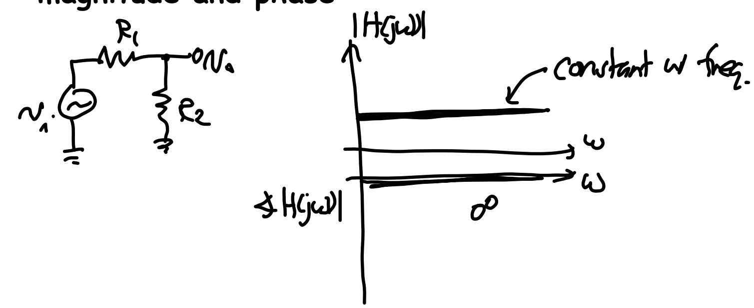
Lecture 4: Amplifiers

- Announcements:
- HW#1 online and due this Friday
- Lab#1 online
- Labs start next week
  - Monday, Sept. 3 is a holiday, so the Monday lab will start one week later
  - The Tuesday lab starts Sept. 4
- Will let in concurrent enrollments end of this week
- ------------------------------------
- Lecture Topics:
  - Finish Bode Plots
  - Amplifiers
  - Amplifier Models (2-port networks)
  - Input R_i
  - Output R_o
  - Ideal Voltage Amplifier
- ------------------------------------
- Last Time:
- Going through procedure for doing a Bode plot
- Now, continue with this …

- For a purely resistive network, the frequency response is constant (i.e., a straight line), both magnitude and phase



- The addition of reactive (energy storage) components, e.g., capacitors, inductors
  - Shapes the frequency response
  - Adds singularities, i.e., poles and zeros
  - Yields the general transfer function:

$$H(s) = \frac{V_o(s)}{V_i(s)} = \underbrace{H_o}_{\text{resistive term}} \frac{(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots(s-p_n)} = H_o \frac{\prod_{j=1}^{m}(s-z_j)}{\prod_{i=1}^{n}(s-z_i)}$$

where

$z_j = \sigma_{z_j} + j\omega_{z_j}$; $\sigma_{z_j} = Re(z_j)$, $\omega_{z_j} = Im(z_j)$

$p_i = \sigma_{p_i} + j\omega_{p_i}$; $\sigma_{p_i} = Re(p_i)$, $\omega_{p_i} = Im(p_i)$

$V_i(s) \triangleq$ input variable (not necessarily a voltage)

V — variable (" " " ")

→ given this, you should be able to plot the Bode plot!

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Bode Plot

① Plot magnitude in decibels (dB) vs. log (freq.)

$$\left[H(s) = H(j\omega)\right] \Rightarrow |H(j\omega)| = H_0 \frac{\prod_{j=1}^{m} |j\omega - z_j|}{\prod_{i=1}^{n} |j\omega - p_i|} =$$

$$= H_0 \frac{\prod_{j=1}^{m} |j(\omega - \omega_{zj}) - \sigma_{zj}|}{\prod_{i=1}^{n} |j(\omega - \omega_{pi}) - \sigma_{pi}|} = H_0 \frac{\prod_{j=1}^{m} \sqrt{(\omega - \omega_{zj})^2 + \sigma_{zj}^2}}{\prod_{i=1}^{n} \sqrt{(\omega - \omega_{pi})^2 + \sigma_{pi}^2}}$$

convert to dB → allows combination by addition rather than products → easier to graph

$$20\log|H(j\omega)| = 20\log H_0 + \sum_{j=1}^{m} \left(20\log|j\omega - z_j|\right) - \sum_{i=1}^{n} \left(20\log|j\omega - p_i|\right)$$

② Plot phase vs. log (freq)

$$\angle H(j\omega) = \sum_{j=1}^{m} \left[\angle(j\omega - z_j)\right] - \sum_{i=1}^{n} \left[\angle(j\omega - p_i)\right]$$

where $\angle(j\omega - s) = \tan^{-1}\left(\frac{\omega - \omega_s}{-\sigma_s}\right)$

$[s = \sigma_s + j\omega_s]$

- Because everything reduces to addition, one can determine the plot for each term then add/substract them together
- Basically, can use superposition for both the magnitude and phase plots

Bode Plot Step-By-Step Procedure

① Get all factors into the form s or $\left(1 + \frac{s}{a}\right)$

e.g., $s + b = b\left(1 + \frac{s}{b}\right)$

② Plot the Bode plot for each factor, one factor at a time. (Note that there are only a few cases.)

③ Sum all decibel magnitude plots to obtain the total magnitude plot.

④ Sum all phase plots to obtain the total phase plot.

Example

$$H(s) = \frac{100s(s+10^5)}{(s+10)(s+10^3)} = \underbrace{\frac{100(10^5)}{(10)(10^3)}}_{10^3 = H_0} \frac{s\left(1 + \frac{s}{10^5}\right)}{\left(1 + \frac{s}{10}\right)\left(1 + \frac{s}{10^3}\right)}$$

Factoring brings out the the gain term

Case: constant value
- constant amplitude = $20\log H_0$
- constant phase = $0°$

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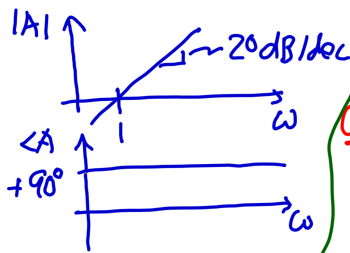
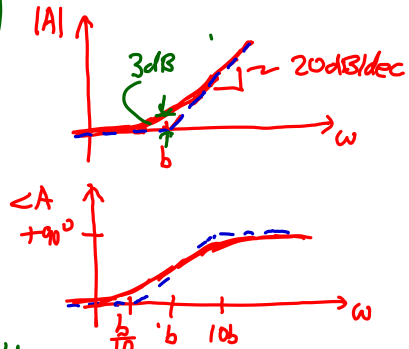
$$H(s) = 10^3 \cdot \frac{s\left(1+\frac{s}{10^5}\right)}{\left(1+\frac{s}{10}\right)\left(1+\frac{s}{10^3}\right)}$$

Case: zero @ origin: $A(s) = s$



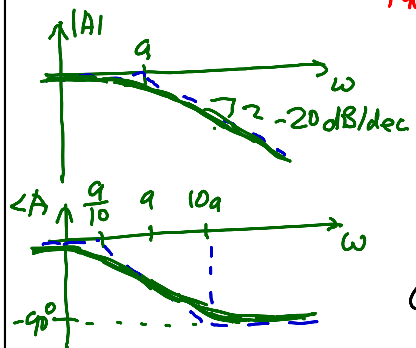
Case: first order zero:

$A(s) = 1 + \frac{s}{b}$



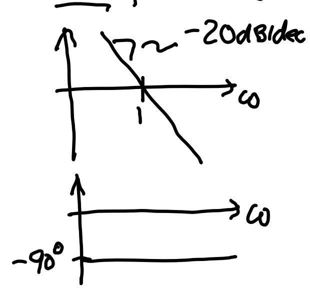
Case: first order pole

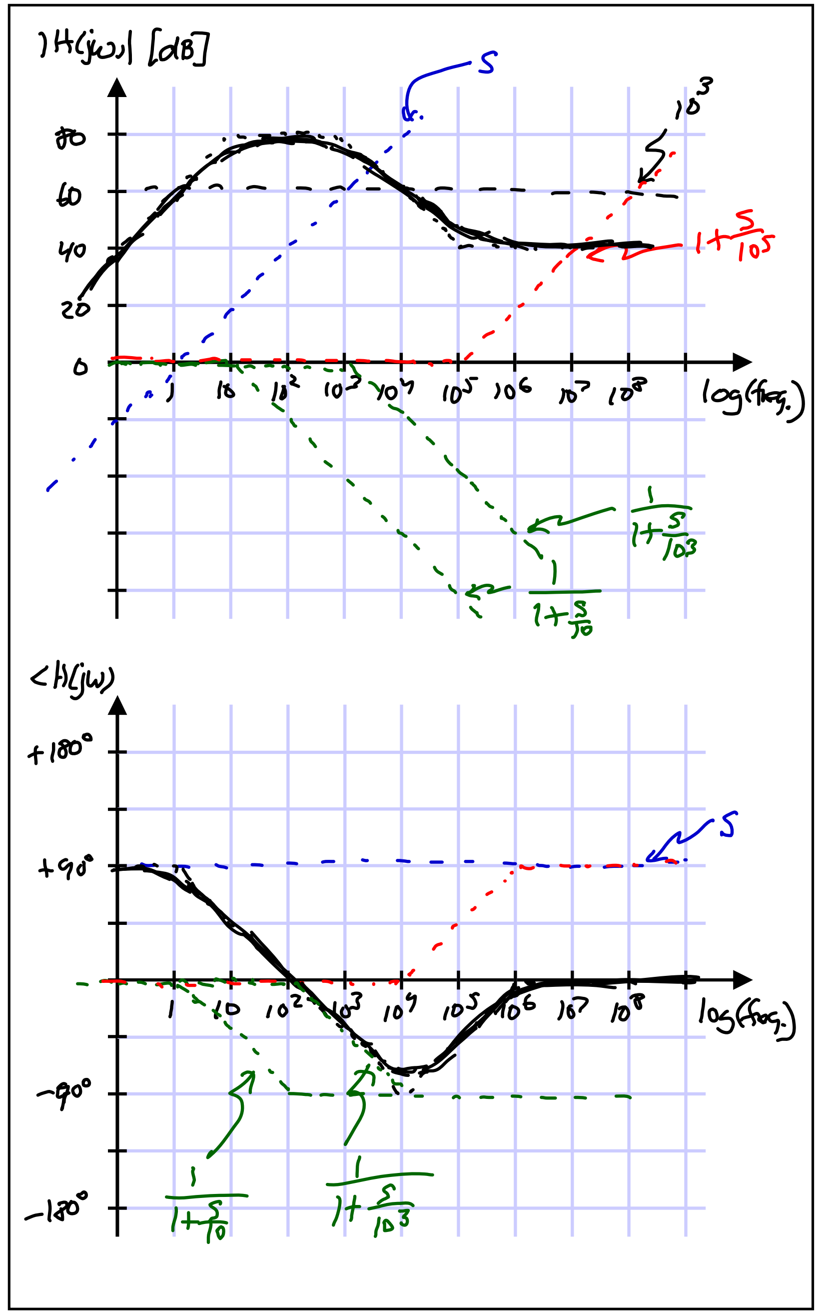
$A(s) = \frac{1}{1+\frac{s}{a}}$



Another useful case (not in this transfer function):

Case: pole @ origin: $A(s) = \frac{1}{s}$





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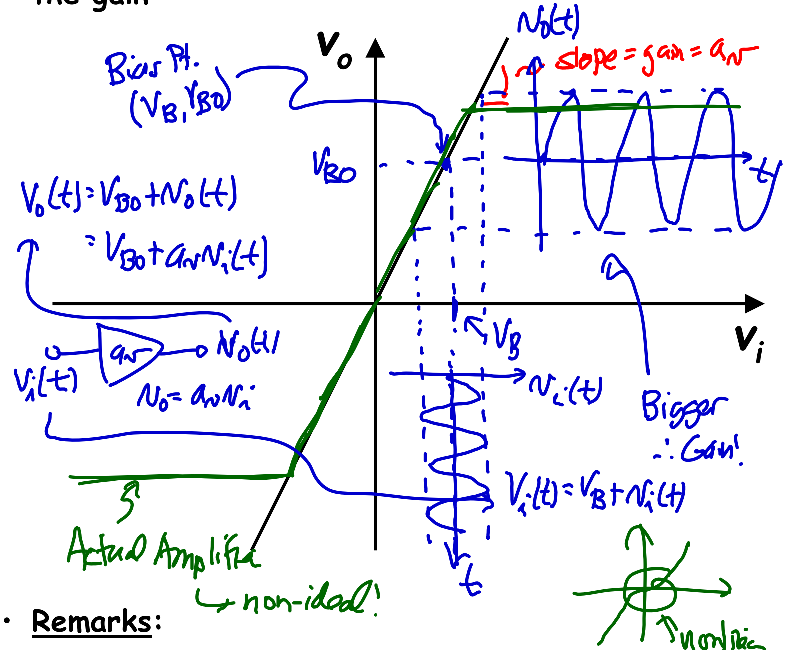
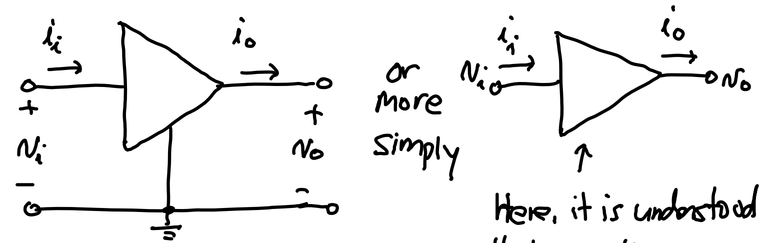
- **Amplification**:
- Really just boils down to creating a transfer function with a large slope, where the slope equals the gain



- **Remarks**:
- The large slope does not come for free
  - ↳ generally requires power if you want power gain
- Ideal amplifier generally has an infinite linear line transfer function
- Power and device non-ideality prevent a truly ideal amplifier
  - ↳ Power rails limit the acceptable input/output range
  - ↳ Device nonlinearity limits the linear range
  - ↳ Noise limits the minimum detectible signal
  - ↳ Parasitic elements, e.g., capacitors, limit the frequency range (i.e., the bandwidth)

$\boxed{\text{Amplifier Models}}$

→ generally given the symbol:



Here, it is understood that everything refers to the same ground

We can interpret a given amplifier as any of four types:

① Voltage Amplifier: $v_i \rightarrow v_o$
(g-parameter model fits best)

② Current Amplifier: $i_i \rightarrow i_o$
(h-parameter model fits best)

③ Transconductance Amplifier: $v_i \rightarrow i_o$
(y-parameter model fits best)

④ Transresistance Amplifier: $i_i \rightarrow v_o$
(z-parameter model fits best)

This class will mainly see these

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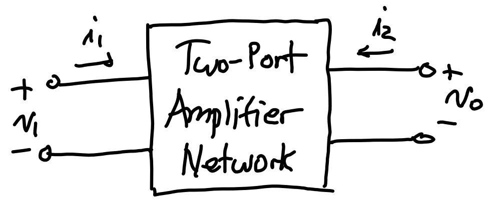
- All of these are equivalent representations, each comprising a gain factor along with an input and output resistance that model the resistance seen looking into the amplifier terminals
- Take for example a voltage amplifier:

$\boxed{\text{Voltage Amplifier}}$

⇒ most appropriate general model is the g-parameter model → Defining Equations:

Note the matrix form →
$$\begin{cases} i_1 = g_{11} v_1 + g_{12} i_2 \\ v_2 = g_{21} v_1 + g_{22} i_2 \end{cases}$$



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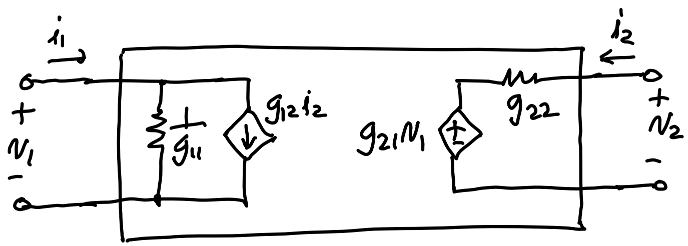
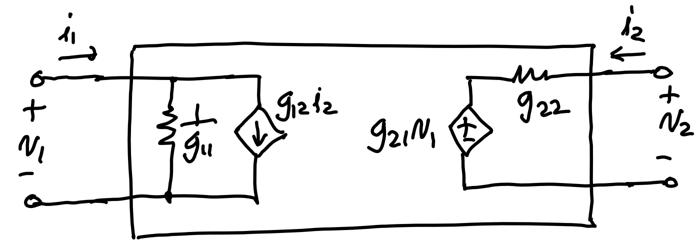




where:

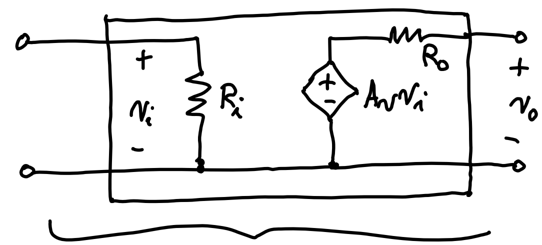
$g_{11} = \left.\frac{i_1}{v_1}\right|_{i_2=0}$ = open-circuit input conductance

$g_{12} = \left.\frac{i_1}{i_2}\right|_{v_1=0}$ = reverse short-circuit current gain

$g_{21} = \left.\frac{v_2}{v_1}\right|_{i_2=0}$ = forward open-circuit voltage gain

$g_{22} = \left.\frac{v_2}{i_2}\right|_{v_1=0}$ = short-circuit output resistance

Assuming a design to amplify in the forward direction → neglect the current source $g_{12} i_2$ in the g-parameter model → the result:



Voltage Amplifier Model

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