Lecture 4, February 24, 2001

EECS 105 Microelectronics Devices and Circuits, Spring 2001

Andrew R. Neureuther

Topics:
Silicon Physics – Carriers,
Process Flow and Layout,

Reading: 2.1-2.4, 2.5.4
4.1.1, 4.5.7, 6.2, 7.1.1, 7.7
The two-ports shown in Chapter 8 illustrate the important idea of modular design.

- Their input and output properties are basically Thevenin and Norton equivalent circuits.

When the output is not isolated from the input, properties like input resistance depend on $I_2$, $V_2$ and the output load.

- This makes finding two-ports much more complex.
- The analysis of the cascaded sections is much more complex.

Most circuit designers just use KCL and KVL.

- (Example: assume $i_{\text{TEST}}$ find $V_{\text{TEST}}$ and $R_{\text{IN}} = V_{\text{TEST}}/I_{\text{TEST}}$)
Expected KCL and KVL Skill Level

By Feb 23

**B Level:** Find $R_{IN}$ when $R_{OUT} = \infty$ and $R_{LOAD}$ present.

**A Level:** Find $R_{IN}$ when $R_{OUT} = \text{finite}$ and $R_{LOAD}$ present.
Silicon Semiconductor: Carriers

Selectively dope regions to create mobile carriers (electrons and holes)

(a) Current flow.
(b) Charge density.
(c) Electric field.
(d) Electrostatic potential.
W2 W L4: Silicon Physics

- Intrinsic silicon
  - Thermal generation of carriers (electrons and holes)
  - Recombination = generation $\sim n_i^2$
- Adding doping
  - Lattice charge and carriers
  - Law of mass action
- Drift current and electrical conductivity
- Diffusion current and Einstein relation
Silicon has 4 electrons in its outer shell

5.00 x 10^{22} \text{ atoms/cm}^3
Intrinsic (no Doping): Thermal Generation

\[ n_i = p_i \text{ due to charge neutrality} \]
\[ n_i = \sqrt{2} \times 10^{10} \text{ atoms/cm}^{-3} \text{ at room temperature} \]
\[ n_i p_i = n_i^2 = 2 \times 10^{20} \text{ atoms/cm}^{-6} \]
Doping: n-type

Selectively changing a region to n-type

Example

\[ N_d = 10^{15} \text{ cm}^{-3} \]

Arsenic has 5 electrons in its outer shell = donor

Immobile + charge (Lattice Charge)
Law of Mass Action to Find Holes

- Thermal energy generates an equal number of new electrons $n$ and holes at the same rate as in intrinsic silicon so $n_{\text{NEW}} p_{\text{NEW}} = n_i^2$
- But n-type dopant (As) has raised density of electrons to $n = N_d$ (Ex. $N_d = 10^{15} \text{ cm}^{-3}$)
- This causes recombination which is proportional to $N_d$ time $p$ to exceed generation until $p$ decreases to $p = \frac{n_i^2}{N_d}$.
- These two equilibrium values are termed $n_o = N_d$ and $p_o = \frac{n_i^2}{N_d}$. (Ex. $p_o = 2 \times 10^{15} \text{ cm}^{-3}$)
Law of Mass Action in Chemistry

- $\text{H}_2\text{O}$ is the product of a reaction of hydrogen ions ($\text{H}^+$) and hydroxyl ions ($\text{OH}^-$).

- In thermal equilibrium, the product of the concentration of hydrogen ions [$\text{H}^+$] and the concentration of hydroxyl ions [$\text{OH}^-$] is a constant at a given temperature.

\[
[\text{H}^+][\text{OH}^-] = K_{\text{eq}}
\]

- In an acid, the hydrogen ion concentration dominates, implying that [$\text{OH}^-$] is much lower.

\[
[\text{OH}^-] = \frac{K_{\text{eq}}}{[\text{H}^+]}\]
Doping: p-type

Selectively changing a region to p-type

Boron has 3 electrons in its outer shell => acceptor

Mobile holes + charge

Immobile - charge (Neg Lattice Charge)
Doping: General Case (EECS 130)

- Assumptions so far
  - \( N_d \) or \( N_a \) much larger than \( n_i \)
  - Only \( N_a \) or \( N_d \) present but not both

- Principles for general case
  - Consider the net doping \( N_d - N_a \)
  - Apply charge neutrality including lattice charge
  - Apply law of mass action \( np = n_i^2 \)

- Result = Equation 2.14, 2.19, 2.24
Thermal Motion and Collisions

- Thermal velocity = $10^7$ cm/s ... *very fast*
  
  » $\frac{1}{2} m v_{TH}^2 = \frac{3}{2} KT$

- But collisions every $100$ A = $10$ nm
  
  » This distance depends on doping (obstacles) and temperature (amount silicon in lattice sites is jumping around)
  
  » Mean free path $\lambda = v_{TH} \tau_c = 10$ nm

- Time between collisions $\tau_c$ is very small
  
  $\tau_c = 10$ nm/$10^7$ cm/s = $10^{-13}$ s = 0.1 ps
Carrier Transport: Drift

An applied electric field causes the carriers to have and added component of motion parallel to the electric field between collisions. Electrons (holes) go in the opposite (same) direction as the electric field.
Physical Basis of Mobility

- Force $F_{el} = -qE$
- Velocity of electron drift
  \[ v_e = \left( \frac{F_{el}}{m_e} \right) \tau_c \]
- Substitute $v_e = \left( -\frac{qE}{m_e} \right) \tau_c$
- Rearrange $v_e = \left( -\frac{q\tau_c}{m_e} \right)E$
- Define the proportionality constant as electron mobility $\mu_e$

  \[ \text{mobility of electron} \quad \mu_e = \frac{q\tau_c}{m_e} \]
Mobility

For doping of $5 \times 10^{16} \text{ cm}^{-3}$

$\mu_e = 1000 \text{ cm}^2/(\text{V s})$

$\mu_p = 400 \text{ cm}^2/(\text{V s})$

Ratio is about a factor of 2.5

Fig. 2.8
Velocity Saturation

Velocity saturates at about $10^7 \text{ cm/s}$

Fields bigger than $10^4 \text{ V/cm}$ do not increase the velocity significantly.

Fig. 2.9

Velocity cm/s

V/cm

V_{dn}, V_{dp} (\text{cm/s})

10^8

10^7

10^6

10^5

10^4

10^3

10^2

10

10^{-1}

10^{-2}

10^{-3}

10^{-4}

10^{-5}

E (\text{V/cm})
Drift Current

- Current density = charge x density x velocity
- Velocity = mobility x electric field
  - Holes move in direction of field => positive current
  - Electrons move counter to direction of field and have negative charge => positive current
- Add the hole and electron contributions to current

\[ J = J_p + J_n = q p v_p + (-q) n v_n \]
\[ J = q p (\mu_p E) - q n (\mu_e E) = (q p \mu_p + q n \mu_e) E = \sigma E \]

Define the conductivity \( \sigma = (q p \mu_p + q n \mu_e) \)
Ohms Law

\[ V = RI \]

\[ R = \rho \left[ \frac{L}{(Wt)} \right] = \left( \frac{1}{\sigma} \right) \left[ \frac{L}{(Wt)} \right] \]

\[ R = \frac{(L/W)}{[1/(ts)]} \]

\[ \rho = \frac{1}{\sigma} = \frac{1}{(qp\mu_p+qn\mu_e)} \]
Carriers also move via diffusion as a consequence of their thermal motion. Move from a high concentration region toward a lower concentration region.
Mean Free Path Length and Diffusion

Flux = # atoms/scm²

Current density = charge times flux

Mean free path length
**Diffusion Current Density**

\[ J_p^{\text{Diff}} = \frac{q}{A \tau_c} \left[ \frac{1}{2} p(x = x_r - \lambda) \lambda A - \frac{1}{2} p(x = x_r - \lambda) \lambda A \right] \]

\[ J_p^{\text{Diff}} = -q(\lambda^2/\tau_c)(dp/dx) = -qD_p(dp/dx) \text{ Fick’s Law} \]

Diffusion constant \( D_p = (\lambda^2/\tau_c) = \text{(distance}^2/\text{time}) \)

EE 130 develops the Einstein relation that shows that the diffusion constant and the mobility are related by \( kT/q \).

\[ D_p = (kT/q)\mu_p \]

With both \( n \) and \( p \)

\[ J^{\text{Diff}} = -qD_p(dp/dx) + qD_n(dn/dx) \]