Small-Signal Model for Common-Mode Inputs

- Now consider other extreme situation: \(v_{i1} = v_{i2} = v_{ic}\)

\[i_1 = i_2 \rightarrow v_x = (2i_1) r_{ob}\] which will be satisfied by splitting the circuit
Common-Mode Voltage Gain

- Solve either circuit to find $a_{cm} = \frac{v_{oc}}{v_{ic}}$

  $$v_{\pi 1} = v_{ic} - v_x$$

  $$v_x = \left(\frac{v_{\pi 1}}{r_{\pi 1}} + g_{m1} v_{\pi 1}\right)(2r_{ob}) \approx g_{m1}(2r_{ob})v_{\pi 1}$$

  substituting and solving for $v_{\pi 1}$

  $$v_{\pi 1} = \frac{v_{ic}}{1 + g_{m1}(2r_{ob})}$$

  the output voltage $v_{oc}$ is

  $$v_{oc} = -(g_{m1}v_{\pi 1})R_C = \frac{-g_{m1}R_C}{1 + g_{m1}(2r_{ob})} v_{ic}$$

- Common-mode voltage gain $a_{cm}$ is

  $$a_{cm} = \frac{-g_{m1}R_C}{1 + g_{m1}(2r_{ob})}$$
Common-Mode Rejection Ratio (CMRR)

- The ratio of differential-mode to common-mode gains is
  \[
  \left| \frac{a_{dm}}{a_{cm}} \right| = CMRR = \left| \frac{-g_m R_C}{-g_m R_C + 1 + g_m(2r_{ob})} \right| = 1 + 2g_m r_{ob}
  \]

  since the product of the transconductance and the internal resistance of the bias current can be on the order of 100-1000, the “differential” amplifier is well-named -- it amplifies \(v_{id}\) much more than \(v_{ic}\)

  to first order, \(a_{cm} = 0\) for differential amplifiers with current-source bias

- Output voltages for general input voltages (including \(a_{cm}\))

  express \(v_{o1}\) and \(v_{o2}\) in terms of common-mode and differential-mode output voltages

  \[
  v_{o1} = v_{oc} + \frac{v_{od}}{2} = a_{cm}v_{ic} + \frac{1}{2}a_{dm}v_{id} = a_{cm}\left(\frac{v_{i1} + v_{i2}}{2}\right) + \frac{1}{2}a_{dm}(v_{i1} - v_{i2})
  \]

  \[
  v_{o2} = v_{oc} - \frac{v_{od}}{2} = a_{cm}v_{ic} - \frac{1}{2}a_{dm}v_{id} = a_{cm}\left(\frac{v_{i1} + v_{i2}}{2}\right) - \frac{1}{2}a_{dm}(v_{i1} - v_{i2})
  \]
Two-Port Differential Model

- Differential input resistance: use differential half-circuit with test source $v_t/2$

$$i_t = \frac{v_t/2}{r_\pi} \quad \rightarrow \quad R_{id} = \frac{v_t}{i_t} = 2r_\pi$$

- Differential output resistance: use differential half-circuit with test source $v_t/2$

$$R_{od} = \frac{v_t}{i_t} = 2R_C$$

Why $v_t/2$? Full differential voltage is applied across amplifier inputs -- 1/2 is dropped across left side.
Two-Port Differential Voltage Model

- Note that neither input nor output is referenced to ground

- Applying the model: must modify source and load into differential form
Frequency Response of Differential Amplifiers

- Consider response to differential-mode and common-mode signals separately

Half-circuit technique reduces differential amplifier to two “single-ended” circuits
Differential-Mode Half Circuit

- Differential-mode half circuit is identical to common-emitter

\[ V_{od} \over V_{id} = \frac{- \left( \frac{r_\pi}{R_S + r_\pi} \right) (g_m R_C)}{1 + j \omega (R_S \left| r_\pi \right| C_\pi + (1 + g_m R_C) C_\mu)} \]
Common-Mode Half Circuits

- Capacitance $C_E$ from common node to ground is due to drain-to-substrate capacitance of bias current transistor.

Common-mode 1/2 circuit: split $C_E$ in two --> $C_E/2$ appears on left side.

At high frequencies, the impedance $1/(j\omega C_E/2)$ --> short circuit, which increases the common-mode gain.

- The common-mode gain's dependence on frequency is found by substituting $2Z_E$ for $2r_{ob}$ in the expression for $a_{cm}$:

$$
\frac{V_{oc}}{V_{ic}} = \frac{-g_m R_C}{1 + g_m 2Z_E}
$$

where

$$
2Z_E = 2r_{ob} \left| \left( \frac{1}{j\omega (C_E/2)} \right) \right| = \frac{2r_{ob}}{1 + j\omega r_{ob} C_E}
$$
Common-Mode Frequency Response (Cont.)

- Since $|Z_E|$ is large, $|g_m Z_E| \gg 1 \implies$
  \[
  \frac{V_{oc}}{V_{ic}} \approx \frac{R_C}{2Z_E} = \frac{R_C}{2r_{ob}} (1 + j\omega r_{ob} C_E)
  \]

  The common-mode gain therefore has a zero at a relatively low frequency --
  \[
  \omega_z = \frac{1}{r_{ob} C_E}
  \]

  Note that a very high value of $r_{ob}$ leads to a lower zero frequency

- The common-mode rejection ratio as a function of frequency is
  \[
  CMRR = \frac{a_{dm}(j\omega)}{a_{cm}(j\omega)} = \frac{CMRR_o}{(1 + j\omega r_{ob} C_E)(1 + j\omega/\omega_1)}
  \]
Magnitude Bode Plots of Diff. Amplifier Gains

- Common-mode gain has a pole at high frequencies.