Lecture 3: Frequency Response

- Announcements:
- HW#1 online and due this Friday
- Discussions this week
- Labs start next week
  - Monday, Sept. 3 is a holiday, so the Monday lab will start one week later
  - The Tuesday lab starts Sept. 4
- Lab#1 will be online soon after lecture today
- Will let in concurrent enrollments end of this week
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- Lecture Topics:
  - Finish Digital Communications Example
  - Review Impedance
  - Frequency Response
  - Bode Plots
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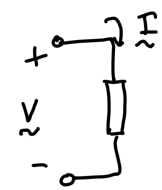
- Last Time:
- Going through an example digital communication transmitter as motivation
- Now, continue with this …

- Review of Analog Circuit Concepts:
- We assume you understand the following concepts from previous courses:
  - Transfer functions
  - Gain (voltage, current, power)
  - Input resistance
  - Output resistance
  - Two-port models for amplifiers
  - Bode plots
  - Ideal op am ckt design and analysis
- We'll review some of these now to jog your memory

$\boxed{\text{Review of Impedance}}$



where $\underset{\sim}{V}$ and $\underset{\sim}{I}$ are phasor variables:

Time — Phasor

$v(t) = V'\cos(\omega t + \phi) \Longleftrightarrow \underset{\sim}{V} = V'e^{j\phi} = V'(\cos\phi + j\sin\phi)$

$i(t) = I'\cos(\omega t + \phi) \Longleftrightarrow \underset{\sim}{I} = I'e^{j\phi}$

and impedance: $Z = \dfrac{\underset{\sim}{V}}{\underset{\sim}{I}}$

Resistor – $R = \dfrac{v(t)}{i(t)} \overset{\text{phasor}}{\Longleftrightarrow} R = \dfrac{\underset{\sim}{V}}{\underset{\sim}{I}}$

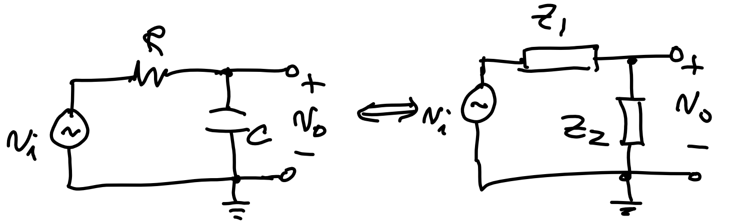
Capacitor – $i(t) = C\dfrac{dv}{dt} \Longleftrightarrow \underset{\sim}{I} = Cj\omega\underset{\sim}{V}$

$\dfrac{\underset{\sim}{V}}{\underset{\sim}{I}} = Z = \dfrac{1}{j\omega C}$ substitute $s = j\omega$ $\rightarrow Z = \dfrac{1}{sC}$

Inductor: $v = L\frac{di}{dt} \Longleftrightarrow \tilde{V} = Lj\omega\tilde{I}$

$$\frac{\tilde{V}}{\tilde{I}} = j\omega L = sL$$

$\boxed{\text{Example: Determine a transfer function}}$



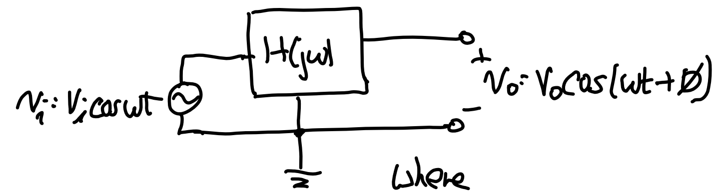
$Z_1 = R$, $Z_2 = \frac{1}{sC}$ } Recognize a voltage divider:

$$\therefore v_o = \frac{Z_2}{Z_1 + Z_2} v_i \Rightarrow \frac{v_o}{v_i}(s) = \frac{\frac{1}{sC}}{R + \frac{1}{sC}}$$

$$\frac{v_o}{v_i}(s) = \frac{1}{1+sRC} = \frac{1}{1+s\tau_p} = \frac{1}{1+\frac{s}{\omega_p}}$$

$[\tau_p = RC]$ $[\omega_p = \frac{1}{RC}]$

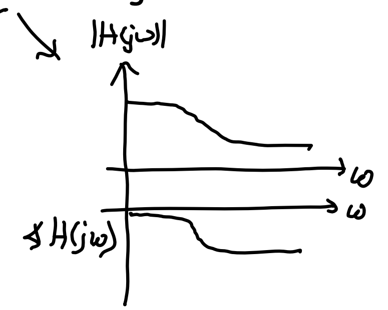
- Frequency Response:
- To measure the frequency response of a given network:
  - Excite the network with a variable-frequency, sinusoidal, constant amplitude source
  - Measure the magnitude and phase of the output signal for different values of input frequency



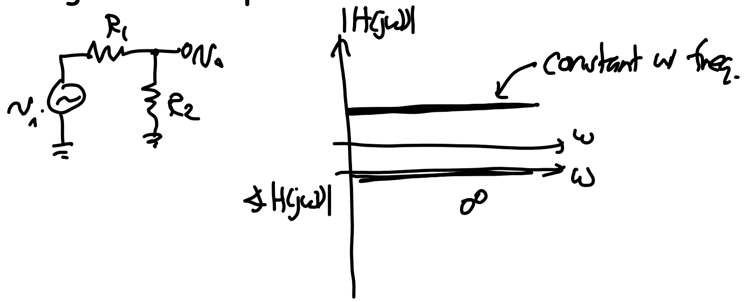
where
$|v_o(j\omega)| = V_o$: magnitude
$\measuredangle v_o(j\omega) = \phi$ = phase

- The frequency response of a given network is commonly described by a plot of magnitude and phase versus frequency. Usually, such a plot is of the transfer function of a given network:

$$H(j\omega) = \frac{v_o(j\omega)}{v_i(j\omega)}$$



- For a purely resistive network, the frequency response is constant (i.e., a straight line), both magnitude and phase



- The addition of reactive (energy storage) components, e.g., capacitors, inductors
  - ↳ Shapes the frequency response
  - ↳ Adds singularities, i.e., poles and zeros
  - ↳ Yields the general transfer function:

$$H(s) = \frac{V_o(s)}{V_i(s)} = \underbrace{H_o}_{\text{resistive term}} \frac{(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots(s-p_n)} = H_o \frac{\prod_{j=1}^{m}(s-z_j)}{\prod_{i=1}^{n}(s-z_i)}$$

where

$$z_j = \sigma_{z_j} + j\omega_{z_j}\ ;\ \sigma_{z_j} = Re(z_j),\ \omega_{z_j} = Im(z_j)$$

$$p_i = \sigma_{p_i} + j\omega_{p_i}\ ;\ \sigma_{p_i} = Re(p_i),\ \omega_{p_i} = Im(p_i)$$

$V_i(s) \triangleq$ input variable (not necessarily a voltage)

V - variable (" " ")

→ given this, you should be able to plot the Bode plot!

$\boxed{\text{Bode Plot}}$

① Plot magnitude in decibels (dB) vs. log (freq.)

$$\left[H(s) = H(j\omega)\right] \Rightarrow |H(j\omega)| = H_o \frac{\prod_{j=1}^{m}|j\omega - z_j|}{\prod_{i=1}^{n}|j\omega - p_i|} =$$

$$= H_o \frac{\prod_{j=1}^{m}|j(\omega-\omega_{z_j}) - \sigma_{z_j}|}{\prod_{i=1}^{n}|j(\omega-\omega_{p_i}) - \sigma_{p_i}|} = H_o \frac{\prod_{j=1}^{m}\sqrt{(\omega-\omega_{z_j})^2 + \sigma_{z_j}^2}}{\prod_{i=1}^{n}\sqrt{(\omega-\omega_{p_i})^2 + \sigma_{p_i}^2}}$$

convert to dB → allows combination by addition rather than products → easier to graph

$$20\log|H(j\omega)| = 20\log H_o + \sum_{j=1}^{m}(20\log|j\omega - z_j|) - \sum_{i=1}^{n}(20\log|j\omega - p_i|)$$