Lecture 4: Amplifiers

- Announcements:
  - HW#1 online and due this Friday
  - Lab#1 online
  - Labs start next week
    - Monday, Sept. 3 is a holiday, so the Monday lab will start one week later
    - The Tuesday lab starts Sept. 4
  - Will let in concurrent enrollments end of this week

- Lecture Topics:
  - Finish Bode Plots
  - Amplifiers
  - Amplifier Models (2-port networks)
  - Input $R_i$
  - Output $R_o$
  - Ideal Voltage Amplifier

- Last Time:
  - Going through procedure for doing a Bode plot
  - Now, continue with this ...

- For a purely resistive network, the frequency response is constant (i.e., a straight line), both magnitude and phase

- The addition of reactive (energy storage) components, e.g., capacitors, inductors
  - Shapes the frequency response
  - Adds singularities, i.e., poles and zeros
  - Yields the general transfer function:

$$H(s) = \frac{V_v(s)}{V_i(s)} = \frac{H_0}{\prod \frac{s - z_i}{s - p_i}} = \frac{\prod (s - z_i)^m}{\prod (s - p_i)^n}$$

where

- $Z_j = \sigma_j + j\omega_j$; $\sigma_j: \text{Re}(Z_j)$, $\omega_j: \text{Im}(Z_j)$
- $P_i = \sigma_i + j\omega_i$; $\sigma_i: \text{Re}(P_i)$, $\omega_i: \text{Im}(P_i)$
- $V_v(s)$ = input variable (not necessarily a voltage)
- $V_i$ = variable (not necessarily a voltage)

Given this, you should be able to plot the Bode plot!
• Because everything reduces to addition, one can determine the plot for each term then add/subtract them together.
• Basically, can use superposition for both the magnitude and phase plots.

**Bode Plot Step-By-Step Procedure**

1. Get all factors into the form $S$ or $\left(1 + \frac{S}{2}ight)$
   
   e.g., $5 + b = b \left(1 + \frac{S}{2}\right)$
2. Plot the Bode plot for each factor, one factor at a time. (Note that there are only a few cases.)
3. Sum all decibel magnitude plots to obtain the total magnitude plot.
4. Sum all phase plots to obtain the total phase plot.

**Example**

\[
H(s) = \frac{100.5(s + 10^3)}{(s + 5)(s + 10)} = \frac{100(10^5)}{(s + 10)(s + 10)} \left(1 + \frac{S}{2}\right) \left(1 + \frac{S}{10}\right) \left(1 + \frac{S}{10^2}\right)
\]

\[
\Rightarrow 10^3 \cdot H_0
\]

Factoring brings out the $H_0$

Gain term

Case: Constant value
- Constant amplitude: $20 \log H_0$
- Constant phase: 0°
$$H(s) = 10^3 \frac{s (1 + \frac{s}{10})}{(1 + \frac{s}{10})(1 + \frac{s}{10^3})}$$

Case: Zero @ origin: $$A(s) = s$$

$$\angle A = 180^\circ$$

Case: First order zero: $$A(s) = 1 + \frac{s}{b}$$

$$\angle A = 90^\circ$$

Case: First order pole

$$A(s) = \frac{1}{1 + \frac{s}{a}}$$

$$\angle A = -90^\circ$$

Another useful case (not in this transfer fuction):

Case: Pole @ origin: $$A(s) = \frac{1}{s}$$

$$\angle A = -20 \text{ dB/dec}$$

$$\angle A = 180^\circ$$

$$\angle A = -90^\circ$$

$$\angle A = -180^\circ$$

$$\angle A = 90^\circ$$

$$\angle A = 180^\circ$$
Amplification:
- Really just boils down to creating a transfer function with a large slope, where the slope equals the gain.

Remarks:
- The large slope does not come for free.
- Generally requires power if you want power gain.
- Ideal amplifier generally has an infinite linear line transfer function.
- Power and device non-ideality prevent a truly ideal amplifier.
  - Power rails limit the acceptable input/output range.
  - Device nonlinearity limits the linear range.
  - Noise limits the minimum detectable signal.
  - Parasitic elements, e.g., capacitors, limit the frequency range (i.e., the bandwidth).

Amplifier Models:
- Generally given the symbol:

We can interpret a given amplifier as any of four types:
1. Voltage Amplifier: \( V_i \to V_o \)
   - \( g \)-parameter model fits best.
2. Current Amplifier: \( i_i \to i_o \)
   - \( h \)-parameter model fits best.
3. Transconductance Amplifier: \( V_i \to i_o \)
   - \( y \)-parameter model fits best.
4. Transresistance Amplifier: \( i_i \to V_o \)
   - \( z \)-parameter model fits best.

This class will mainly see these.
• All of these are equivalent representations, each comprising a gain factor along with an input and output resistance that model the resistance seen looking into the amplifier terminals.
• Take for example a voltage amplifier:

**Voltage Amplifier**

**g-parameter model** — Defining Equations:

Note the matrix form:

\[ i_1 = g_{11} V_1 + g_{12} i_2 \]
\[ N_2 = g_{21} V_1 + g_{22} i_2 \]

where:

\[ g_{uu} = \frac{\dfrac{i_1}{V_1}}{i_2 = 0} = \text{open-circuit input conductance} \]
\[ g_{12} = \frac{\dfrac{i_1}{V_1}}{i_2 = 0} = \text{reverse short-circuit current gain} \]
\[ g_{21} = \frac{\dfrac{N_2}{V_1}}{i_2 = 0} = \text{forward open-circuit voltage gain} \]
\[ g_{22} = \frac{\dfrac{N_2}{i_2}}{V_1 = 0} = \text{short-circuit output resistance} \]

Assuming a design to amplify in the forward direction — neglect the current source \( g_{12} i_2 \) in the **g-parameter model** — the result:

![Voltage Amplifier Model Diagram](image-url)