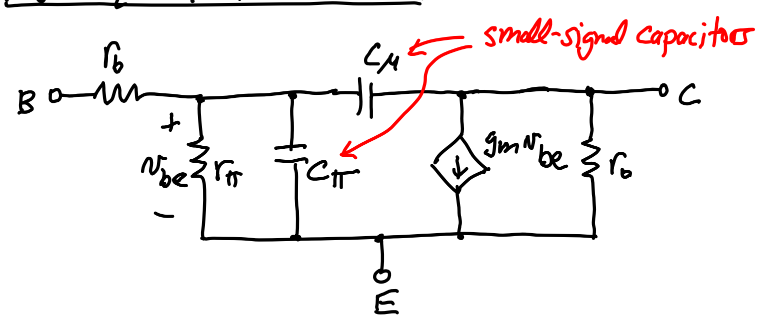
Lecture 26: High Frequency Circuit Analysis

- **Announcements**:
- HW#8 online and due Friday via Gradescope
- Lab#5 due Tuesday, Oct. 30, 5 p.m.
- I am on travel (as indicated in the schedule shown on the first day)
  - This is a video lecture
- -----------------------------------
- Lecture Topics:
  - Measuring High Frequency Model for BJT
  - MOS High Frequency Model
  - Brute Force CE HF Analysis
  - Open Circuit Time Constant (OCTC) Analysis
- -----------------------------------
- **Last Time**:
- Generated the high frequency small-signal model for a BJT
- Examining methods to extract the model experimentally
- Now, continue with this …

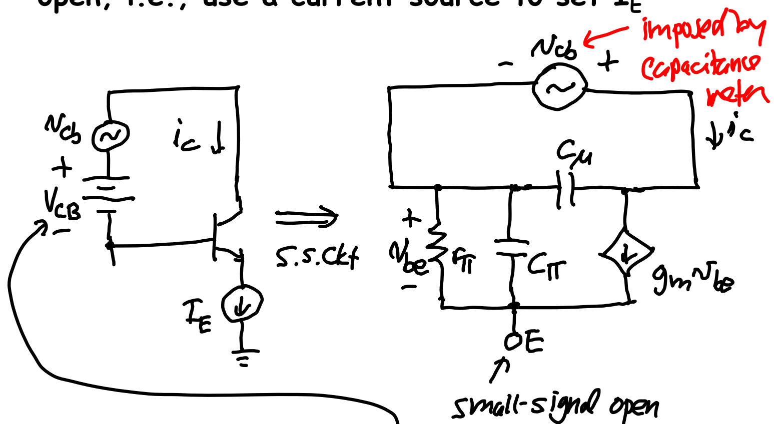
$\boxed{\text{High Frequency Hybrid-}\pi\text{ Model}}$



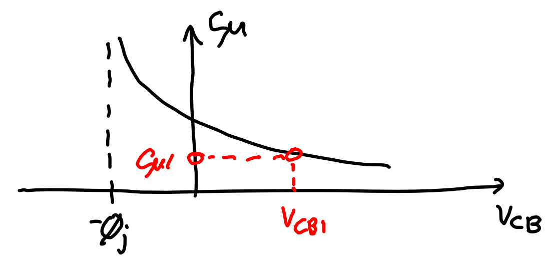
$$\boxed{C_\pi = C_b + C_{je} = \tau_F g_m + \frac{C_{jeo}}{\sqrt{1+\frac{V_{EB}}{\phi_{je}}}}}$$

$\boxed{\text{Determining } C_\pi \text{ and } C_\mu}$

- Can experimentally determine C_μ by measuring the small-signal capacitance between the base and collector terminals with the emitter incrementally open, i.e., use a current source to set I_E

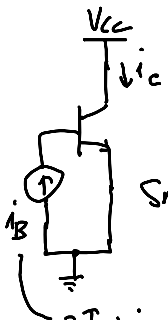


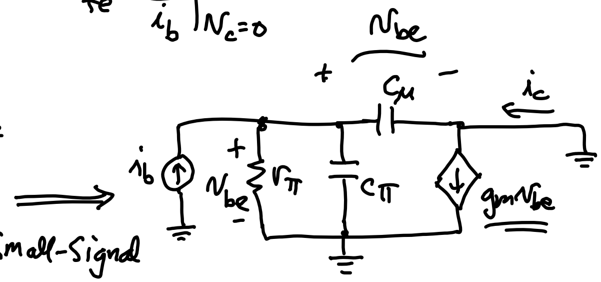
$\Rightarrow$ can measure C_μ vs. V_{CB}



- To find C_π, find an expression for C_π in terms of C_μ and known measurable parameters
- One parameter we can conveniently measure is the short-circuit current gain:

$$h_{fe} = \frac{i_c}{i_b}\bigg|_{v_c=0}$$





Find $\frac{i_c}{i_b}\Big|_{v_c=0}$:

$$v_{be} = i_b\left(r_\pi \| \frac{1}{sC_\pi} \| \frac{1}{sC_\mu}\right)$$

even for fairly large ω's

$$[g_m \gg sC_\mu]$$

$$i_c = g_m v_{be} - sC_\mu v_{be} = (g_m - sC_\mu)v_{be} \approx g_m v_{be}$$

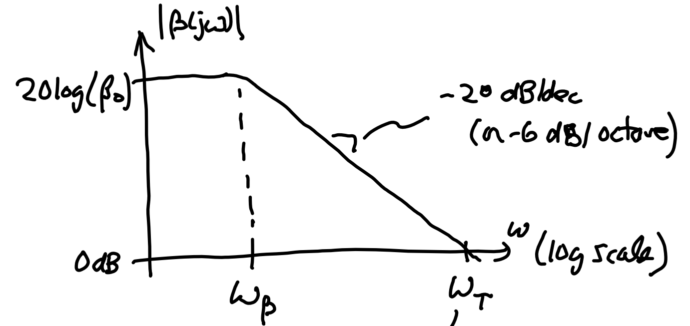
$$= g_m\left(r_\pi \| \frac{1}{sC_\pi} \| \frac{1}{sC_\mu}\right) i_b$$

$$\frac{i_c}{i_b} = \frac{g_m}{\frac{1}{r_\pi} + s(C_\pi + C_\mu)} = \frac{g_m r_\pi}{1 + s r_\pi (C_\pi + C_\mu)} = \frac{\beta_o}{1 + s r_\pi (C_\pi + C_\mu)}$$

$[\beta_o = g_m r_\pi]$
(low freq β)

$$\boxed{\beta(j\omega) = \frac{\beta_o}{1 + \frac{j\omega}{\omega_\beta}} \qquad \omega_\beta = \frac{1}{r_\pi (C_\pi + C_\mu)}}$$

Plot $|\beta(j\omega)|$: (Bode plot)



$\triangleq$ defined as the radian frequency where $|\beta(j\omega)| = 1$

$$|\beta(j\omega_T)| \doteq \frac{\beta_o}{\omega_T r_\pi (C_\pi + C_\mu)} = 1$$

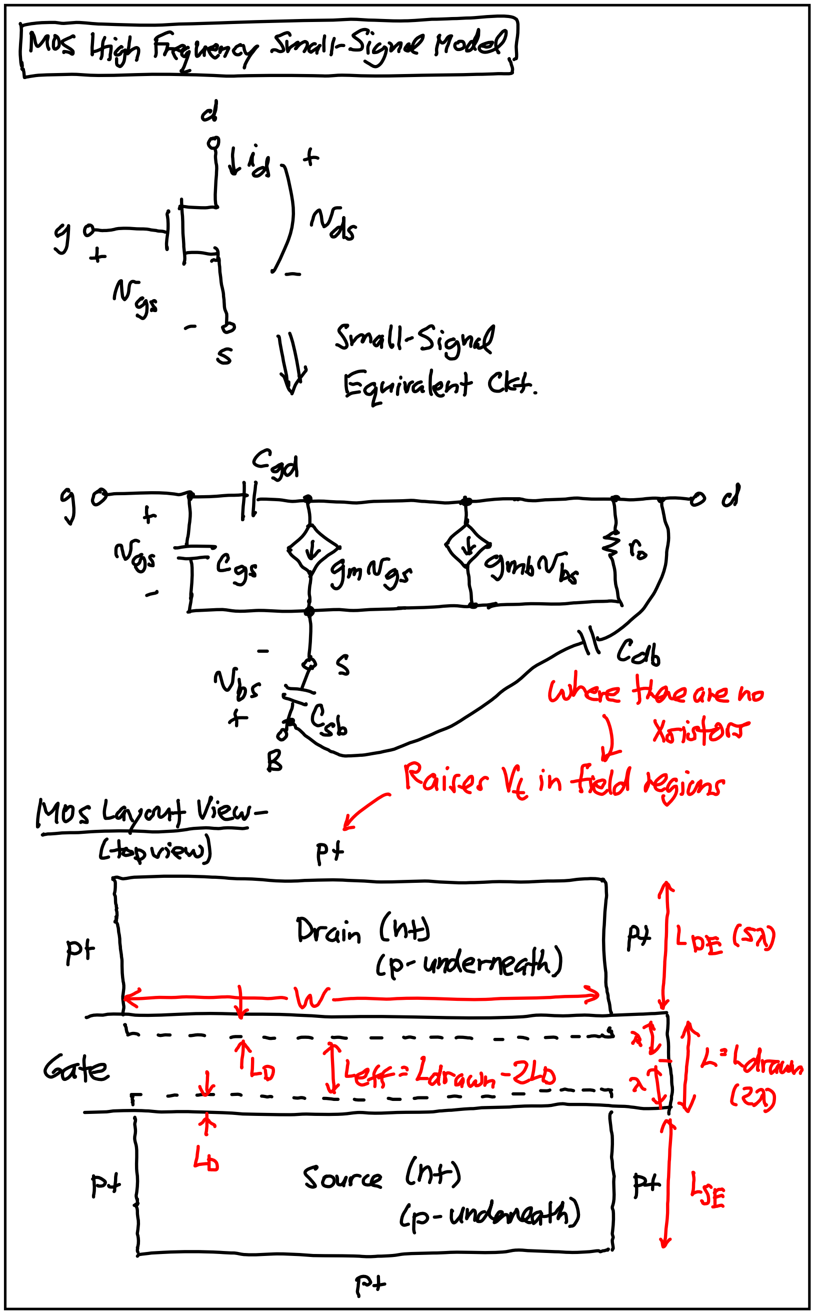
$$\boxed{\omega_T = \frac{g_m}{C_\pi + C_\mu}} \Rightarrow f_T = \frac{\omega_T}{2\pi}$$

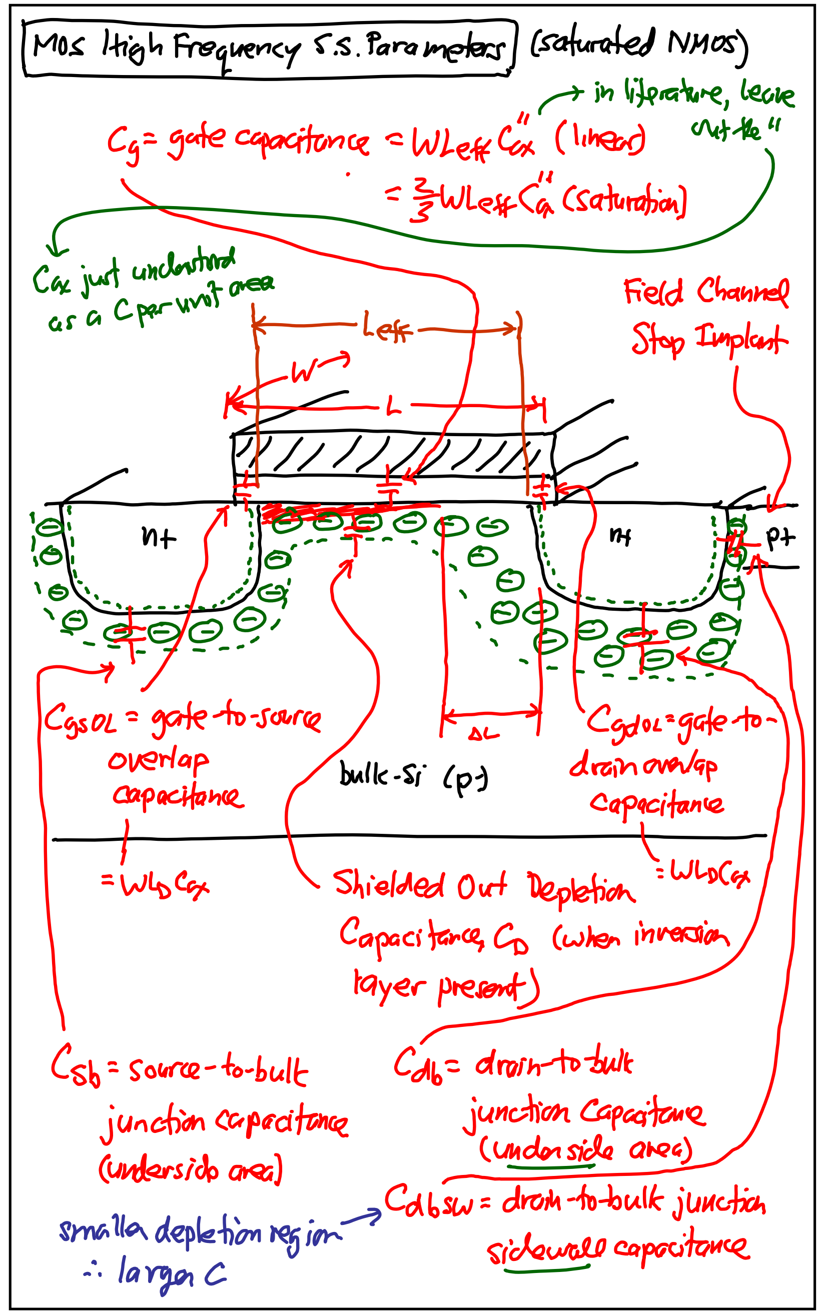
Figure of Merit for the frequency performance of a transistor

For Si bipolar: $f_T \sim 100\,\text{MHz} \rightarrow 50\,\text{GHz}$

Also, note: $\boxed{\omega_T = \beta_o \omega_\beta}$

$$\boxed{C_\pi = \frac{g_m}{\omega_T} - C_\mu}$$


MOS High Frequency Small-Signal Model
d
i_d
+
v_{ds}
g
+
v_{gs}
-
s
Small-Signal Equivalent Ckt.
C_{gd}
g
d
+
v_{gs}
-
C_{gs}
$g_m v_{gs}$
$g_{mb} v_{bs}$
r_o
-
v_{bs}
+
S
C_{sb}
B
C_{db}
where there are no Xsistors
Raises V_t in field regions
MOS Layout View - (top view)
p+
Drain (n+) (p- underneath)
W
L_{DE} (5λ)
Gate
L_D
$L_{eff} = L_{drawn} - 2L_D$
λ
$L = L_{drawn}$ (2λ)
Source (n+) (p- underneath)
L_{SE}


MOS High Frequency S.S. Parameters (saturated NMOS)
in literature, leave out the "
C_g = gate capacitance = $W L_{eff} C_{ox}''$ (linear)
= $\frac{2}{3} W L_{eff} C_{ox}''$ (saturation)
C_{ox} just understood as a C per unit area
Field Channel Stop Implant
L_{eff}
W
L
n+
n+
p+
C_{gsOL} = gate-to-source overlap capacitance
= $W L_D C_{ox}$
ΔL
bulk-Si (p-)
C_{gdOL} = gate-to-drain overlap capacitance
= $W L_D C_{ox}$
Shielded Out Depletion Capacitance, C_D (when inversion layer present)
C_{sb} = source-to-bulk junction capacitance (underside area)
C_{db} = drain-to-bulk junction capacitance (underside area)
smaller depletion region ∴ larger C
C_{dbsw} = drain-to-bulk junction sidewall capacitance

Gate-to-Source Capacitor, C_{gs}: (saturated MOS)

$$C_{gs} = C_{OL} + \frac{2}{3} W L_{eff} C_{ox}$$

$C_{OL} = W L_D C_{ox}$

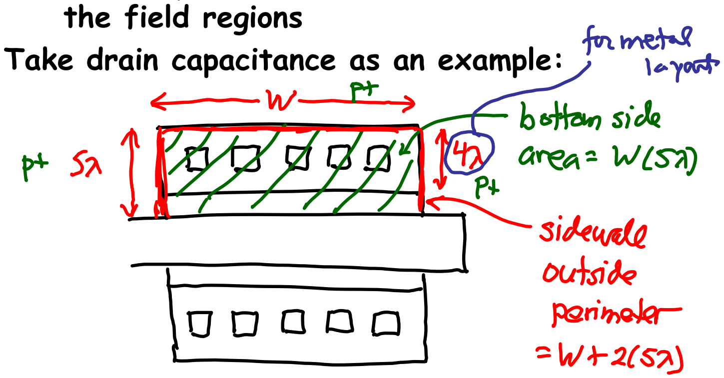
$\frac{2}{3} W L_{eff} C_{ox}$: accounts for the fact that the inversion charge is only under a portion of the gate

Gate-to-Drain Capacitance, C_{gd}:

$$C_{gd} = C_{OL} \quad \text{(no inversion charge near the drain in the saturation region)}$$

$C_{OL} = W L_D C_{ox}$

- Source/Drain Junction Capacitance, C_{sb} & C_{db}:
- These are depletion capacitors associated with bulk-to-drain and bulk-to-source pn junctions
- Bottom capacitance per unit area differs from sidewall capacitance due to higher p+ bulk doping at the sidewalls
- The higher doping is near the silicon surface and designed to raise the threshold voltage in field areas, i.e., areas between transistors
  - This way unwanted inversion does not occur in the field regions
- Take drain capacitance as an example:



$$C_{db} = \frac{C_{db0}}{\sqrt{1 + \frac{V_{DB}}{\phi_j}}}, \text{ where:}$$

$C_{db0} \triangleq$ depletion capacitance w/ $V_{SB} = 0V$

$= (\text{junction bottom-side area}) \times C_{j0}$

$\quad + (\text{junction outside perimeter}) \times C_{jsw0}$

$= W(5\lambda) \times C_{j0} + (W + 2(5\lambda)) \times C_{jsw0}$

and where:

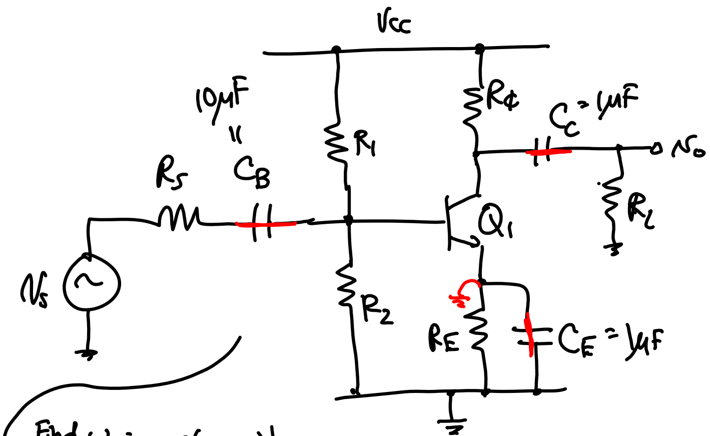
$$C_{j0} = \sqrt{\frac{q \epsilon_s N_B}{2\phi_j}} \quad \text{and} \quad C_{jsw0} = \sqrt{\frac{q \epsilon_s N_{CS}}{2\phi_j}} \times x_j$$

N_B: bulk doping concentration

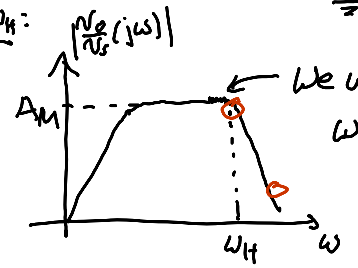
N_{CS}: channel-stop implant doping concentration

x_j: S/D junction depth

C.E. High Freq. Analysis (Brute Force)



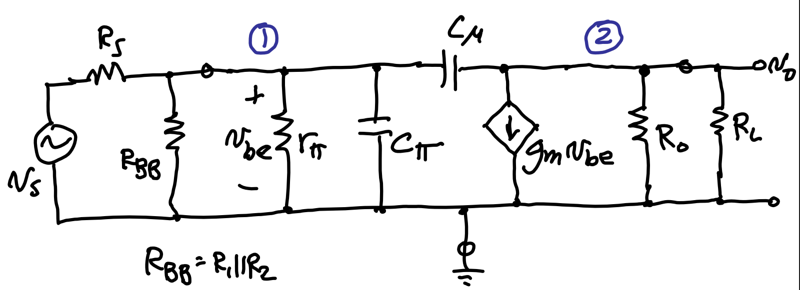
Find ω_H:



We want this frequency, which then gives us

$$\frac{v_o}{v_s}(s) = A_M \frac{1}{1+\frac{s}{\omega_H}}$$

Small-Signal Ckt.



$R_{BB} = R_1 || R_2$

$R_o = r_o || R_C$

KCL ①: $\frac{v_s - v_{be}}{R_s} = \frac{v_{be}}{r_\pi || R_{BB}} + v_{be}(sC_\pi) + (v_{be} - v_o)(sC_\mu)$

KCL ②: $(v_{be} - v_o)(sC_\mu) = g_m v_{be} + \frac{v_o}{R_o || R_L} = g_m v_{be} + R''$

$[R'' = R_o || R_L]$

Rearrange:

$$\frac{v_s}{R_s} = v_{be}\left[\underbrace{\frac{1}{R_s} + \frac{1}{r_\pi || R_{BB}}}_{\frac{1}{r_\pi || R_s || R_{BB}} = \frac{1}{R'}} + s(C_\pi + C_\mu)\right] - v_o(sC_\mu)$$

$[R' = r_\pi || R_s || R_{BB}]$

...math...

$$\boxed{\frac{v_o}{v_s}(s) = \underbrace{-\frac{g_m R' R''}{R_s}}_{A_M} \underbrace{\left\{\frac{(1 - s\frac{C_\mu}{g_m})}{1 + s[R'(C_\pi + C_\mu) + R'' C_\mu + g_m C_\mu R' R''] + s^2 R' R'' C_\pi C_\mu}\right\}}_{F_H(s)}}$$

$$A_M = \underbrace{-g_m \frac{(r_\pi || R_{BB})(r_o || R_C || R_L)}{R_s + r_\pi || R_{BB}}}_{\text{constant} \triangleq \text{midband gain}}$$

$F_H(s)$ models the freq. response

*

*

$$F_H(s) = \frac{1 - s\frac{C_\mu}{g_m}}{1 + s\left[R'(C_\pi + C_\mu) + R''C_\mu + g_m C_\mu R'R''\right] + s^2 R'R''C_\pi C_\mu}$$

$$= \frac{1 - \frac{s}{z_1}}{\left(1 - \frac{s}{p_1}\right)\left(1 - \frac{s}{p_2}\right)} = \frac{1 + \frac{s}{\omega_z}}{\left(1 + \frac{s}{\omega_{p1}}\right)\left(1 + \frac{s}{\omega_{p2}}\right)} = \frac{N(s)}{D(s)}$$

Zeros:

$\Rightarrow$ identify a RHP zero: $z_1 = +\frac{g_m}{C_\mu} \rightarrow \omega_z = \frac{g_m}{C_\mu}$

Note that $\frac{g_m}{C_\mu} \gg \omega_T$ (since $\frac{g_m}{C_\mu} \gg \frac{g_m}{C_\pi + C_\mu}$ and $C_\pi \gg C_\mu$)

$\therefore$ ω_z is a very high freq.!

$\hookrightarrow$ can ignore relative to the lower freq. poles

Poles:

$\Rightarrow$ often, $\omega_{p1} \ll \omega_{p2}$ (i.e., one pole frequency is much lower than other)

(ω_{p1} dominates over ω_{p2})

$$D(s) = \left(1 + \frac{s}{\omega_{p1}}\right)\left(1 + \frac{s}{\omega_{p2}}\right)$$

$$= 1 + s\left(\frac{1}{\omega_{p1}} + \cancel{\frac{1}{\omega_{p2}}}\right) + \frac{s^2}{\omega_{p1}\omega_{p2}}$$

$$= 1 + \frac{s}{\omega_{p1}} + \frac{s^2}{\omega_{p1}\omega_{p2}} \quad \Big\} \text{ Dominant pole approximation}$$

Comparing w/ the denominator of $F_H(s)$:

$$\omega_{p1} = -p_1 \cong \frac{1}{R'(C_\pi + C_\mu) + R''C_\mu + g_m C_\mu R'R''}$$

$$\boxed{\omega_{p1} \cong \frac{1}{\left(C_\pi + C_\mu\left(1 + g_m R'' + \frac{R''}{R'}\right)\right)R'}}$$

$$\omega_{p1}\omega_{p2} \cong \frac{1}{R'R''C_\pi C_\mu} \Rightarrow \omega_{p2} \cong \frac{\left[C_\pi + C_\mu\left(1 + g_m R'' + \frac{R''}{R'}\right)\right]\cancel{R'}}{\cancel{R'}R''C_\pi C_\mu}$$

$$\therefore \omega_{p2} = \frac{g_m}{C_\pi} + \frac{1}{R''C_\mu} + \frac{1}{C_\pi}\left(\frac{1}{R'} + \frac{1}{R''}\right)$$

$\frac{g_m}{C_\pi} > \omega_T \left(= \frac{g_m}{C_\pi + C_\mu}\right)$

$\therefore$ ω_{p2} is indeed at a very high freq. $\gg \omega_{p1}$

$\therefore$ satisfies our dominant pole approx. ✓

Take a closer look @ the form of ω_{P1}:

$$\omega_{P1} \cong \frac{1}{\underbrace{R'C_\pi}_{\tau_{\pi 0} = R_{\pi 0} C_\pi} + \underbrace{(R' + R'' + g_m R' R'')C_\mu}_{\tau_{\mu 0} = R_{\mu 0} C_\mu}} \quad *$$

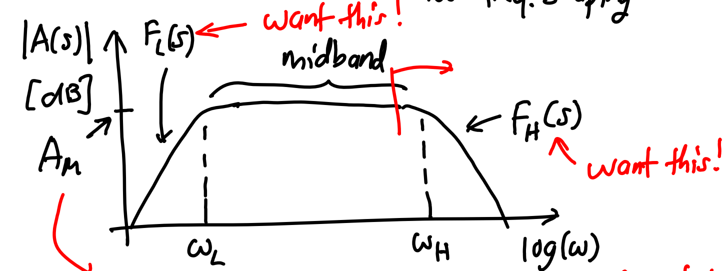
Can think of the denominator as the sum of time constants associated w/ each of the capacitor in the ckt!

Freq. Response

Recall that the transfer function of a general amplifier can be expressed as a function of frequency via:

$$A(s) = A_M F_L(s) F_H(s) \qquad s = j\omega$$

A_M: midband gain; $F_L(s)$: low freq. shaping; $F_H(s)$: high frequency shaping



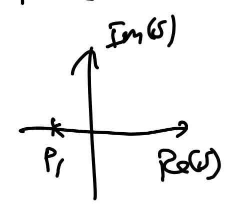
we're already found this using small-signal analysis

High Freq. Response Determination Using Open Ckt. Time Constant (OCTC) Analysis

In general:

$$F_H(s) = \frac{1 + a_1 s + a_2 s^2 + \cdots + a_{n_z} s^{n_z}}{1 + b_1 s + b_2 s^2 + \cdots + b_{n_p} s^{n_p}}, \quad n_p > n_z$$

$$= \frac{\prod_{j=1}^{n_z}\left(1 - \frac{s}{z_j}\right)}{\prod_{i=1}^{n_p}\left(1 - \frac{s}{p_i}\right)} = \frac{\prod_{j=1}^{n_z}\left(1 + \frac{s}{\omega_{zj}}\right)}{\prod_{i=1}^{n_p}\left(1 + \frac{s}{\omega_{pi}}\right)}$$



from which:

$$b_1 = \frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}} + \cdots + \frac{1}{\omega_{pn_p}} = \sum_{i=1}^{n_p} \frac{1}{\omega_{pi}} = \sum_{k=1}^{n_p} \tau_{pk}$$

b_1: coeff. of the 1st order term

Through network theory, one can prove that: (see Gray & Meyer, Chpt. 7)

$$\sum_{i=1}^{n_p} \tau_{pi} = \sum_j C_j R_{jo} = \sum_j \tau_{jo} = b_1$$

where C_j are capacitors in the H.F. ckt., i.e., small ones

$R_{jo} \triangleq$ driving pt. resistance seen between the terminals of C_j determined with

1. all small (<1nF) capacitors open-circuited
2. all independent sources eliminated (i.e., short voltage sources, open current sources)
3. short all large (coupling/bypass) capacitors (i.e., >1μF or >1nF)

In calculating the H.F. response, we use the dominant pole approximation:

(i) $\omega_{p1} \ll \omega_{p2}, \ldots, \omega_{pn_p}$

(ii) $F_H(s) \cong \dfrac{1}{1+b_1 s} = \dfrac{1}{1+\frac{s}{\omega_H}}$

(iii) $\boxed{\omega_H \cong \dfrac{1}{b_1} = \dfrac{1}{\sum_j C_j R_{jo}} = \dfrac{1}{\sum_j \tau_{jo}}}$

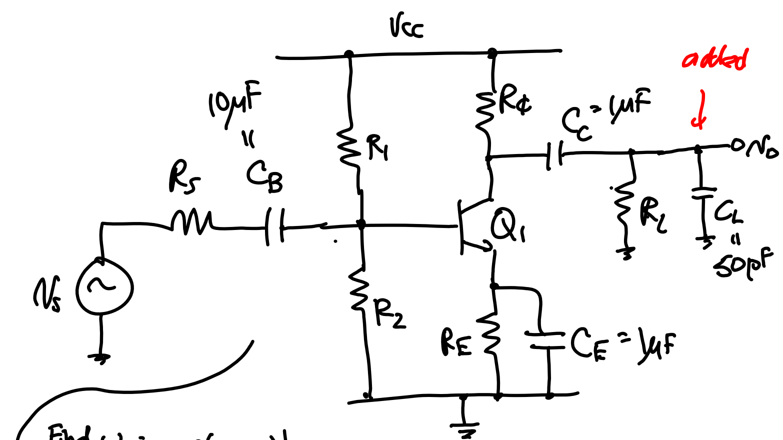
When there is no dominant pole, an approximate expression for ω_H is:

$$\omega_H \approx \frac{1}{\sqrt{\frac{1}{\omega_{p1}^2} + \frac{1}{\omega_{p2}^2} + \cdots - \frac{1}{\omega_{z1}^2} - \frac{1}{\omega_{z2}^2} - \cdots}}$$

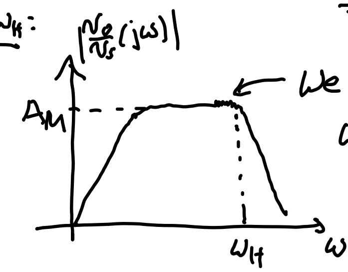
(just FYI)

- Now, use OCTC analysis on the CE amplifier to find the upper cut-off frequency, ω_H

C.E. High Freq. Analysis Using OCTC



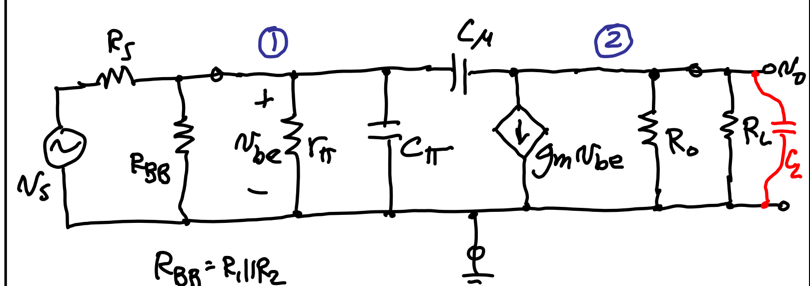
Find ω_H:



We want this frequency, which then gives us

$$\frac{v_o}{v_s}(s) = A_M \frac{1}{1+\frac{s}{\omega_H}}$$

Small-Signal Ckt.



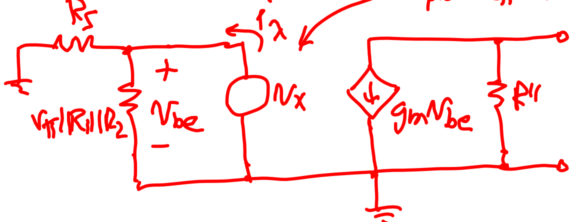
$R_{BB} = R_1 \| R_2$

$R_o = r_o \| R_c$

(a) Determine $\tau_{\pi o} = C_\pi R_{\pi o}$

⇒ open ckt. all C's, zero out all sources, and determine the driving point impedance, $R_{\pi o}$:

replace C_π w/ test source

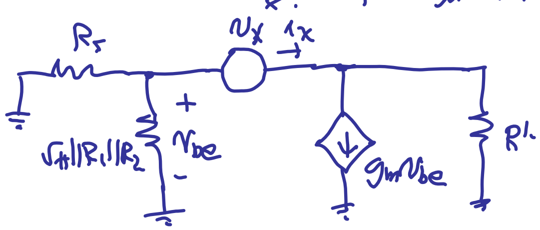


$$R_{\pi o} = \frac{v_x}{i_x} = r_\pi \| R_1 \| R_2 \| R_S = R' \quad \text{(by inspection)}$$

$$\therefore \boxed{\tau_{\pi o} = C_\pi R'}$$

(b) Determine $\tau_{\mu o} = C_\mu R_{\mu o}$:

⇒ need $R_{\mu o}$: replace C_μ w/ test source



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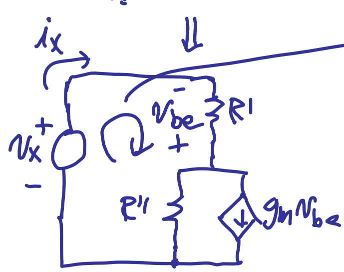


$$R_{\mu o} = \frac{v_x}{i_x}$$

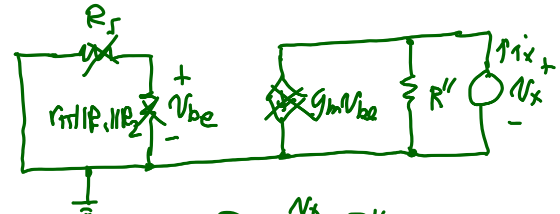
* KVL: $v_x = i_x R' + R''(i_x - g_m v_{be})$

$[v_{be} = -i_x R'] \rightarrow v_x = i_x R' + R''(i_x + i_x R' g_m)$

$$\therefore R_{\mu o} = \frac{v_x}{i_x} = R' + R'' + g_m R' R''$$

$$\therefore \boxed{\tau_{\mu o} = C_\mu (R' + R'' + g_m R' R'')}$$

(c) Determine $\tau_{Lo} = C_L R_{Lo}$:



$$R_{Lo} = \frac{v_x}{i_x} = R''$$

$$\therefore \boxed{\tau_{Lo} = C_L R''}$$

Thus:

$$\boxed{\omega_H = \frac{1}{\sum_j \tau_{jo}} = \frac{1}{C_\pi R' + C_\mu (R' + R'' + g_m R' R'') + C_L R''}}$$

= *