EE 105: Microelectronic Devices & Circuits
Lecture 39w: Propagation Delay II

Lecture 39: Propagation Delay II

- Announcements:
- HKN course evaluation facilitators took the first 15 minutes of class
- HW#11 online and due during RRR week
- Lab 6 online and due 5 p.m., Friday, Dec. 7
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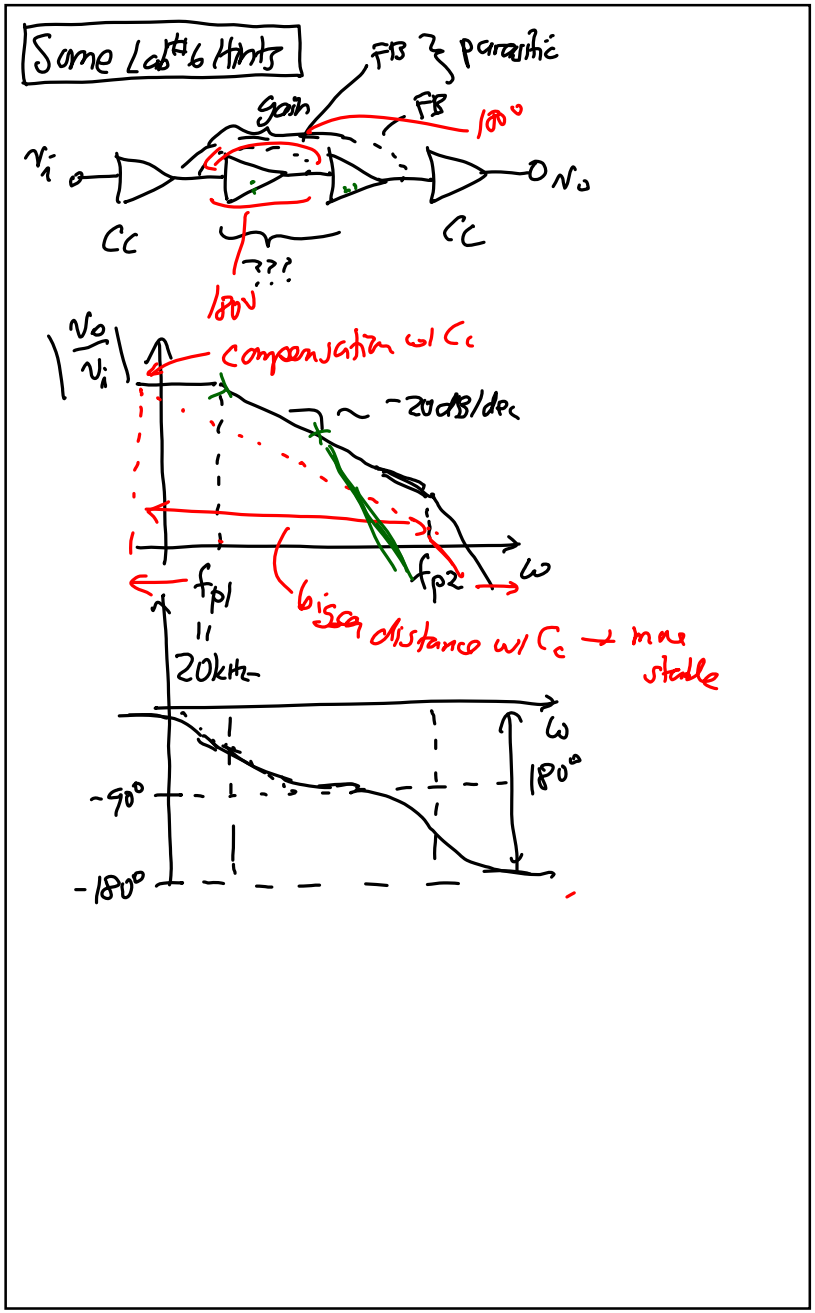
- Lecture Topics: (over the next few days)
  - Static CMOS Inverter Behavior
    - V_{OL} and V_{OH}
    - V_{IL} and V_{IH}
  - Dynamic CMOS Inverter Behavior
    - Propagation Delay
    - Capacitance
  - Astable Ring Oscillator
  - CMOS Inverter Propagation Delay
- ----------
- Last Time:
- Just starting to derive expressions for propagation delay
- Now continue with this …



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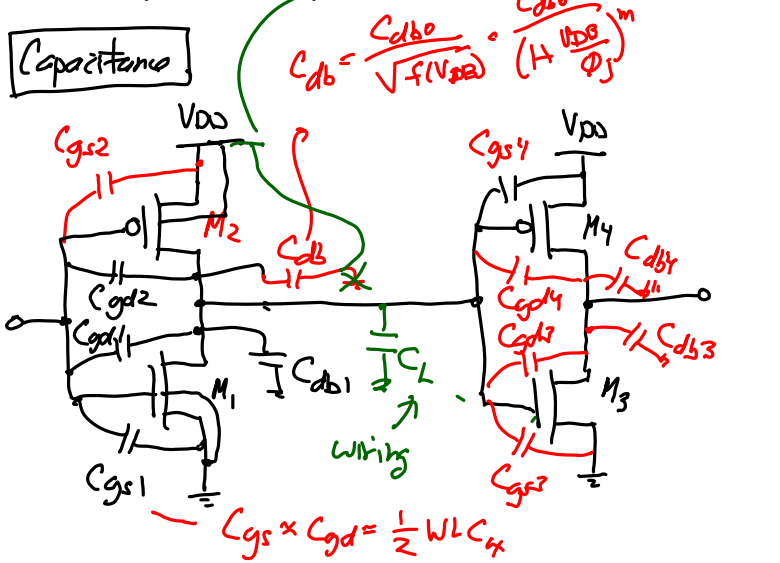
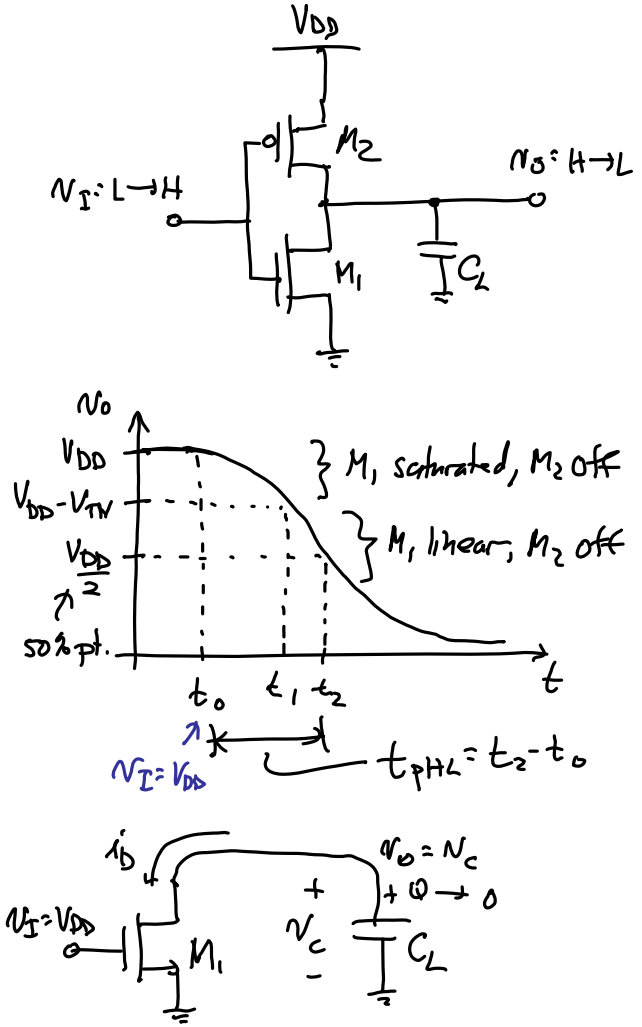
- Remarks
- Propagation delay is the delay experienced by a signal passing through a gate as measured between the 50% transition points between input and output waveforms
- In general, a gate displays different response times for rising and falling input waveforms
- Thus, define:
  - t_{pLH}: response time of a gate making a low→high output transition
  - t_{pHL}: response time of a gate making a high→low output transition
- Propogation delay then defined as the average of t_{pLH} and t_{pHL}
- What causes switching delay?
  - Finite current transistor current drive (i.e., finite on resistance R_{on}
  - Output node capacitance

Capacitance



Propagation Delay in CMOS Inverter



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Case: $v_O \geq V_{DD} - V_{TN} \rightarrow M_1$ saturated

$$i_{D(sat)} = \frac{K_N}{2}(v_{GS} - V_{TN})^2 = -C_L \frac{dv_C}{dt}$$

$$dt = \frac{-2C_L dv_C}{K_N(v_{GS} - V_{TN})^2} \Rightarrow \frac{-2C_L dv_C}{K_N(V_{DD} - V_{TN})^2}$$

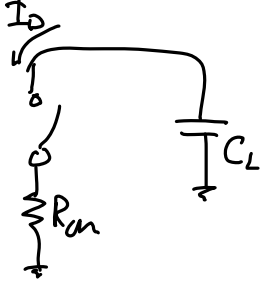
$$[v_{GS} = v_{OH} = V_{DD} = v_I]$$

$$\int_{t_0}^{t_1} dt = \int_{V_{DD}}^{V_{DD}-V_{TN}} -\frac{2C_L}{K_N(V_{DD} - V_{TN})^2}$$

$$(t_1 - t_0) = -\frac{2C_L}{K_N(V_{DD} - V_{TN})^2}(V_{DD} - V_{TN} - V_{DD})$$

$$(t_1 - t_0) = \frac{2C_L}{K_N(V_{DD} - V_{TN})} \frac{V_{TN}}{V_{DD} - V_{TN}} = 2C_L R_{on} \frac{V_{TN}}{V_{DD} - V_{TN}}$$

effective "on resistance" of the transistor $\rightarrow R_{on} = \frac{1}{K_N(V_{DD} - V_{TN})}$



Case: $v_O < V_{DD} - V_{TN} \rightarrow M_1$ linear

$$i_{D(lin)} = -C_L \frac{dv_C}{dt}$$

$$K_N\left(v_{GS} - V_{TN} - \frac{v_{DS}}{2}\right) v_{DS} = -C_L \frac{dv_C}{dt}$$

$$[v_{DS} = v_C, v_{GS} = V_{DD}] \Rightarrow \left\{K_N\left(V_{DD} - V_{TN} - \frac{v_C}{2}\right) v_C = -C_L \frac{dv_C}{dt}\right\} \times 2$$

$$\int_{t_1}^{t_2} \frac{K_N}{2C_L} dt = -\int_{V_1}^{V_2} \frac{dv_C}{[2(V_{DD} - V_{TN}) - v_C] v_C}$$

$$\left[\int \frac{dx}{(a-x)x} = \int \frac{dx}{a(a-x)} + \int \frac{dx}{ax} = \frac{1}{a}\int\left[\frac{1}{a-x} + \frac{1}{x}\right]dx = \frac{1}{a}\ln\left(\frac{x}{a-x}\right)\right]$$

$$[V_2 = \frac{V_{DD}}{2}, V_1 = V_{DD} - V_{TN}] \Rightarrow$$

$$\frac{K_N}{2C_L}(t_2 - t_1) = -\frac{1}{2(V_{DD} - V_{TN})} \ln\left[\frac{v_C}{2(V_{DD} - V_{TN}) - v_C}\right]\Bigg|_{V_{DD} - V_{TN}}^{\frac{V_{DD}}{2}}$$

$$\vdots \text{ algebra}$$

$$= -\frac{1}{2(V_{DD} - V_{TN})} \ln\left[\frac{V_{DD}}{4V_{DD} - 4V_{TN} - V_{DD}}\right]$$