Lecture 27: Low Frequency Circuit Analysis

- **Announcements**:
- HW#8 online and due Friday via Gradescope
- Extend Lab#5 due date by one week
  - Now due Tuesday, Nov. 6, 5 p.m.
- Hopefully, you watched Monday's video lecture
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- Lecture Topics:
  - Short Circuit Time Constant (SCTC) Analysis
  - Intro. to Inspection Analysis
  - C.E. Design Project Hints
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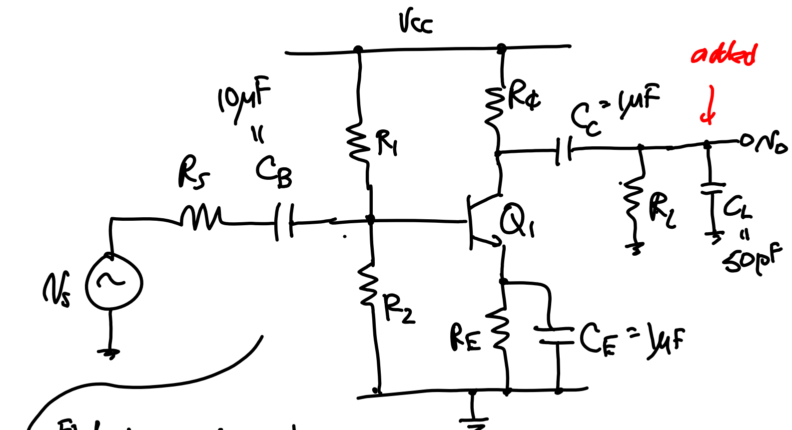
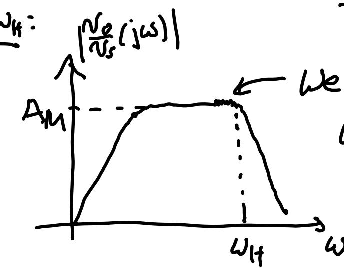
- **Last Time**:
- Finished OCTC analysis for high frequency

C.E. High Freq. Analysis Using OCTC



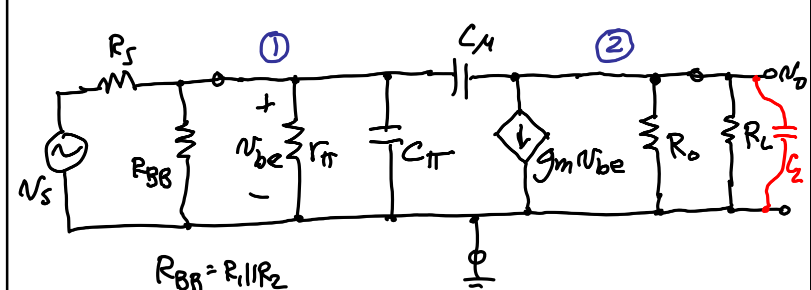
Find ω_H:



We want this frequency, which then gives us

$$\frac{v_o}{v_s}(s) = A_M \frac{1}{1+\frac{s}{\omega_H}}$$

Small-Signal Ckt.



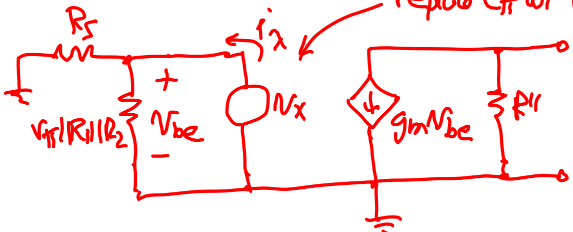
$R_{BB} = R_1 || R_2$

$R_o = r_o || R_C$

(a) Determine $\tau_{\pi o} = C_\pi R_{\pi o}$

⇒ open ckt. all C's, zero out all sources, and determine the driving point impedance, $R_{\pi o}$:

replace C_π w/ test source

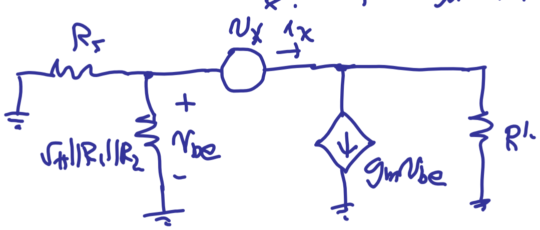


$R_{\pi o} = \frac{v_x}{i_x} = r_\pi || R_1 || R_2 || R_S = R'$ (by inspection)

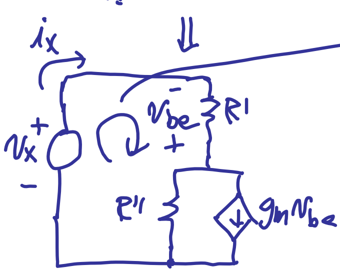
$\therefore \boxed{\tau_{\pi o} = C_\pi R'}$

(b) Determine $\tau_{\mu o} = C_\mu R_{\mu o}$:

⇒ need $R_{\mu o}$: replace C_μ w/ test source



⇓



$R_{\mu o} = \frac{v_x}{i_x}$

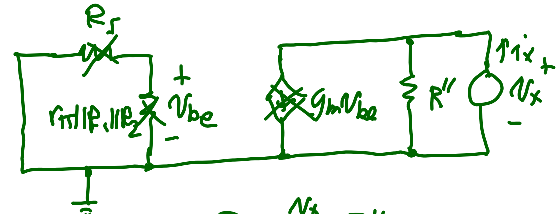
* KVL: $v_x = i_x R' + R''(i_x - g_m v_{be})$

$[v_{be} = -i_x R'] \rightarrow v_x = i_x R' + R''(i_x + i_x R' g_m)$

$\therefore R_{\mu o} = \frac{v_x}{i_x} = R' + R'' + g_m R' R''$

$\therefore \boxed{\tau_{\mu o} = C_\mu (R' + R'' + g_m R' R'')}$

(c) Determine $\tau_{Lo} = C_L R_{Lo}$:



$R_{Lo} = \frac{v_x}{i_x} = R''$

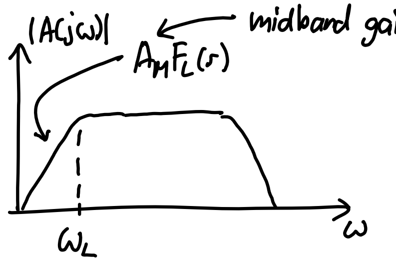
$\therefore \boxed{\tau_{Lo} = C_L R''}$

Thus:

$$\boxed{\omega_H = \frac{1}{\sum_j \tau_{jo}} = \frac{1}{\underbrace{C_\pi R' + C_\mu (R' + R'' + g_m R' R'') + C_L R''}_{= *}}}$$

Low Freq. Amplifier Response Using Short Circuit Time Constant Analysis (SCTC)

Recall:



In general, for the low freq. response:

$$F_L(s) = \frac{s^{n_L} + d_1 s^{(n_L-1)} + \cdots}{s^{n_L} + e_1 s^{(n_L-1)} + \cdots}, \quad n_L = \#\text{poles} = \#\text{zeros}$$

We can express the coefficient e_1 by:

$$e_1 = \omega_{P1} + \omega_{P2} + \cdots + \omega_{Pn_L}$$

For the case of a dominant pole:
↳ i.e., the highest freq. pole

$$F_L(s) \cong \frac{s}{s+\omega_L} = \frac{s}{s+e_1} \rightarrow e_1 \cong \omega_{P1} = \omega_L$$

Similar analysis to that used for OCTC...

$$\therefore \boxed{\omega_L \cong e_1 = \sum_j \omega_{Pj} = \sum_j \frac{1}{C_j R_{js}} = \sum_j \frac{1}{\tau_{js}}}$$

where $C_j \triangleq$ various large (>10nF) capacitors in the ckt. (e.g., the bypass caps.)

$R_{js} \triangleq$ driving point resistance seen between the terminals of C_j determined with:

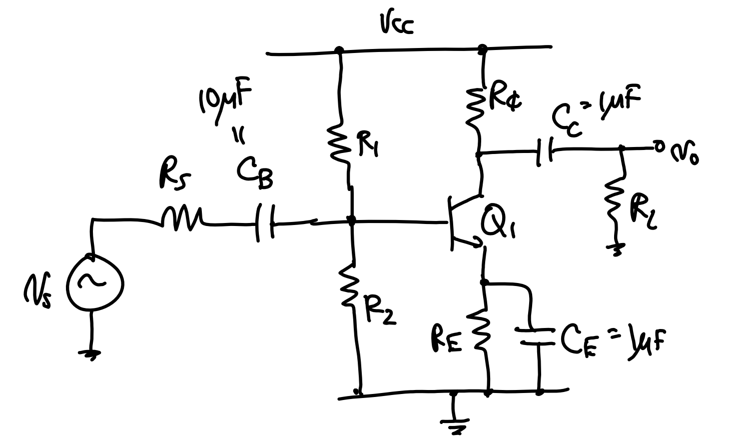
For reading, can go to Sedra & Smith

1. all large capacitors short-circuited, except C_j, which is replaced by the test voltage source for R determination
2. all independent sources eliminated (i.e., short voltage sources, open current sources)
3. open all H.F. capacitors (i.e., small caps in the pF range, or < 1nF)

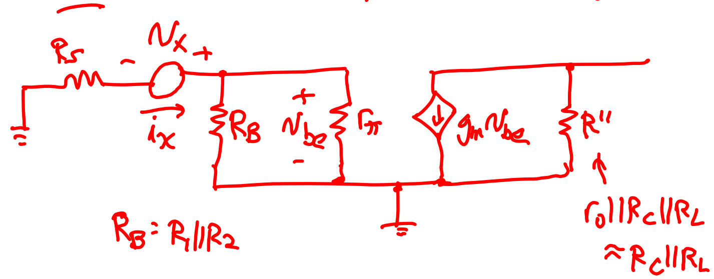
Again, for the case where there are no dominant poles, a reasonable approximation is:

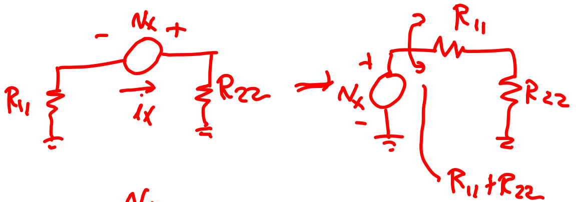
$$\omega_L \cong \sqrt{\omega_{P1}^2 + \omega_{P2}^2 - 2\omega_{Z1}^2 - 2\omega_{Z2}^2}$$

Ex: Determine the L.F. response of the C.E. Amplifier



(a) C_B: short ckt. C_E & C_C, then determine R_{BS}

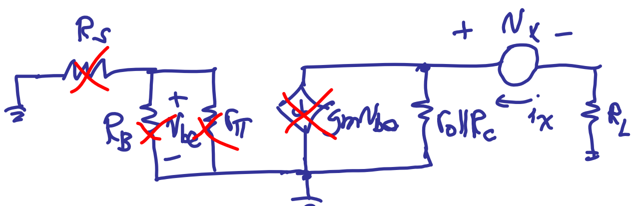




$$R_{BS} = \frac{v_x}{i_x} = R_S + r_\pi \| R_B$$

$\therefore$ $\boxed{\tau_{BS} = C_B(R_S + r_\pi \| R_B)} \rightarrow \boxed{\omega_{BS} = \frac{1}{\tau_{BS}} = \frac{1}{C_B(R_S + r_\pi \| R_B)}}$ $= \omega_{p1}$

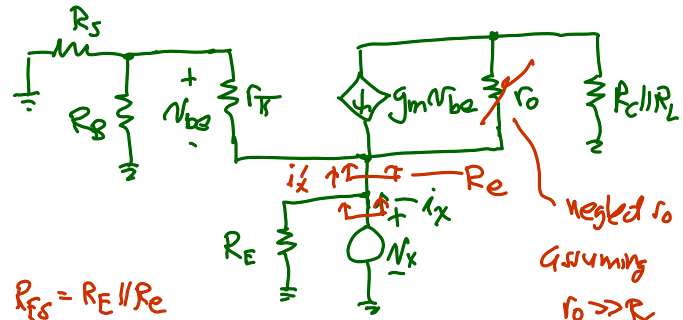
(b) C_C: short ckt. C_B & C_E, then determine R_{CS}
& zero out v_s



$$R_{CS} = R_L + r_o \| R_C \simeq R_L + R_C$$
$[r_o >> R_C]$

$\boxed{\tau_{CS} = C_C(R_L + R_C)} \rightarrow \boxed{\omega_{CS} = \frac{1}{\tau_{CS}} = \frac{1}{C_C(R_L+R_C)}} = \omega_{p2}$

(c) C_E: short C_B & C_C; zero out v_s; determine R_{ES}



$R_{ES} = R_E \| R_e$

R_e: $v_{be} = \frac{-r_\pi v_x}{r_\pi + (R_S \| R_B)}$

$$i_x' = \frac{v_x}{r_\pi + (R_S \| R_B)} - g_m v_{be} = v_x \left(\frac{1}{r_\pi + (R_S \| R_B)} + \frac{\overbrace{g_m r_\pi}^{\beta}}{r_\pi + (R_S \| R_B)} \right)$$

$$R_e = \frac{v_x}{i_x'} = \frac{r_\pi + (R_S \| R_B)}{\beta + 1}$$

$$R_{ES} = R_E \| R_e = R_E \| \frac{r_\pi + R_S \| R_B}{\beta + 1}$$

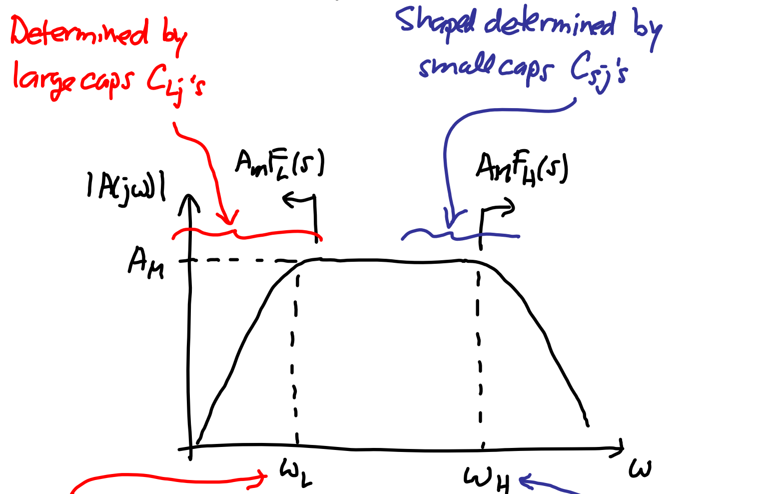
$$\boxed{\tau_{ES} = C_E \left(R_E \| \frac{r_\pi + R_S \| R_B}{\beta+1} \right)}$$

$$\boxed{\omega_{ES} = \frac{1}{C_E \left(R_E \| \frac{r_\pi + R_S \| R_B}{\beta+1} \right)}} = \omega_{p3}$$

$$\boxed{\omega_L \simeq \sum_j \frac{1}{\tau_j} = \sum_j \omega_{pj}}$$

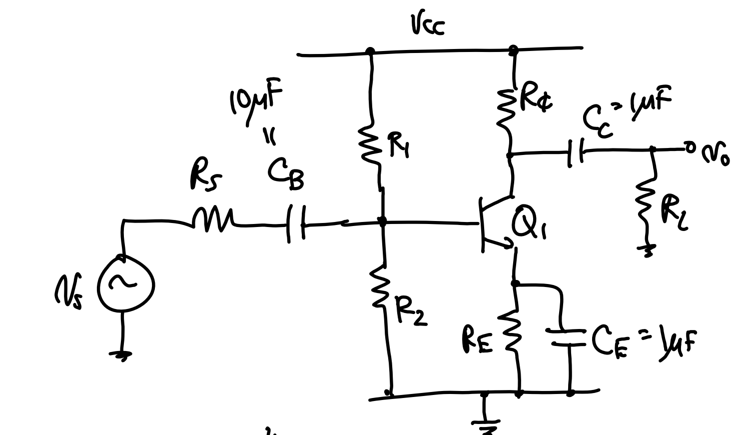
Summarize:

- Which capacitors to use for OCTC? Which for SCTC?
- Separate caps into two categories:
  - ↳ Large caps → C_{Lj}'s
  - ↳ Small caps → C_{Sj}'s

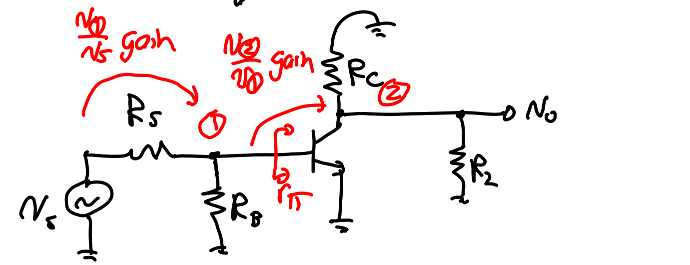


- Find using SCTC
  - ↳ Get time constants using capacitors that contribute to low frequency poles and zeros, which generally means bypass or coupling caps
  - ↳ Open smaller capacitors (e.g., hybrid-π ones)
  - ↳ Use C_{Lj}'s; open C_{Sj}'s
- Find using OCTC
  - ↳ Get time constants using capacitors that contribute high freq. poles & zeros, which generally means hybrid-π or any small caps
  - ↳ Short larger caps (e.g., bypass or coupling capacitors)
  - ↳ Use C_{Sj}'s; short C_{Lj}'s

$\boxed{\text{Intro. to Inspection Analysis}}$ (w/ Lab#5 hints)



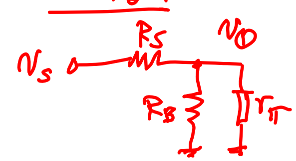
⇓ S.S.Ckt.



⇒ get gains from node-to-node, then combine.

⇒ account for load resistance for each gain calculation for each stage

1st Stage:



$$\frac{v_1}{v_s} = \frac{r_\pi \| R_B}{R_S + r_\pi \| R_B}$$

2nd Stage: