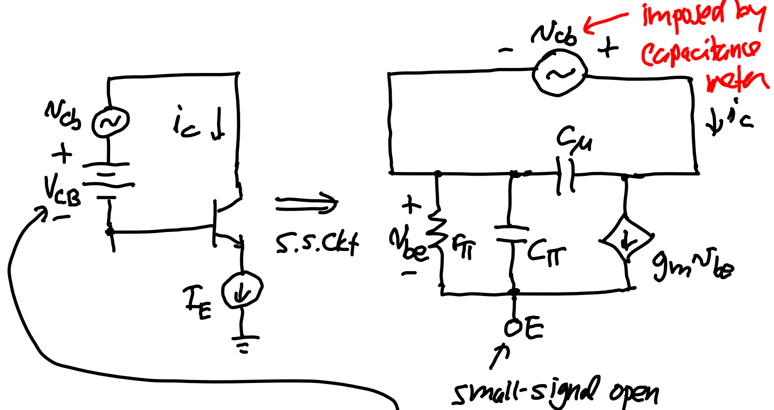
$$\therefore C_b \propto I_C$$

Putting it all together:

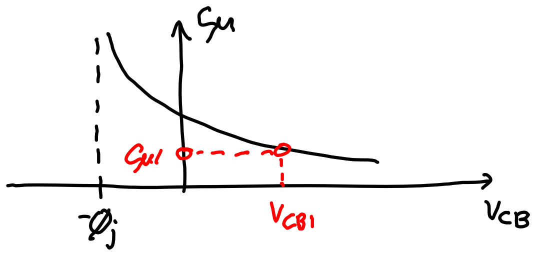
$$\boxed{C_\pi = C_b + C_{je} = \tau_F g_m + \frac{C_{jeo}}{\sqrt{1 + \frac{V_{EB}}{\phi_{je}}}}}$$

Determining C_π and C_μ

- Can experimentally determine C_μ by measuring the small-signal capacitance between the base and collector terminals with the emitter incrementally open, i.e., use a current source to set I_E

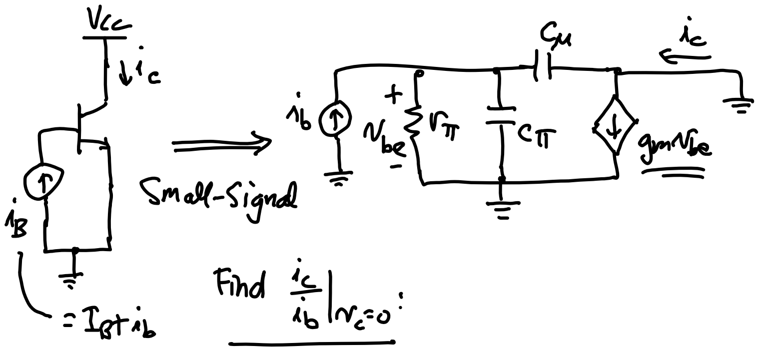


⇒ can measure C_μ vs. V_{CB}



- To find C_π, find an expression for C_π in terms of C_μ and known measurable parameters
- One parameter we can convenienty measure is the short-circuit current gain:

$$h_{fe} = \left.\frac{i_c}{i_b}\right|_{v_c=0}$$



Find $\left.\frac{i_c}{i_b}\right|_{v_c=0}$:

$$v_{be} = i_b \left(r_\pi \| \frac{1}{sC_\pi} \| \frac{1}{sC_\mu}\right)$$

$[g_m \gg sC_\mu]$

$$i_c = g_m v_{be} - sC_\mu v_{be} = (g_m - sC_\mu) v_{be} \approx g_m v_{be}$$