Lecture 2

- Last time:
  - Course overview
  - Sinusoidal signals
  - *RC* circuit with sinusoidal input voltage
- Today :
  - Finish *RC* circuit: introduce deciBel (dB)
  - Introduce phasor representation of sinusoids

Amplitude: a new representation

- We are interested in *very* small ratios (e.g., $V_c/V_s = 0.0001$)

- Therefore, we use a log plot … but we also define a new function called the deciBel (after Alex. Graham Bell)

$$(V_c/V_s)_{\mathrm{dB}} = 20 \log_{10} (V_c/V_s)$$

- Examples: $V_c/V_s = 0.0001 \rightarrow (V_c/V_s)_{\mathrm{dB}} = -80$ dB

  $V_c/V_s = 0.707 \rightarrow (V_c/V_s)_{\mathrm{dB}} = -3$ dB

A Better Technique

- It is much more efficient to work with *imaginary exponentials* as "representing" sinusoids, since these functions are direct solutions of linear differential equations:

$$\frac{d}{dt}(e^{j\omega t}) = j\omega(e^{j\omega t})$$

- Note that EEs use $j = (-1)^{1/2}$ rather than i, since the symbol i is already taken for current

Using Imaginary Exponentials

From the example in Lecture 1:

$$RC\frac{dv_c}{dt} + v_c = v_s$$

Substitute:

$$v_s(t) = v_s e^{j\omega t}$$

$$v_c(t) = v_c e^{j(\omega t+\phi)}$$

Result:

$$\tau(j\omega)v_c e^{j(\omega t+\phi)} + v_c e^{j(\omega t+\phi)} = v_s e^{j\omega t}$$

Finding the Amplitude Ratio

$$[\tau(j\omega)v_c e^{j\phi} + v_c e^{j\phi}]e^{j\omega t} = v_s e^{j\omega t}$$

$$[j\omega\tau e^{j\phi} + e^{j\phi}]v_c = v_s$$ … use to find amplitude and phase

Amplitude Ratio: $$\frac{v_c}{v_s} = \frac{1}{j\omega\tau e^{j\phi} + e^{j\phi}} = \frac{e^{-j\phi}}{(1 + j\omega\tau)}$$

Answer is a real number, so take magnitude

$$\frac{v_c}{v_s} = \frac{1}{\sqrt{1 + (\omega\tau)^2}}$$

Finding the Phase

$$[j\omega\tau e^{j\phi} + e^{j\phi}] = v_s / v_c$$ (a real number)

Use Euler's formula to convert to rectangular form:

$$j\omega\tau(\cos\phi + j\sin\phi) + (\cos\phi + j\sin\phi) = v_s / v_c$$

Collect real and imaginary parts; latter must be zero:

$$\mathrm{Im}(\cdot) = \omega\tau\cos\phi + \sin\phi = 0$$

$$\tan\phi = -\omega\tau$$

Finding the “Real” Waveform

- How to connect the imaginary exponential solution to the measured waveform *v(t)*? Conventionally, *v(t)* is the *real part* of the of the imaginary exponential

$$\mathrm{Re}(ve^{j(\omega t+\phi)}) = v\cos(\omega t + \phi)$$

Pushing This Idea Further …

There are two parameters needed to define a sinusoidal signal:

* magnitude
* phase

Why not work with a *complex number* as the signal and eliminate the imaginary exponential from the analysis (it cancelled out)?

Define the complex number consisting of the amplitude and phase a sinusoidal signal as a **phasor**

$$v(t) = v\cos(\omega t + \phi) \Leftrightarrow v(t) = Ve^{j\omega t}$$

$$V = ve^{j\phi}$$