Problem Set #7: Solutions

\[ V_{DD} = 9 \text{ V} \quad I_0 = 1 \text{ mA} \]
\[ V_4 = 1 \text{ V} \]
\[ \beta = 0 \]
\[ k_c = 2 \text{ mA/V}^2 \]

Assume saturation:
\[ I_0 = \frac{1}{2} \mu C_\text{ox} \frac{W}{L} (V_{DD} - V_0)^{(1+\beta)} \]
\[ 1 \text{ mA} = (1 \text{ mA}/V^2)(V_{DD} - 1)^2 \]
\[ V_{DD} = 2 \text{ V} \]

To stay in saturation,
\[ V_{DS} > 1 \text{ V} \]

\[ R_0 \approx R_0 \text{ should each have } \approx \frac{1}{2} V_{DD} \text{ drop across them:} \]
\[ V_0 = V_{DD} - \frac{1}{2} V_{DD} = \frac{1}{2} V_{DD} = I_0 R_0 \]
\[ V_3 = \frac{1}{2} V_{DD} = I_0 R_3 \]
\[ V_6 = 9 \text{ V} - \frac{1}{2} (9 \text{ V}) = 4.5 \text{ V} \]
\[ V_5 = \frac{1}{2} (9 \text{ V}) = 4.5 \text{ V} \]
\[ V_7 = V_3 + V_4 = 3 \text{ V} + 2 \text{ V} = 5 \text{ V} \]
\[ V_2 = V_7 + V_5 = 5 \text{ V} + 4.5 \text{ V} = 9.5 \text{ V} \]

\[ \frac{R_{dd}}{R_{d1} + R_{d2}} = \frac{5}{9} \quad ; \quad R_{d1} < R_{d2} \Rightarrow R_{d2} = 2.2 \text{ k\Omega} \]
\[ R_{d1} = \frac{9}{5} R_{d2} - R_{d2} = \frac{4}{5} (2.2 \text{ k\Omega}) = 1.6 \text{ k\Omega} \Rightarrow R_{d1} = 18 \text{ k\Omega} \]

\[ V_m = 5 \text{ V} \quad I_0 = 1 \text{ mA} \]
\[ V_0 = 0.5 \text{ V} \quad h_n = 8 \text{ mA/V} \]
\[ \lambda = 0 \]

To allow \( V_4 \) to swing by 2V due to leaving saturation:
\[ V_0 \in (V_0 - V_4) + 2 \text{ V} \]
\[ V_0 \in (1.5 - 0.8) + 2 \text{ V} \]
\[ V_m = 2.5 \text{ V} \]

Assuming \( I_0 = I_1 = 22 \text{ mA} \):
\[ V_{VOS} = V_0 - I_0 R_0 \]
\[ R_0 = \frac{(5 - 2.5)}{0.5} \text{ V} = 2.5 \text{ k\Omega} \]

\[ R_{d1} = \frac{R_{d1}}{R_{d1} + R_{d2}} = \frac{V_0}{V_6} \]
\[ R_{d2} = R_{d0} \left( \frac{2.5}{1.5} \right) = (22 \text{ mA})(0.13) = 2.8 \text{ k\Omega} \]
3. [\text{S45 P45 24}]

\[ k = \frac{10}{\text{mA}}/\text{V} \]
\[ V_{in} = 0.05 \text{V} \]
\[ I_{in} = \frac{V_{in}}{k} = \frac{0.05}{0.05} = \frac{1}{2} \text{mA} \]
\[ I_{s} = 300 \text{mA} \]

\[ V_{out} = V_{in} + V_{2} = (V_{in} + V_{2}) + (20-\text{mA}) \text{mA}(V_{in}) \]
\[ I_{dren} = \frac{2}{2} \left(0.05\text{mA}\right)^{2} = 242 \mu\text{A} \]

Then the change in total drain current, \(\Delta I_{dren} = I_{dren} - I_{0} = 42\mu\text{A} \)
\[ \Delta I_{dren} = 42 \mu\text{A} \]

Similarly, for \(\Delta V_{in} = -20 \mu\text{V} \)
\[ I_{dren} = (5\text{mA})(0.01\text{V}) = 100 \mu\text{A} \]
\[ \Rightarrow \Delta I_{dren} = -38 \mu\text{A} \]

To estimate \(g_{m} \) from these two values, take the total output current signal swing and divide by the input voltage signal swing.
\[ g_{m} \approx \frac{\Delta I_{dren}}{\Delta V_{in}} = \frac{42 \mu\text{A} - (-38 \mu\text{A})}{20 \mu\text{V}} = 28 \times 10^{-6} \text{ S} \]
\[ g_{m} = 2.8 \text{ S} \]

\[ \text{[Eqn. 9.33]} \]
\[ g_{m} = \frac{2I_{dren}}{V_{in}} \]

Both methods yield the same result!

4. [5 6 5 7.26]

0.20 V peak output signal, i.e.,

\[ V_{dc} = 0.20 \text{V} \]

We've told that the load can be used as a drain resistor... remember that in small-signal differential DC nodes go to ground:

\[ V_{d} = 0 \]

As you can see, any drain resistor contributes to the small-signal load

So our amplifier should look something like this:

\[ V_{in} = 1.8 \text{V} \]
\[ V_{out} = 20 \text{V} \]

Note that there are many possible solutions to this design problem, but by minimizing \(V_{in} \) (i.e., by increasing \(g_{m} \)) and \( I_{dren} \), then we maximize the \( V_{in} \) needed for a given \( g_{m} \) (or gain).

This in turn minimizes the \( V_{out} \) needed to achieve that \( g_{m} \), which can help reduce parasitic capacitances & self-driving elements.
\( V_i = 1 \text{ V} \)
\( r_e = 4 \text{ m\Omega} / \text{V}^2 \)

\( V_a = 15 \text{ V} \)
\[ \frac{6 \text{ mA}}{5 \text{ mA} + 10 \text{ mA}} = 0.5 \text{ V} \]

Assuming \( I_o = 0.5 \text{ mA} \):
\[ V_o = I_o \cdot R_a = (0.5 \text{ mA})(2 \text{ k}\Omega) = 3.5 \text{ V} \]

\[ V_{oa} = V_a - V_i = 6 - 3.5 = 2.5 \text{ V} \]

\( g_m = \frac{2I_o}{V_{oa}} = \frac{2(0.5 \text{ mA})}{1.5 - 1} \approx 2.5 \text{ mS} \)

\( r_e = \frac{1}{\lambda g_m} = \frac{1}{\lambda \cdot 2.5 \text{ mS}} \approx 100 \text{ k}\Omega \)

\( r_o = 200 \text{ k}\Omega \)

For small-signal analysis, circuit all capacitors & ground all DC supplies.

\( V_s' = \frac{V_s}{R_s} = \frac{200 \text{ V}}{200 \text{ k}\Omega} = 1 \text{ V} \)

\( V_s' = \frac{R_{in} + R_{out}}{R_{in} + R_{out} + R_s} = \frac{3.33 \text{ k}\Omega}{3.33 \text{ k}\Omega + 200 \text{ k}\Omega} = 0.943 \text{ V} \)

\( V_o' = -(V_i - V_s') \left( \frac{r_e}{16 \text{ k}\Omega + 16 \text{ k}\Omega} + \frac{200 \text{ k}\Omega}{200 \text{ k}\Omega + 8 \text{ k}\Omega} \right) = -(0.943 \text{ V}) \left( \frac{8 \text{ k}\Omega}{208 \text{ k}\Omega} \right) = -15.38 \text{ V} \)

\( V_s'' = \frac{V_s}{g_m} \quad V_o'' = (0.943 \text{ V})(-15.38) \)

\( V_s'' = -14.61 \text{ V} = 23.24 \text{ dB} \)
The integral of the current over the time interval is related to the area under the curve, which is represented by the definite integral:

\[ \int I \, dt = \text{Area under the curve} \]

The derivative of the integral with respect to time is the original function, which is the current itself:

\[ \frac{d}{dt} \left( \int I \, dt \right) = I \]

This is a fundamental result of calculus, known as the Fundamental Theorem of Calculus. It states that the derivative of the integral of a function is the function itself, provided the function is integrable.