Lecture 27: Low Frequency Circuit Analysis

Announcements:
- HW#8 online and due Friday via Gradescope
- Extend Lab#5 due date by one week
  🔄 Now due Tuesday, Nov. 6, 5 p.m.
- Hopefully, you watched Monday’s video lecture

Lecture Topics:
- Short Circuit Time Constant (SCTC) Analysis
- Intro. to Inspection Analysis
- C.E. Design Project Hints

Last Time:
- Finished OCTC analysis for high frequency

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C.E. High Freq. Analysis Using OCTC

Find ωₜₐₚ:
\[ V_{oc} (jω) \]

We want this frequency, which then gives us
\[ \frac{N_{eq}}{N_q} (s) = A_M (1 + \frac{s}{ωₜₐₚ}) \]

Small Signal Ckt.

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(a) Determine $T_{DAC} = C_T R_m$

- Open C'T, zero out all sources, and determine the driving point impedance, $R_{DAC}$:

\[
R_{DAC} = \frac{N_A}{I_x} = \frac{1}{R_1 R_2 (1 + I_F R_S)} = R' \quad \text{(by inspection)}
\]

\[
T_{DAC} = C_T R'
\]

(b) Determine $T_{LO} = C_L R_L$

- Need $R_{DAC}$: Replace C'T with test source

\[
\frac{R_{DAC}}{I_x} = \frac{N_A}{I_x} = R' + R'' + g_m R' R''
\]

\[
T_{LO} = C_L R''
\]

(c) Determine $\omega_n = \frac{C_T}{T_{DAC}}$

\[
\frac{1}{\omega_n} = \frac{1}{C_T R'} + \frac{1}{C_L R''} + \frac{1}{g_m R' R''}
\]

Thus:

\[
\omega_n = \frac{1}{\frac{1}{T_{DAC}} + \frac{1}{T_{LO}}}
\]
Low Freq. Amplifier Response Using Short Circuit Time Constant Analysis (SCTC)

Recall: \( A_{in}(s) \) midband gain

In general, for the low freq. response:

\[ F(s) = \frac{s^{n_c} + c_1 s^{(n_c-1)} + \ldots}{s^{n_c} + e_1 s^{(n_c-1)} + \ldots} \]

\( n_c \) poles = \# zeros

We can express the coefficient \( e_1 \) by:

\[ e_1 = \omega_p + \omega_{p_2} + \ldots + \omega_{p_{n_c}} \]

For the case of a dominant pole:

4 i.e., the highest freq. pole

\[ F(s) \approx \frac{s}{s + \omega_L} = \frac{\frac{1}{j}}{\frac{1}{j} + \frac{1}{C_j R_{js}}} = \frac{1}{j \omega_L} \]

\[ \omega_L \approx e_1 = \frac{1}{j} \omega_p = \frac{1}{j} \frac{1}{C_j R_{js}} = \frac{1}{j} C_j \]

where \( C_j \) is a very large (>10nF) capacitor in the circuit (e.g., the bypass cap).

\( R_{js} \) is the driving point resistance seen between the terminals of \( C_j \) determined with:

1) all large capacitors short-circuited, except \( C_j \), which is replaced by the test voltage source for \( R_{js} \) determination.

2) all independent sources eliminated (i.e., short voltage source, open current source)

3) open all H.F. capacitors (i.e., small caps in the PF range, or <1nF)

Again, for the case where there are no dominant poles, a reasonable approximation is:

\[ \omega_L \approx \sqrt{\omega_p^2 + \omega_{p_2}^2 - 2 \omega_p \omega_{p_2}} \]

Ex: Determine the L.F. response of the C.E. Amplifier

\[ C_E = 1 \mu F \]

\[ C_f = 2 \mu F \]

\[ R_1 = 1 k \]

\[ R_2 = 1 k \]

\[ R_3 = 1 k \]

\[ R_4 = 1 k \]

\[ R_5 = 1 k \]

\[ R_6 = 1 k \]

\[ V_{cc} \]

\[ R_{in} \]

\[ R_{out} \]

(a) \( R_E \): short chkt. \( C_f \) \& \( C_E \), then determine \( R_{in} \)
Lecture 27w: Low Frequency Circuit Analysis

\[ I_{BS} = C_B \left( R_S + \frac{r_n}{r_b} \right) \]

\[ I_{CS} = C_C \left( R_C + R_e \right) \]

(c) \[ C_E \]: Short \( C_B \) \& \( C_E \); zero out \( N_x \); determine \( R_{Es} \)

\[ R_{Es} = \frac{R_E}{R_{LL}} \]

\[ R_E = \frac{N_x}{r_t} \left( R_{LL} + R_E \right) \]

\[ \frac{1}{C_E} = \frac{1}{C_{Es}} = \frac{1}{C_E \left( R_{LL} + R_E \right)} \]

\[ \omega_P = \frac{1}{\sqrt{L_C R_C}} \]

\[ \omega_L = \frac{1}{\sqrt{L_L}} \]

\[ \omega_L \approx \frac{1}{\sqrt{L_L}} \]

\[ \omega_P \approx \frac{1}{\sqrt{C_E \left( R_{LL} + R_E \right)}} \]

\[ \omega_P \approx \frac{1}{\sqrt{L_L}} \]
Summarize:

- Which capacitors to use for OCTC? Which for SCTC?
- Separate caps into two categories:
  - Large caps → $C_{Lj}'s$
  - Small caps → $C_{Sj}'s$

Determined by large caps $C_{Lj}'s$. Shaped determined by small caps $C_{Sj}'s$.

- Find using SCTC
  - Get time constants using capacitors that contribute to low frequency poles and zeros, which generally means bypass or coupling caps
  - Open smaller capacitors (e.g., hybrid-$\pi$ ones)
  - Use $C_{Lj}'s$: open $C_{Sj}'s$

- Find using OCTC
  - Get time constants using capacitors that contribute high freq. poles & zeros, which generally means hybrid-$\pi$ or any small caps
  - Short larger caps (e.g., bypass or coupling capacitors)
  - Use $C_{Sj}'s$: short $C_{Lj}'s$

Intro. to Inspection Analysis (w/ Labs' hints)

- $C_{Sj}$
- $C_{Lj}$
- $R_5$
- $R_1$
- $R_2$
- $R_3$
- $R_4$
- $R_6$
- $R_7$
- $C_E = 1\mu F$

$1st$ Stage:

Gain from node to node, then combine. Account for load resistance in each gain calculation for each stage.
2nd Stage: