Lecture 6: Finite Gain & Bandwidth

- **Announcements**:
- HW#2 online and due Friday via Gradescope
- Lab#2 online
- Lots of SPICE this week
- Approved all concurrent enrollments
- Next Wednesday, 9/12: I will be out of town.
  - So no ground lecture that day
  - Make up will probably need to be by video recording and put online
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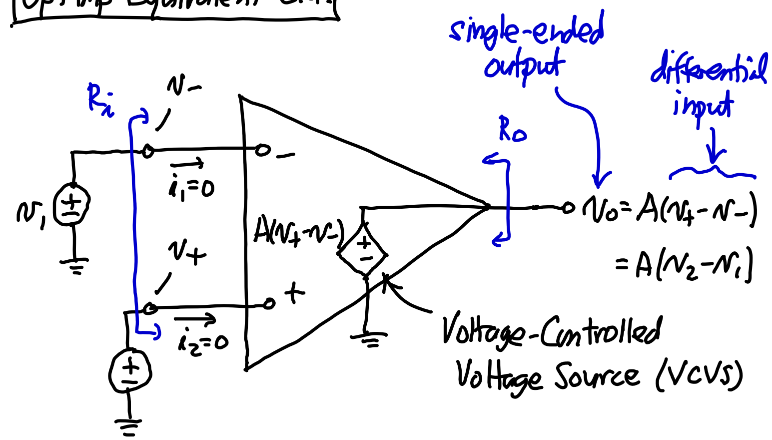
- Lecture Topics:
  - Ideal Op Amp Circuits
  - Non-Ideal Op Amp Bode Plot
  - Finite Gain & Bandwidth
  - Closed Loop Amplifier Freq. Response
    - —Non-Inverting Amplifier
    - —Inverting Amplifier
- ------------------------------------
- **Last Time**:
- Op amp circuit example
- Now, continue with this …

Perhaps the best way to define an op amp is via its equivalent ckt:

$\boxed{\text{Op Amp Equivalent Ckt.}}$



Properties of Ideal Op Amps:

① $R_i = \infty \longrightarrow$ ④ $i_+ = i_- = 0$

② $R_o = 0$

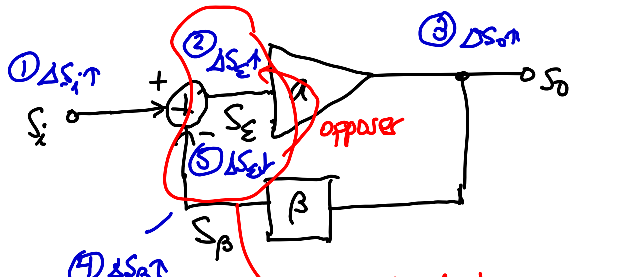
③ $A = \infty \longrightarrow$ ⑤ $v_+ = v_-$, assuming v_o = finite

Why? $\infty(v_+ - v_-) = v_o$ = finite

$\therefore v_+ - v_- = 0 \rightarrow \underbrace{v_+ = v_-}_{\text{Source of "virtual ground"}}$

Big Assumption!
↳ only holds when we have negative feedback!

Negative Feedback

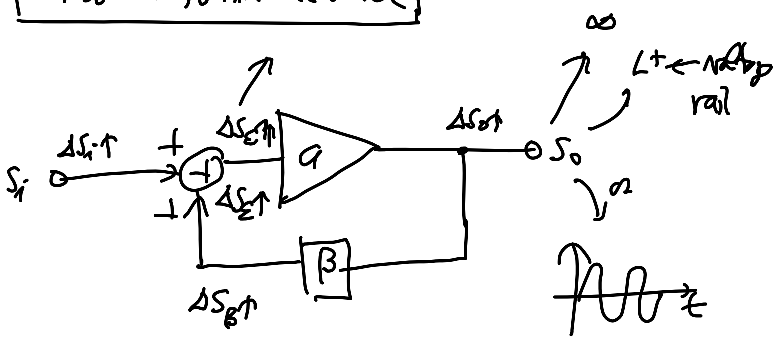


Neg. FB acts to oppose or subtract from the input.

$$\left.\begin{aligned} S_o &= aS_\varepsilon \\ S_\varepsilon &= S_i - \beta S_o \end{aligned}\right\} \quad S_o = a(S_i - \beta S_o), \quad S_o(1+a\beta) = aS_i \rightarrow \boxed{\frac{S_o}{S_i} = \frac{a}{1+a\beta}}$$

$$[a\to\infty] \Rightarrow \frac{S_o}{S_i} \approx \frac{a}{a\beta} = \frac{1}{\beta} = \text{finite!}$$

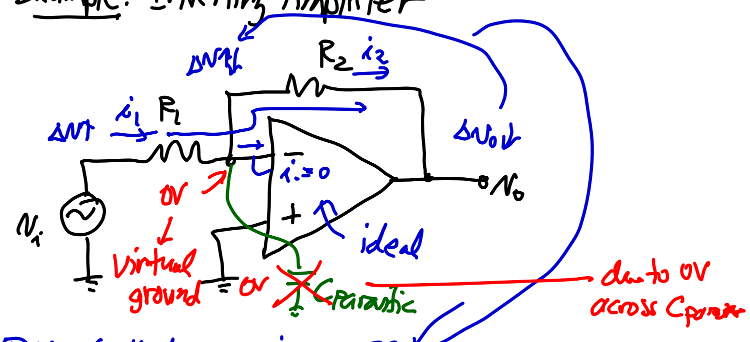
Contrast w/ Positive Feedback



- **Remarks**: (on neg. FB)
- Neg. FB can insure v_o = finite even with a=infinity
- Overall closed-loop gain (or transfer function) is dependent only on external components (e.g., β)
- Overall closed-loop gain S_o/S_i is independent of amplifier gain a
- This is very important, since it's easy to get large amplifier gain, but it's hard to get an exact value
  - ↳ If you're shooting for a=50,000, you might get 47,000 or 60,000 instead
  - ↳ But it won't matter much in the feedback ckt.

Op Amp Ckts.

Example. Inverting Amplifier



① Verify that there is neg. FB! → ∴ ideal op amp rules apply.

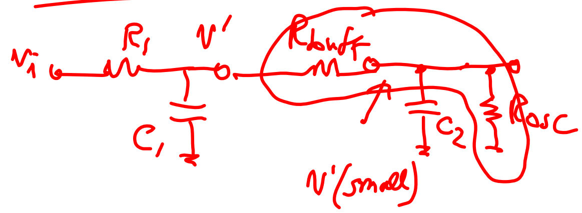
② ∴ v_o = finite → $v_+ = v_-$ → virtual ground

③ $i_- = 0$, $i_1 = i_2$

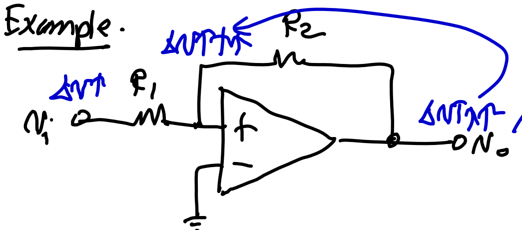
$$\left.\begin{aligned} i_1 &= \frac{v_i - 0}{R_1}, \quad \frac{v_i}{R_1} = i_2 \\ v_o &= -i_2R_2 \end{aligned}\right\} \quad v_o = -\left(\frac{v_i}{R_1}\right)R_2 = -\frac{R_2}{R_1}v_i$$

$$\therefore \boxed{\frac{v_o}{v_i} = -\frac{R_2}{R_1}}$$

Your Lab#1:



Example.



$v_o = L^+$ or L^- depending on initial conditions

① verify neg. FB — ✗

$\therefore$ $v_o \neq$ finite $\rightarrow$ $v_+ \neq v_-$

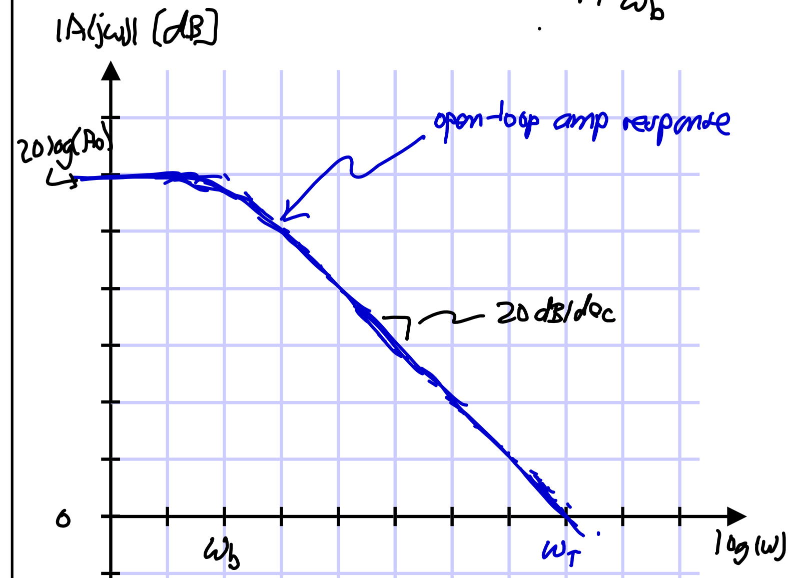
$\Rightarrow$ this ckt. will rail out!

Non-Ideal Op Amps

Finite Op Amp Gain & Bandwidth

For an ideal op amp, $A = \infty$.

In reality, the gain goes as: $A(s) = \dfrac{A_o}{1+\frac{s}{\omega_b}}$



$\omega_T \triangleq$ unity gain frequency: freq. @ which $|A(j\omega)| = 1$ ($= 0$dB)

At ω_T, $|A(j\omega_T)| = 1 = \dfrac{A_o}{\sqrt{1+\left(\frac{\omega_T}{\omega_b}\right)^2}} \Rightarrow \dfrac{A_o}{\frac{\omega_T}{\omega_b}} = 1 \rightarrow \boxed{\omega_T = A_o\omega_b}$

$[\omega_T \gg \omega_b]$

gain Bandwidth

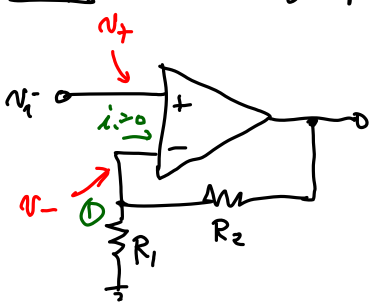
[For $\omega \gg \omega_b$] $\Rightarrow$ $A(s) = \dfrac{A_o}{\frac{s}{\omega_b}} = \dfrac{A_o\omega_b}{s} = \dfrac{\omega_T}{s} \Longrightarrow \dfrac{f_T}{f}$

ω_T in datasheets

(an op amp ultimately is an integrator w/ time constant $\tau = \frac{1}{\omega_T}$)

Frequency Response of Closed Loop Amplifier

Example. Non-Inverting Amplifier



$v_o = A(s)(v_+ - v_-)$

$v_- = v_+ - \dfrac{v_o}{A(s)}$

$v_- = v_i - \dfrac{v_o}{A(s)}$

Find an expression for gain as a function of freq.

① Brute Force Determination:

KCL①: $\dfrac{v_o - v_-}{R_2} = \dfrac{v_-}{R_1} \rightarrow \dfrac{v_o}{R_2} = v_-\left(\dfrac{1}{R_1} + \dfrac{1}{R_2}\right)$

$\dfrac{v_o}{R_2} = \left(v_i - \dfrac{v_o}{A(s)}\right)\left(\dfrac{1}{R_1} + \dfrac{1}{R_2}\right) \rightarrow \dfrac{v_o}{v_i}(s) = \dfrac{1 + \frac{R_2}{R_1}}{1 + \frac{1}{A(s)}\left(1 + \frac{R_2}{R_1}\right)}$

$\left[A(s) = \dfrac{A_o}{1 + \frac{s}{\omega_b}}\right] \Rightarrow \boxed{\dfrac{v_o}{v_i}(s) = \left(1 + \dfrac{R_2}{R_1}\right)\dfrac{1}{1 + \dfrac{s}{A_o\omega_b\left(\frac{R_1}{R_1 + R_2}\right)}}}$

