Lecture 37: CMOS Inverter

- Announcements:
- HW#11 online and due during RRR week
- Lab 6 online and due 5 p.m., Friday, Dec. 7
- No ground class today, due to smoke
- This is a video recorded lecture
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- Lecture Topics: (over the next few days)
  - Static CMOS Inverter Behavior
    - V_{OL} and V_{OH}
    - V_{IL} and V_{IH}
  - Dynamic CMOS Inverter Behavior
    - Propagation Delay
    - Capacitance
  - Astable Ring Oscillator
  - CMOS Inverter Propagation Delay
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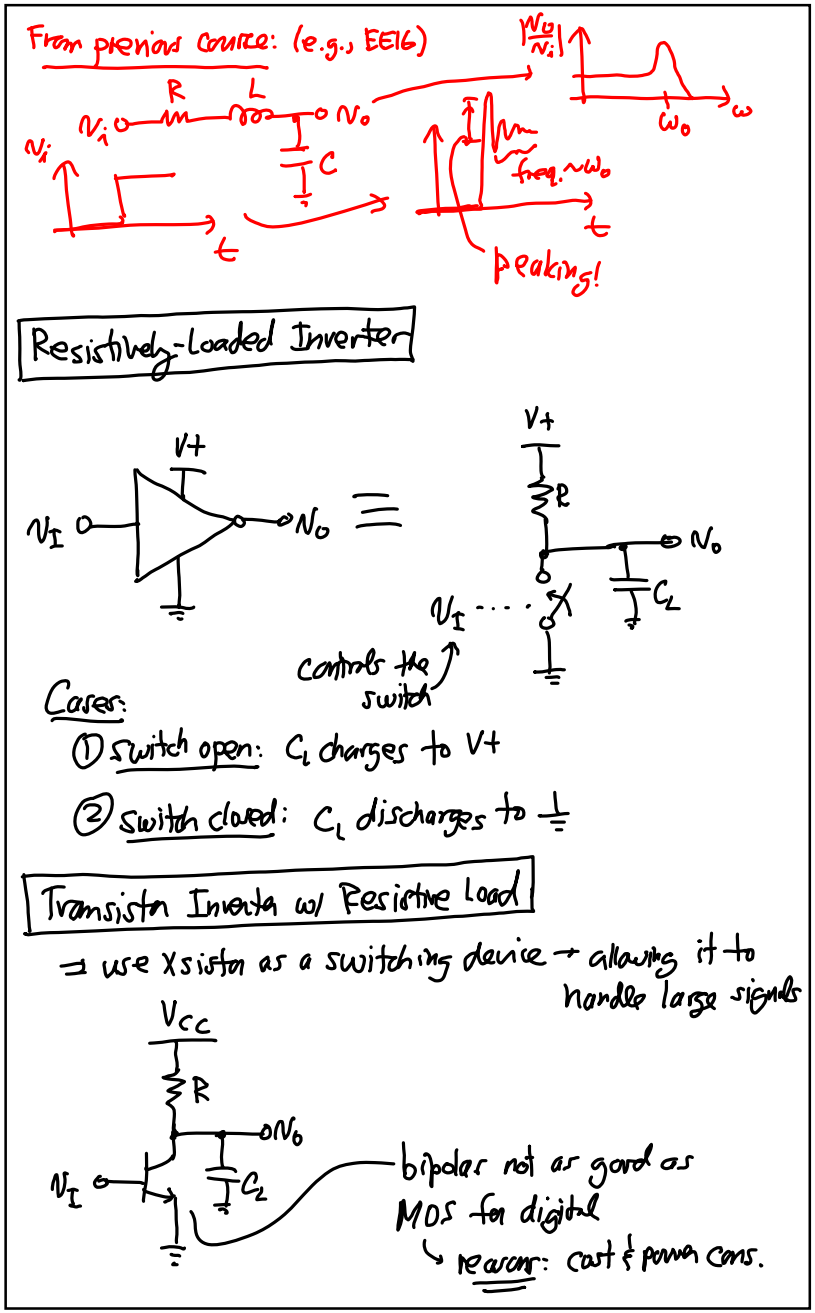
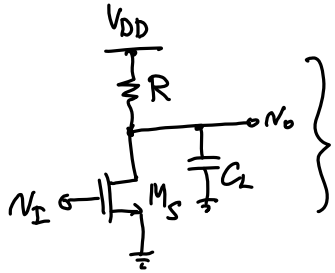
- Last Time:
- A first pass on differential pairs
  - Next pass will be in EE140 …
- Then started digital circuits



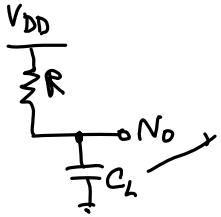


Find V_{OH} & V_{OL}.

Resistively-loaded MOS Inverter

V_{OH}: $v_I \le V_{IL}$ → say = 0V

⇒ M_S off

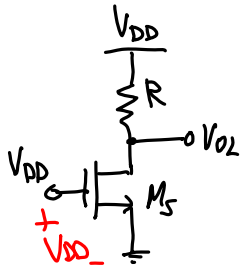


C_L charges to $V_O = V_{DD} = V_{OH}$

V_{OL}: $v_I \ge V_{IH}$ → say = V_{DD}

⇒ M_S on

⇒ in steady-state:



For M_S: $V_{GS} - V_{TNS} = V_{DD} - V_{TNS} > V_{OL}$

∴ M_S in linear!

$V_{OL} = V_{DD} - I_D R$

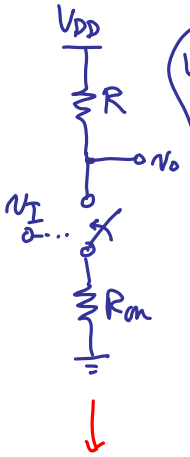
$I_D = \mu_n C_{ox} \left(\frac{W}{L}\right)_S \left(V_{DD} - V_{TNS} - \frac{V_{OL}}{2}\right) V_{OL}$

$V_{OL} = V_{DD} - \mu_n C_{ox} \left(\frac{W}{L}\right)_S (V_{DD} - V_{TNS}) V_{OL} + \frac{\mu_n C_{ox} \left(\frac{W}{L}\right)_S}{2} V_{OL}^2$

⇒ solve quadratic for V_{OL}

⇒ or can get a less accurate (but still good) value...

⇒ ... by defining an on-resistance, R_{on}, for the switching device:

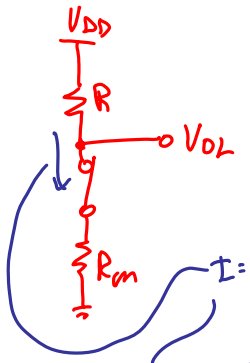


$V_{OL} = V_{DD} \frac{R_{on}}{R + R_{on}} = V_{DD} \left(\frac{1}{1 + \frac{R}{R_{on}}}\right)$

$R_{on} = \frac{V_{DS}}{i_D} = \frac{V_{OL}}{\mu_n C_{ox} \left(\frac{W}{L}\right)_S \left(V_{DD} - V_{TNS} - \frac{V_{OL}}{2}\right) V_{OL}}$

$R_{on} = \frac{1}{\mu_n C_{ox} \left(\frac{W}{L}\right)_S \left(V_{DD} - V_{TNS} - \frac{V_{OL}}{2}\right)}$

When $v_O = V_{OL}$



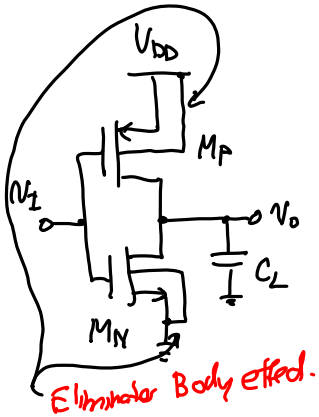
$I = \frac{V_{DD}}{R + R_{on}}$ ⇒ in digital, any current is too much!

say I = 1μA → okay for 100 transistors = 100μA

↳ But for 1 billion → 1000A → way too much

↳ Thus, need lower power consumption → CMOS

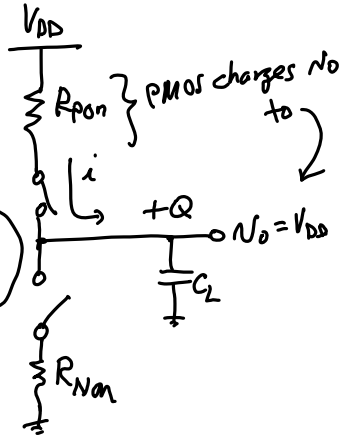
The CMOS Inverter



Get V_{OH} & V_{OL}!

① Case: $V_I < V_{IL}$

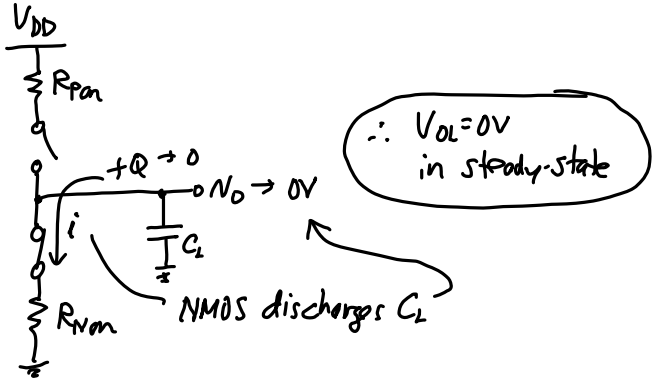
M_P on, M_N off



$\therefore V_{OH} = V_{DD}$ in steady-state

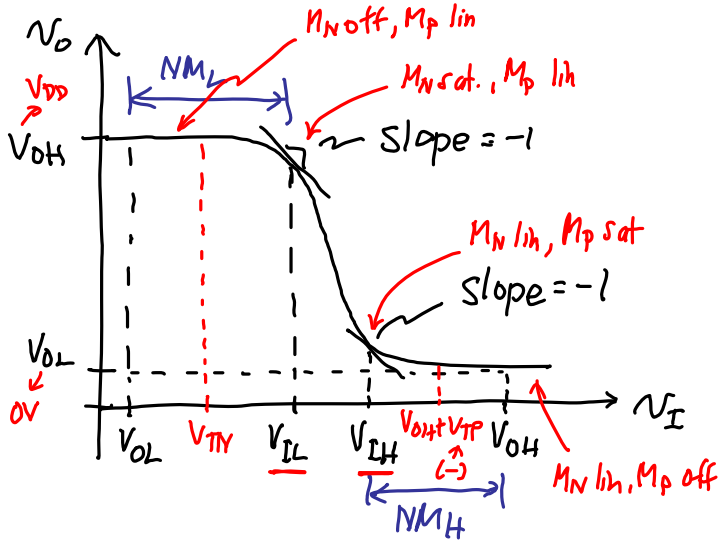
② Case: $V_I > V_{IH}$

M_P off, M_N on



$\therefore V_{OL} = 0V$ in steady-state

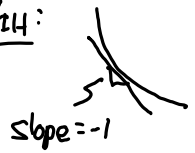
Determination of the Voltage Transfer Characteristic (VTC)



Already know $V_{OL} = 0V$, $V_{OH} = V_{DD}$. } for the CMOS inverter

Now, find $V_{IL}, V_{IH}, NM_L, NM_H$.

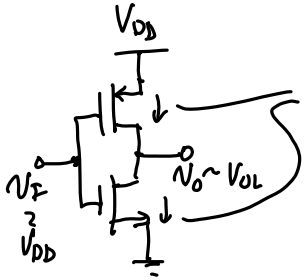
V_{IH}:



M_N linear w/ $V_{GS} = V_I$, $V_{DS} = V_O$

M_P saturated w/ $V_{SG} = V_{DD} - V_I$,

$V_{SD} = V_{DD} - V_O$



must be equal @ steady-state

Thus: (equating drain currents)

$i_{DSN}(lin) = i_{SDP}(sat.)$

$$\underbrace{\mu_n C_{ox}\left(\frac{W}{L}\right)_n}_{k_n}\left[V_I - V_{TN} - \frac{V_O}{2}\right]V_O = \underbrace{\frac{\mu_p C_{ox}}{2}\left(\frac{W}{L}\right)_p}_{k_p}\left(V_{DD} - V_I + V_{TP}\right)^2$$

$$\left[K_R = k_n/k_p\right] \Rightarrow K_R\left(2V_I - 2V_{TN} - V_O\right)V_O = \left(V_{DD} - V_I + V_{TP}\right)^2$$

Solve for V_O: ... algebra ...

$$V_O = V_I - V_{TN} \pm \sqrt{\left(V_I - V_{TN}\right)^2 - \frac{\left(V_{DD} - V_I + V_{TP}\right)^2}{K_R}}$$

$\left[\frac{dV_O}{dV_I} = -1,\ V_I = V_{IH}\right] \Rightarrow$ Lots of math ... using mathematica ...

$$V_{IH} = \frac{2K_R\left(V_{DD} - V_{TN} + V_{TP}\right)}{\left(K_R - 1\right)\sqrt{1 + 3K_R}} - \frac{\left(V_{DD} - K_R V_{TN} + V_{TP}\right)}{K_R - 1}$$

$$= \frac{2K_R\left(V_{DD} - V_{TN} - |V_{TP}|\right)}{\left(K_R - 1\right)\sqrt{1 + 3K_R}} - \frac{\left(V_{DD} - K_R V_{TN} - |V_{TP}|\right)}{K_R - 1}$$

→ one condensed form seen in literature

Problem: Not useful for the popular case when $K_R = 1$, i.e., $K_N = K_P$

For $K_R = 1$, better to go back to the original constraining equations.

For $K_R = 1$:

$$\left(2V_I - 2V_{TN} - V_O\right)V_O = \left(V_{DD} - V_I + V_{TP}\right)^2$$

$$\left(2V_I - 2V_{TN} - V_O\right)\frac{dV_O}{dV_I} + \left(2 - \frac{dV_O}{dV_I}\right)V_O = 2\left(V_{DD} - V_I + V_{TP}\right)\frac{dV_O}{dV_I}$$

$\left[\frac{dV_O}{dV_I} = -1,\ V_I = V_{IH}\right] \Rightarrow$

$$\left(2V_I - 2V_{TN} - V_O\right)(-1) + 3V_O = -2V_{DD} + 2V_I - 2V_{TP}$$

↓ math

$$2V_O + V_{TN} + V_{TP} + V_{DD} = 2V_{IH}$$

$$V_{IH} = \frac{2V_O + V_{TN} + V_{TP} + V_{DD}}{2} \rightarrow V_O = \frac{1}{2}\left(2V_{IH} - V_{TN} - V_{TP} - V_{DD}\right)$$

Lots of math ...

$\left[V_I = V_{IH}\right]$

$$V_{IH} = \frac{5V_{DD} + 3V_{TN} + 5V_{TP}}{8}$$

↓

$$NM_H = V_{OH} - V_{IH} = V_{DD} - V_{IH}$$

$$\therefore\ NM_H = \frac{3V_{DD} - 3V_{TN} - 5V_{TP}}{8}$$

Important Case:

⇒ NMOS & PMOS have identical strength

⇒ $V_{TN} = V_t$, $V_{TP} = -V_t$ (symmetrical V_t magnitudes)

$$V_{IH} = \frac{1}{8}(5V_{DD} - 2V_t), \quad NM_H = \frac{1}{8}(3V_{DD} + 2V_t)$$

- Remarks:
- $K_R = 1$ can can be an important case when one wants the switching point to be at $V_{DD}/2$
- To get this case, one must size the PMOS bigger than the NMOS to give the former equal strength
- But if area is important (for cost reasons), then a designer will choose to use minimum-size devices, in which case the PMOS will not be as strong as the NMOS
  - The switch point will shift a bit from the midpoint of the supply
  - K_R not equal to 1

V_{IL}: Volt — slope = -1 — V_{IL}

M_N saturated w/ $V_{GS} = v_I$, $v_{DS} = v_O$

M_P linear w/ $V_{SG} = V_{DD} - v_I$, $v_{SD} = V_{DD} - v_O$

[Again, equate drain currents] ⇒

$$i_{SDP}(\text{lin}) = i_{DSN}(\text{sat.})$$

$$K_P\left(V_{DD} - v_I + \overset{(-)}{V_{TP}} - \frac{V_{DD} - v_O}{2}\right)(V_{DD} - v_O) = \frac{K_n}{2}(v_I - V_{TN})^2$$

$\left[\frac{dv_O}{dv_I} = -1, v_I = V_{IL}\right]$ ⇒ Again, lots of math...

$$V_{IL} = \frac{2\sqrt{K_R}(V_{DD} - V_{TN} + V_{TP})}{(K_R - 1)\sqrt{K_R + 3}} - \frac{(V_{DD} - K_R V_{TN} + V_{TP})}{K_R - 1}$$

$$= \frac{2\sqrt{K_R}(V_{DD} - V_{TN} - |V_{TP}|)}{(K_R - 1)\sqrt{K_R + 3}} - \frac{(V_{DD} - K_R V_{TN} - |V_{TP}|)}{K_R - 1}$$

→ Again, not useful for $K_R = 1$.

Using a similar procedure to V_{IH}, can get for $K_R = 1$:

$$V_{IL} = \frac{3V_{DD} + 5V_{TN} + 3V_{TP}}{8}, \quad NM_L = V_{IL}$$

... and for $K_R = 1$ & $V_{TN} = V_t$, $V_{TP} = -V_t$

$$V_{IL} = \frac{1}{8}(3V_{DD} + 2V_t) = NM_L$$