

Lecture 7: Non-Ideal Op Amp Circuits

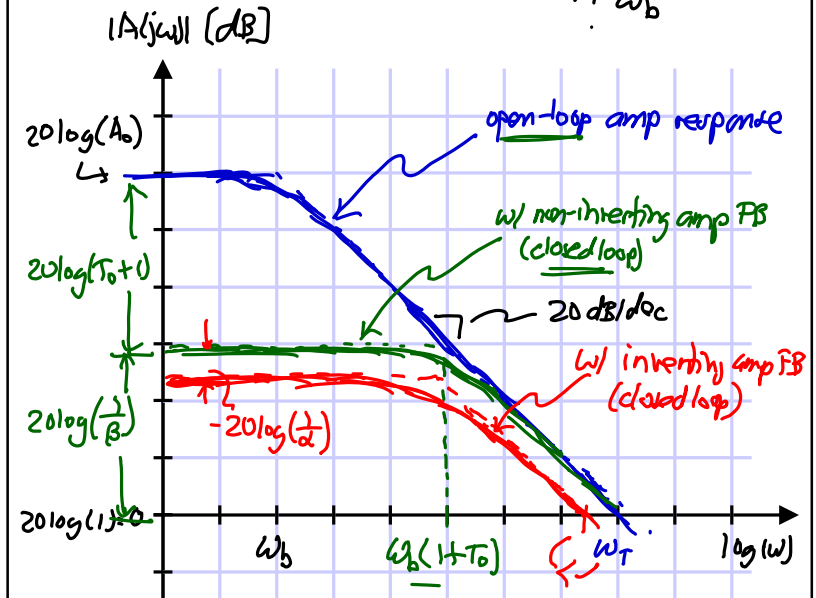
- Announcements:
- HW#3 online and due Friday via Gradescope
- Late Homework Policy:
 - ↳ -10% per day
 - ↳ It's -10% one second after Friday noon
 - ↳ -20% on second after Saturday noon
 - ↳ ...
- Regrades:
 - ↳ Submit HW w/ a note with reason
 - ↳ HW will go back to reader, which will take time
- We will NOT drop a HW
 - ↳ It's only 10% of your grade and it's better that you at least do some problems in a HW
- Next Wednesday, 9/12: I will be out of town.
 - ↳ So no ground lecture that day
 - ↳ Make up will probably need to be by video recording and put online
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- Lecture Topics:
 - ↳ Closed Loop Amplifier Freq. Response Using Finite Gain-BW Op Amps
 - Non-Inverting Amplifier
 - Inverting Amplifier
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- Last Time:
- Non-inverting amplifier using finite gain-BW op amp
- Now, continue with this ...

Non-Ideal Op Amps

Finite Op Amp Gain & Bandwidth

For an ideal op amp, $A = \infty$.

In reality, the gain goes as: $A(s) = \frac{A_0}{1 + \frac{s}{\omega_b}}$



$\omega_T \triangleq$ unity gain frequency = freq. @ which $|A(j\omega)| = 1$ ($= 0 \text{ dB}$)

$$\text{At } \omega_T, |A(j\omega)| = 1 = \frac{A_0}{\sqrt{1 + \left(\frac{\omega_T}{\omega_b}\right)^2}} \Rightarrow \frac{A_0}{\frac{\omega_T}{\omega_b}} = 1 \rightarrow \boxed{\omega_T = A_0 \omega_b}$$

[$\omega_T \gg \omega_b$]

↑ gain ↑ Bandwidth

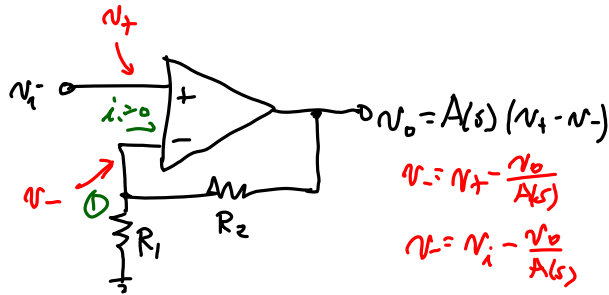
(For $\omega \rightarrow \omega_b$) $\Rightarrow A(s) = \frac{A_0}{s} : \frac{A_0 \omega_b}{s} = \frac{\omega_T}{s} \Rightarrow \frac{f_T}{f}$

ω_T in data sheets

(an op amp ultimately is an integrator w/ time constant $\tau = \frac{1}{\omega_T}$)

Frequency Response of Closed Loop Amplifier

Example. Non-Inverting Amplifier



Find an expression for gain as a function of freq.

① Brute Force Determination:

KCL: $\frac{v_o - v_-}{R_2} = \frac{v_-}{R_1} \rightarrow \frac{v_o}{R_2} = v_- \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$

$\frac{v_o}{R_2} = \left(v_i - \frac{v_o}{A(s)} \right) \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \rightarrow \frac{v_o}{v_i}(s) = \frac{1 + \frac{R_2}{R_1}}{1 + \frac{1}{A(s)} \left(1 + \frac{R_2}{R_1} \right)}$

$\left[A(s) = \frac{A_0}{1 + \frac{s}{\omega_b}} \right]$

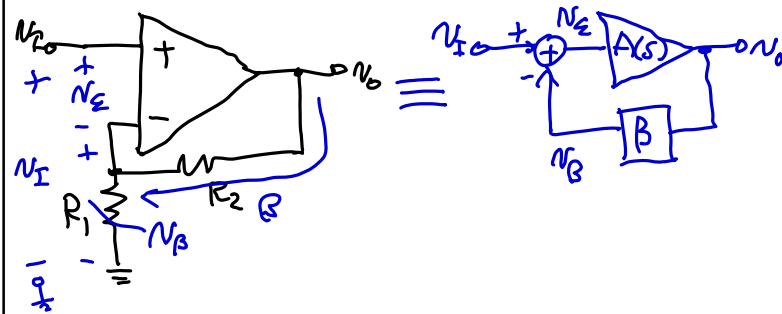
* $\frac{v_o}{v_i} = \frac{1 + \frac{R_2}{R_1}}{1 + \frac{1}{A_0} \left(1 + \frac{R_2}{R_1} \right)} \frac{1}{1 + \frac{s}{\omega_b} \frac{1}{A_0} \left(1 + \frac{R_2}{R_1} \right)}$ } finite gain & BW

$\left[A_0 \text{ large} \rightarrow \frac{1}{A_0} \left(1 + \frac{R_2}{R_1} \right) \ll 1 \right] \rightarrow$ Here, we're essentially saying that the op amp gain is close to ideal & that the non-ideality is merely finite BW.

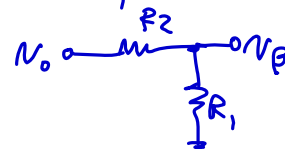
$\frac{v_o}{v_i}(s) = \left(1 + \frac{R_2}{R_1} \right) \frac{1}{1 + \frac{s}{A_0 \omega_b} \left(\frac{R_1 + R_2}{R_1} \right)}$

Near Ideal Gain, but Finite BW

② More insightful way to do this.



What is β ?

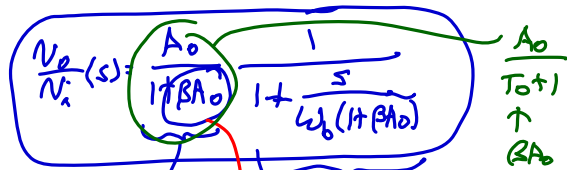


$\beta = \frac{v_B}{v_o} = \frac{R_1}{R_1 + R_2}$

Recall fr previous FB analysis:

$$\frac{V_o}{V_i}(s) = \frac{A(s)}{1 + \beta A(s)}$$

$$\left[A(s) = \frac{A_o}{1 + \frac{s}{\omega_b}} \right] \rightarrow \frac{V_o}{V_i}(s) = \frac{\frac{A_o}{1 + \frac{s}{\omega_b}}}{1 + \beta \left(\frac{A_o}{1 + \frac{s}{\omega_b}} \right)}$$



DC gain term
(midband gain)

Frequency Shaping Term

$T_o = \beta A_o \triangleq$ "loop gain" @ $\omega = 0$
(i.e., @ DC)

If $A_o \rightarrow \infty$ or
if $\beta A_o \gg 1$ } \Rightarrow DC gain = $\frac{1}{\beta}$

Plug in β :

$$\left[\beta A_o \gg 1 \right] \Rightarrow \frac{V_o}{V_i}(s) \approx \frac{1}{\beta} \frac{1}{1 + \frac{s}{\omega_b \beta A_o}}$$

$$= \left(1 + \frac{R_2}{R_1} \right) \frac{1}{1 + \frac{s}{\omega_b A_o \left(\frac{R_1}{R_1 + R_2} \right)}}$$

Observations:

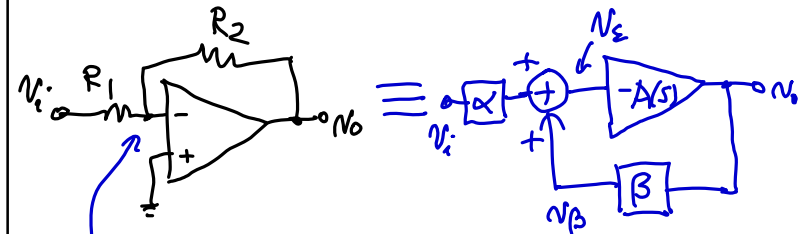
① Closed loop DC gain = $\frac{A_o}{1 + \beta A_o} = \frac{A_o}{1 + T_o} \approx \frac{A_o}{T_o}$
 i.e., the closed loop gain is reduced from the open loop gain by $1 + T_o \rightarrow$ show this on graph
 [$T_o \gg 1$]

② Alternatively, closed loop DC gain $\approx \frac{A_o}{\beta A_o} = \frac{1}{\beta}$ [$T_o \gg 1$]

③ ω_{-3dB} has increased from $\omega_b \rightarrow \omega_b (1 + \beta A_o) = \omega_b (1 + T_o)$
 To draw the Bode plot, just find the dc gain, draw a horizontal line across, then follow the open loop response after running into it!

④ Gain-BW Product = $\frac{A_o}{1 + \beta A_o} \omega_b (1 + \beta A_o) = A_o \omega_b = \omega_T$
 \therefore the Gain-BW product remains the same for the open & closed loop FB cases!

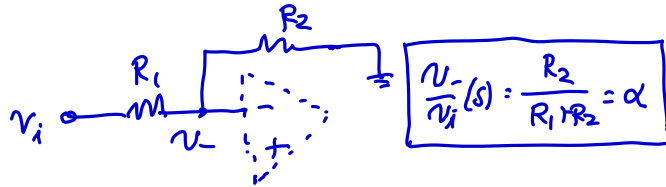
Example - Inverting Amplifier



signals from v_o & v_i sum here
 how much of each appears depends on α & β

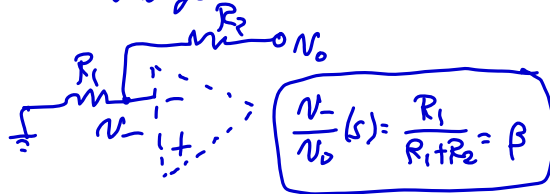
Determine α :

- ① Ground output & open loop
- ② Feed v_i forward to get T.F.



Determine β :

- ① Ground input, open loop
- ② Feedback v_o to get T.F.



Now, get the T.F. for the system block diagram:

$$\left. \begin{aligned} v_o &= -A(s)v_{\Sigma} \\ v_{\Sigma} &= \alpha v_i + \beta v_o \end{aligned} \right\} \begin{aligned} v_o &= -\alpha A(s)v_i - \beta A(s)v_o \\ v_o(1 + \beta A(s)) &= -\alpha A(s)v_i \end{aligned}$$

$$\therefore \frac{v_o}{v_i}(s) = -\frac{\alpha A(s)}{1 + \beta A(s)}$$

$$\left[A(s) = \frac{A_o}{1 + \frac{s}{\omega_b}} \right] \Rightarrow \frac{v_o}{v_i}(s) = \frac{-\alpha \frac{A_o}{1 + \frac{s}{\omega_b}}}{1 + \frac{\beta A_o}{1 + \frac{s}{\omega_b}}}$$

$$\frac{v_o}{v_i}(s) = \frac{-\alpha A_o}{1 + \beta A_o} \frac{1}{1 + \frac{s}{\omega_b(1 + \beta A_o)}}$$

V_{os} fn Lab:

