Lecture 19: Small-Signal Analysis

- Announcements:
  - HW#6 online and due Friday via Gradescope
  - Those in the Wednesday lab section should finish their Lab#3 by going to the lab when the stations are free
  - Per Piazza, Lab#3 is due on Friday at 8 p.m. for everyone
  - Lab#4 this week
  - Midterm 1 moved to Wednesday, Oct. 16, 4-5 p.m., in our regular room
  - My Monday Office Hours are 5-6 p.m. today and thereafter
- Lecture Topics:
  - Small Signal Analysis
    - Linearizing Non-Linear Elements
    - DC and Small-Signal AC Components
    - Taylor Series Approximation
- Last Time:
  - Going through biasing for discrete MOS transistors
  - Now, continue with this …

For practical amplifier requirements: compromise:

1. \( V_{BB} \approx \frac{1}{3} V_{CC} \)
2. \( V_{CE} \approx \frac{1}{3} V_{CC} \)
3. \( V_{e} = I_{e} R_{e} = \frac{1}{3} V_{CC} \)
4. \( 0.1 I_{E} < I_{BSA} < I_{E} \)

MOS Biasing

\( => \) use a similar biasing strategy for discrete MOS cir.

\[
\begin{align*}
V_{DD} & - V_{DD} \left( \frac{R_2}{R_1 + R_2} \right), \quad I_G = 0 \rightarrow V_G = V_{GS} \\
KVL: \quad V_G & = V_{GS} + I_{DS} R_S \quad (1) \\
V_{DD} & = I_{DS} R_P + V_{DS} + I_{DS} R_S + V_{FS} \quad (1)
\end{align*}
\]
To find the DC operating point: (by hand)

1. Assume saturation:
   \[ I_{GS} = \frac{1}{2} M_{n} C_{ox} \frac{W}{L} (V_{GS} - V_{T})^2 (1 + \lambda V_{DS}) \]
   \[ V_{T} = f(V_{SB}) = V_{T0} + g(V_{2}\overline{2} - V_{2}\overline{2}) \]
   \[ \Rightarrow \text{using (2): } V_{GS} = V_{T0} + \frac{1}{2} M_{n} C_{ox} \frac{W}{L} (V_{GS} - V_{T})^2 R_{S} \]  

2. Solve for \( V_{GS} \) assuming \( V_{T} = V_{T0} \).

3. \( V_{S} = V_{GS} + V_{SS} \rightarrow V_{SB} = V_{S} - V_{SS} \rightarrow \text{find } V_{T}(V_{SB}) = V'_{T} \)

4. Plug \( V'_{T} = V_{T}(V_{SB}) \) into (3) \( \rightarrow \text{Get } V_{GS} \)

5. Back to (3) \( \rightarrow \text{iterate to convergence} \)

6. Check operating pt. \( \rightarrow \text{saturated?} \)
   - if yes \( \rightarrow \text{done} \)
   - if no \( \text{assume linear & start over} \)

= tedious, but effective fn discrete (i.e., off-chip)
= MOS ckt
= on-chip, we generally use current mirrors...

- For discrete (off-chip) circuits, avoid transistors
  - Discrete transistors are more expensive than discrete resistors
- For integrated (on-chip) circuits, avoid resistors
  - Resistors take up much more space than transistors, and space is money
  - Use lots of transistors and few if any resistors
A method to solve nonlinear problems by linearizing them around a specific coordinate.

As we’ve seen, transistors are nonlinear devices. We’ve already experienced difficulty solving for DC operating points for nonlinear transistors. So solving circuits with more complex inputs, e.g., sinusoids or sums of them, will become even more difficult.

Need some way to simplify these problems

Small-Signal Analysis

Take a two-terminal nonlinear resistor circuit as an example:

Example: Two-Terminal Non-Linear Ckt.

First, look at a linear ckt:

\[ V_Q = V_B - i_Q R_B \]
\[ i_Q = i_Q R_L \]

Solve two simultaneous linear equations:

\[ V_Q = \left( \frac{R_L}{R_L + R_B} \right) V_B \]

With non-linear ckt., things become more difficult:

\[ V_Q = V_B - i_Q R_B \]
\[ i_Q = f(V_Q) \]

Must solve two simultaneous equations:

\[ V_B = V_B - f(V_Q) R_B \]

Not easy to solve!

Generally, can’t get into closed form.

\[ \text{e.g., } i = f(V) = 2V^2 + V^3 \Rightarrow V_Q = V_B - (2V_Q + V_Q^3) R_B \]
- Need a convenient method to solve this nonlinear circuit, i.e., need some way to linearize it
- Such linearization is possible for analog circuits when the signals can break down into DC and small-signal AC components

Using this notation:

\[ V_{B} + N_{b} = V_{Q} + N_{Q} - f [ V_{Q} + N_{Q} ] R_{G} \]

For this: Use Taylor series approx. for \( f(\alpha) \) around the bias pt. \( V_{Q} \):

\[ f(\alpha) \approx f(a) + f'(a)(\alpha - a) + \frac{f''(a)(\alpha - a)^2}{2!} + \cdots + \frac{f^{(n)}(a)(\alpha - a)^n}{n!} \]

Review Taylor Review:

- Taylor function \( f(x) \) can be approximated for points near \( a \) by:
- \( f(a) \approx f(a) + f'(a)(x - a) + \frac{f''(a)(x - a)^2}{2!} + \cdots + \frac{f^{(n)}(a)(x - a)^n}{n!} \)
- \( f'(a) \) is the 1st derivative of \( f(x) \)
- \( f'(a) \) is the linear term
- \( f'(a)(x - a) \) is the total linear term
- \( f'(a)(x - a) \) is the bias point
- \( f'(a)(x - a) \) is easy to solve!
\[
V_Q + V_N = V_B + N_b - I_Q R_B - \frac{R_B}{R_{s.s.}} N_Q
\]

\( R_{s.s.} = \frac{1}{\frac{dI}{dV}} \) small signal resistance

\( I_Q = f(V_Q) \)

Can split this into two equations \( \rightarrow \) two e.f.t.

**DC Components:** \( V_Q = V_B - I_Q R_B \)

**Small-Signal AC Components:** \( N_Q = N_b - \frac{R_B}{R_{s.s.}} N_Q \rightarrow \) linear!

\( R_{s.s.} \rightarrow \frac{1}{\frac{dI}{dV}} V_Q \)

\( \Rightarrow \) all from analysis! \( \Rightarrow \) Good!