## Lecture 30: Transistor Terminal Gains

- Announcements:
- HW#9 online and due Friday via Gradescope
- Lab#5 due Tuesday, Nov. 6, 5 p.m.
  - If you are using excel or matlab to compute equations for Lab#5 → you will regret doing this when taking midterm 2
  - You need to design by hand at least 3 times before going to a computer
  - This is the only way to get familiar with the process
  - You cannot see the trade-offs without analyzing by hand
  - You need to put the work in to be able to recognize things
- Midterm coming up
  - Friday next week, Nov. 9, @ 5 p.m., in 277 Cory (just like last time)
  - More info this coming Friday
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- Lecture Topics:
  - Generally-Loaded Transistor
    - —Terminal Resistances
    - —Terminal-to-Terminal Gains
    - —Inspection Analysis Sheet
    - —Examples
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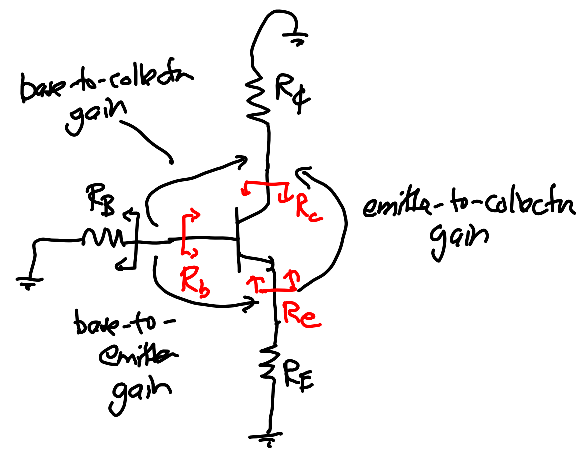
- Last Time:
- Derived inspection analysis terminal resistances
- Now proceed to terminal-to-terminal gains

## Generally Loaded Transistor

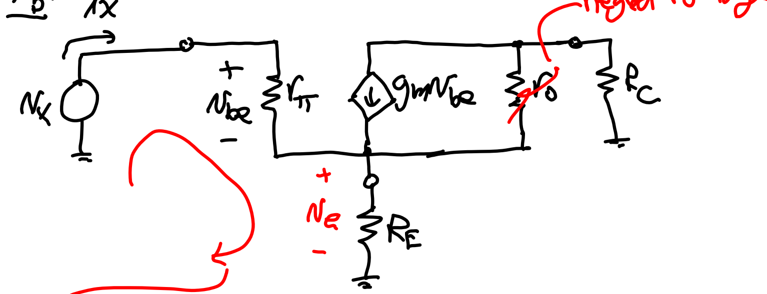



Find the terminal resistances: (i.e., the resistances seen looking into each terminal)

$R_e$: we've already derived this in association w/ finding $R_{ES}$ for $\omega_L$ using SCTC analysis!

$$\boxed{R_e = \frac{r_\pi + R_B}{\beta+1}} \approx \underbrace{\frac{1}{g_m}}_{= r_e} + \frac{R_B}{\beta+1} \quad [r_o \gg R_C]$$

$R_b$:




KVL: $v_x = v_{be} + v_e$

$v_{be} = i_x r_\pi$

$v_e = (i_x + g_m v_{be}) R_E = i_x (1 + g_m r_\pi) R_E$

$v_x = i_x r_\pi + i_x(1 + \overbrace{g_m r_\pi}^{\beta}) R_E$

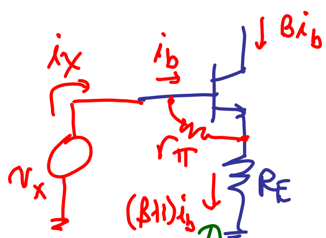
$= i_x (r_\pi + (\beta+1) R_E)$

will be a big R!

good for a V→V amplifier

$$\boxed{R_b = \frac{v_x}{i_x} = r_\pi + (\beta+1) R_E \approx r_\pi (1 + g_m R_E)}$$

$[\beta = g_m r_\pi \gg 1]$



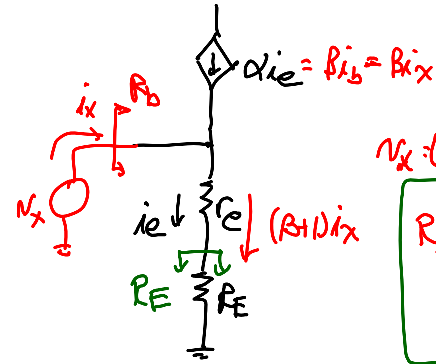

following the currents

$v_x = i_x r_\pi + (\beta+1) i_x R_E$

$R_b = \frac{v_x}{i_x} = r_\pi + \underbrace{(\beta+1) R_E}$

this comes about due to amplification of $i_b$

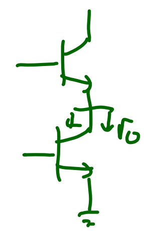
T-Model:



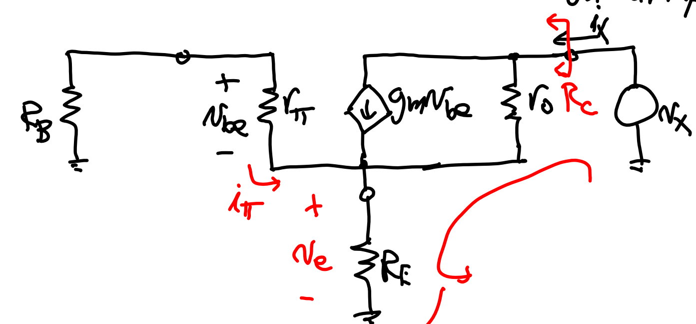

$v_x = (\beta+1) i_x (r_e + R_E)$

$$\boxed{R_b = \frac{v_x}{i_x} = (\beta+1)(r_e + R_E) = (\beta+1)\left(\frac{1}{g_m} + R_E\right)}$$

yet another form of the same thing...




Find $R_c$: (note that $R_B$ can influence, so include in our analysis)




KVL: $v_x = r_o (i_x - g_m v_{be}) + v_e$ → *

[Usually, for cases that matter, $R_E \ll R_B + r_\pi$]

∴ most of $i_x$ flows thru $R_E$:

for this class

∴ $v_e \approx i_x R_E$

$$i_\pi = -\frac{v_e}{r_\pi + R_B} = -\frac{i_x R_E}{r_\pi + R_B}$$

$$\therefore v_{be} = \frac{i_x r_\pi R_E}{r_\pi + R_B}$$

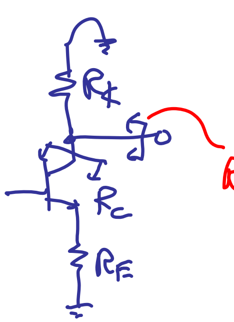
$$* \quad v_x = i_x\left(1+\frac{g_m r_\pi R_E}{r_\pi + R_B}\right) r_o + i_x R_E$$

$$\therefore R_c = \frac{v_x}{i_x} = r_o\left(1+\frac{g_m R_E}{1+(R_B/r_\pi)}\right) + R_E$$

$$\therefore \boxed{R_c \cong r_o\left(1+\frac{g_m R_E}{1+(R_B/r_\pi)}\right) \approx r_o(1+g_m R_E)}$$

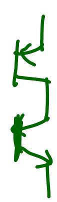
[for $r_\pi \gg R_B$]

even bigger than $r_o$!

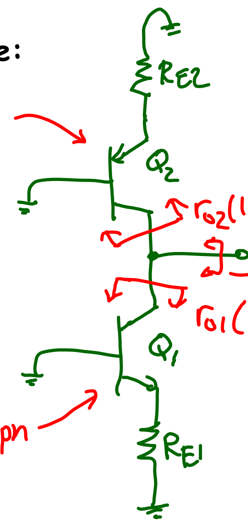



$R_o = R_C \| R_c \approx R_c$

huge



- **Remarks**:
- $R_c$ ~ (1+(0.04)(1k))(100kΩ) ~ 4.1MΩ (this is huge)
- Rarely use $R_c$ in discrete circuits, since it is generally much larger than $R_C$
- In integrated circuits, however, the loading can be very large, especially if it comes from another transistor
- For example:



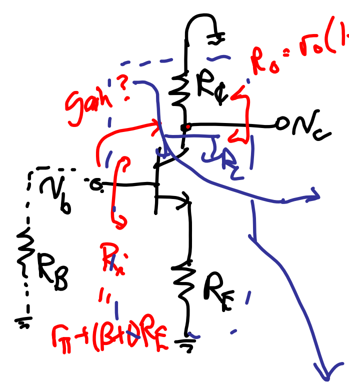

$r_{o2}(1+g_{m2}R_{E2}) \sim 4M\Omega$

$(4M\Omega)\|(4M\Omega) = 2M\Omega$

$r_{o1}(1+g_{m1}R_{E1}) \sim 4M\Omega$

**Base-to-Collector Gain**

$$R_o = r_o\left(1+\frac{g_m R_E}{1+\frac{R_B}{r_\pi}}\right) \| R_c \propto R_c$$

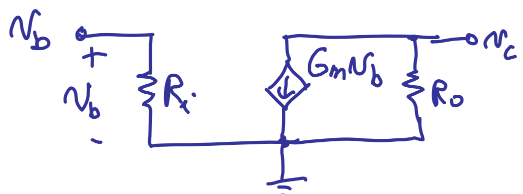



$R_i = r_\pi + (\beta+1)R_E$

$$r_o\left(1+\frac{g_m R_E}{1+\frac{R_B}{r_\pi}}\right)$$

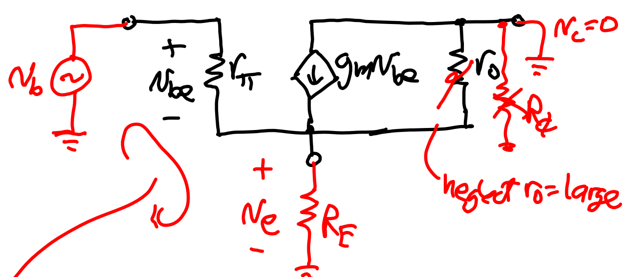
$$\frac{v_c}{v_b} = -G_m R_o = -\left(\frac{g_m}{1+g_m R_E}\right)(R_C \| R_c)$$

convert to an "equivalent Xsistor" y-parameter model




All we need is $G_m \triangleq$ short-ckt transconductance

$$G_m = \frac{i_c}{v_b}\Big|_{v_c=0}$$




$$v_e = \left(\frac{v_{be}}{r_\pi} + g_m v_{be}\right)R_E = v_{be}\left(\frac{1}{r_\pi}+g_m\right)R_E$$

$$v_b = v_{be} + v_e = v_{be}\left(1+\left(\frac{1}{r_\pi}+g_m\right)R_E\right)$$

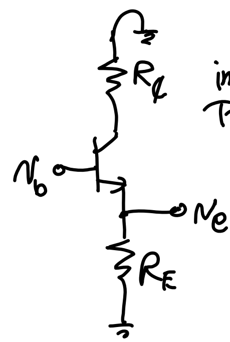
$$i_c = g_m v_{be}$$

$$G_m = \frac{i_c}{v_b}\Big|_{v_c=0} = \frac{g_m \cancel{v_{be}}}{\cancel{v_{be}}\left(1+\underbrace{\left(\cancel{\tfrac{1}{r_\pi}}+g_m\right)}_{\frac{1}{r_\pi} \ll g_m}R_E\right)} = \boxed{\frac{g_m}{1+g_m R_E} = G_m}$$

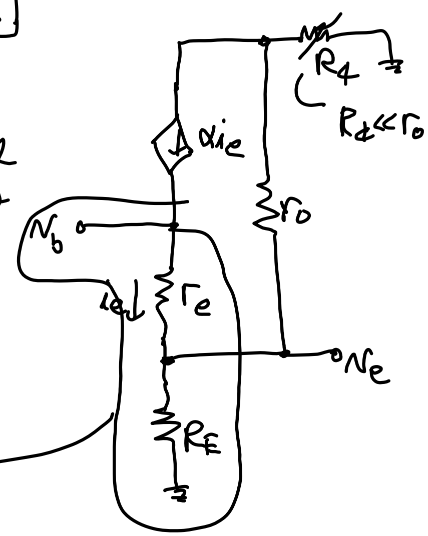
$$\text{Gain} = \frac{v_c}{v_b} = -G_m R_o$$

$$G_m = \frac{g_m}{1+g_m R_E} = \frac{g_m}{1+\frac{R_E}{r_e}}$$

Base-to-Emitter Gain




insert T-model




Voltage Divider!

[$r_o >> R_E$]  kΩ

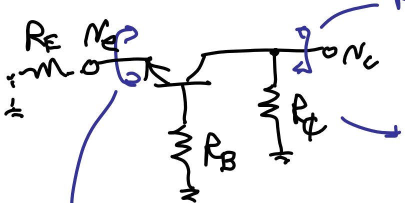
$$\boxed{\frac{v_e}{v_b} \approx \frac{R_E \| r_o}{r_e + R_E \| r_o} \approx \frac{R_E}{r_e + R_E} = \frac{g_m R_E}{1+g_m R_E} \approx 1}$$

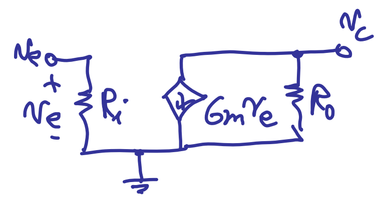
$$\frac{\alpha}{g_m} \approx \frac{1}{g_m} \approx 25\Omega$$

$I_C = 1mA$

## Emitter-to-Collector Gain

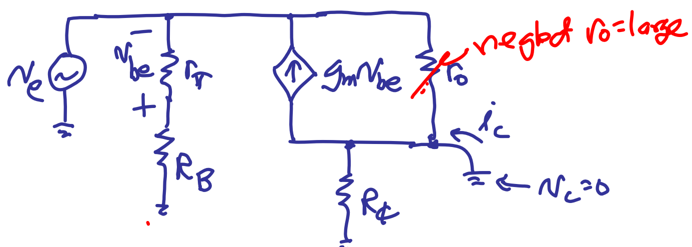
$R_o = R_C \,\|\, r_o\left(1 + \frac{g_m R_E}{\left(1+\frac{R_B}{r_\pi}\right)}\right)$




$R_i = \frac{r_\pi + R_B}{\beta+1}$

Again, all we need is $\underline{G_m = \frac{i_c}{v_e}\Big|_{v_c=0}}$ ≐ short-ckt transconductance




$$v_{be} = -\frac{v_e r_\pi}{r_\pi + R_B}$$

$$\therefore\ i_c = g_m v_{be} = -\frac{g_m r_\pi}{r_\pi + R_B} v_e$$

$$\boxed{G_m = \frac{i_c}{v_e}\Big|_{v_c=0} = -g_m\left(\frac{r_\pi}{r_\pi + R_B}\right)} \rightarrow G_m = -g_m \quad R_B = 0$$

$$\text{Thus, } \boxed{\frac{v_c}{v_e} = -G_m R_o \underset{\text{for the initial ckt}}{=} g_m\left(\frac{r_\pi}{r_\pi + R_B}\right) R_C \overset{\text{if } R_B = 0}{=} g_m R_C}$$

- Now go through the Inspection Analysis Handout