Lecture 3

- Last time:
  - Imaginary exponentials: simplify the math
  - Phasor: complex "prefactor" for $e^{j\omega t}$
- Today :
  - Complex number review
  - Circuit analysis with phasors

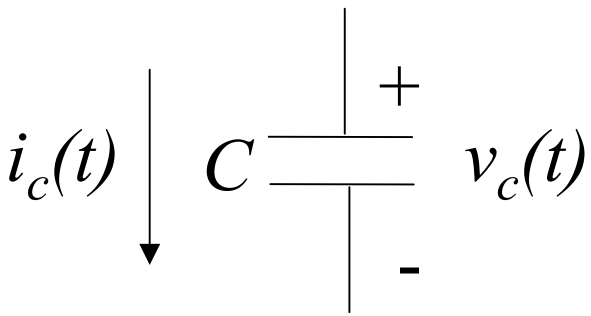
Complex Number Summary

- Rectangular form: $z = x + jy$
  - Magnitude $|z| =$
  - Phase $\angle z =$
- Polar form:
- Useful results (easily shown in polar form):

$|z_1 \, z_2| =$ $\angle(z_1 z_2) =$

Question: $\sqrt{j} =$

Using Phasors: Capacitor Current



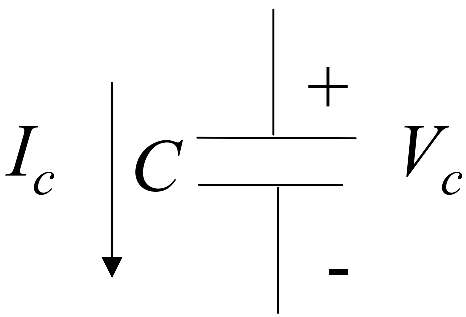
$$i_c(t) = C\frac{dv_c}{dt}$$

$$i_c(t) = I_c e^{j\omega t} \qquad v_c(t) = V_c e^{j\omega t}$$

Result:

Impedance of a Capacitor

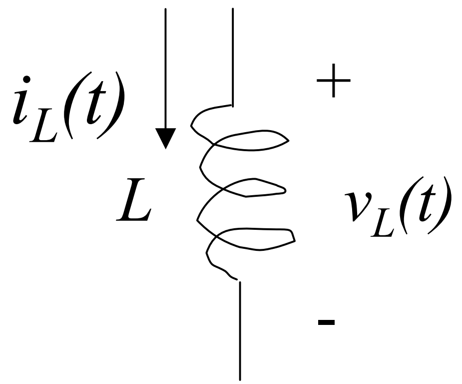
Definition: the impedance *Z* of a two-terminal circuit element is the ratio of the phasor voltage to the phasor current (positive reference convention)



$$Z_c =$$

Admittance: $Y_c = 1 / Z_c =$

Using Phasors: Inductor Voltage



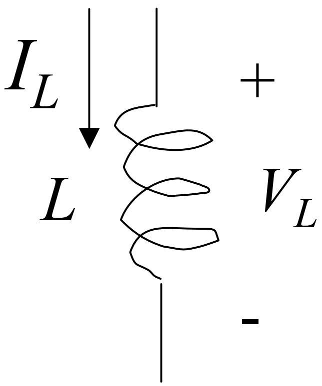
$$v_L(t) = L\frac{di_L}{dt}$$

$$v_L(t) = V_L e^{j\omega t}$$

$$i_L(t) = I_L e^{j\omega t}$$

Result:

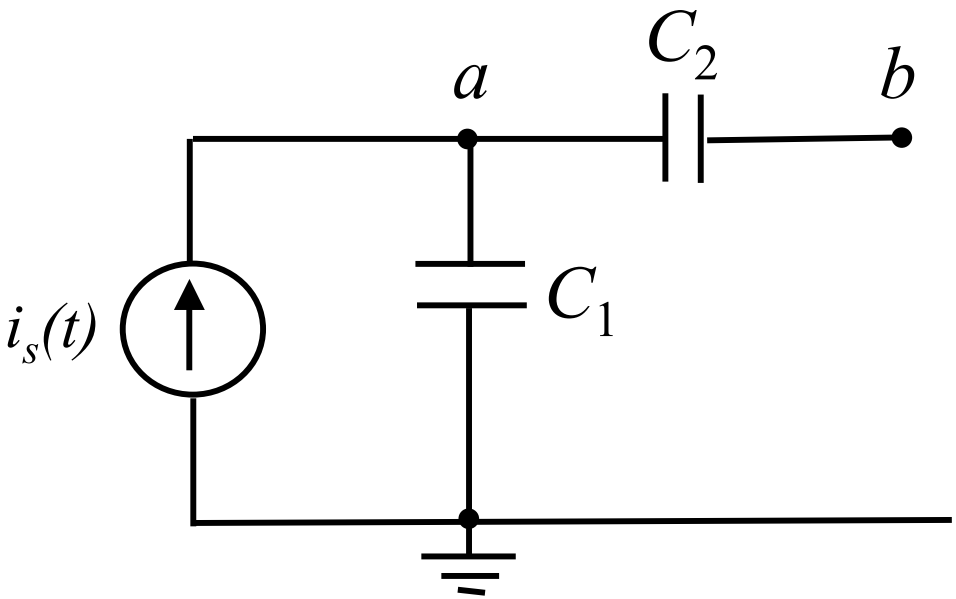
Inductor Impedance



$Z_L =$

Admittance: $Y_L = 1 / Z_L =$

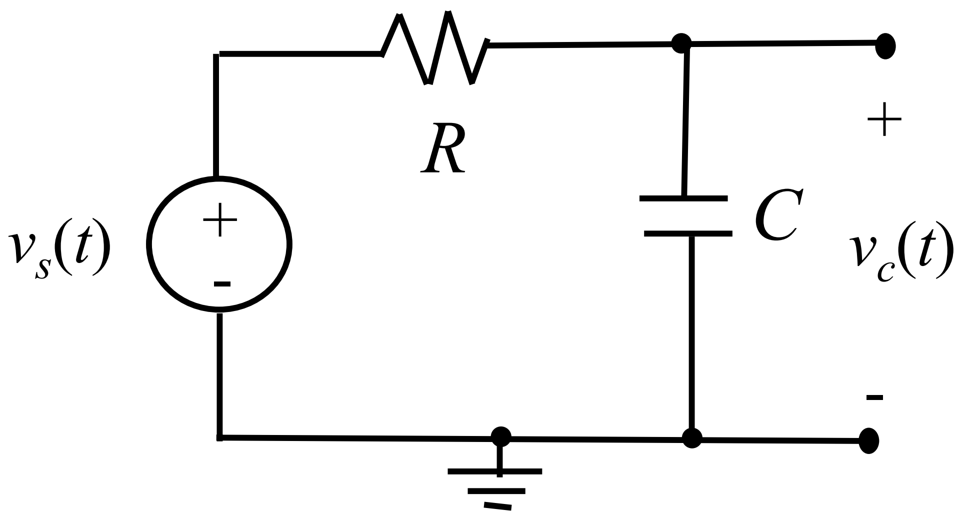
Kirchhoff's Current Law Example



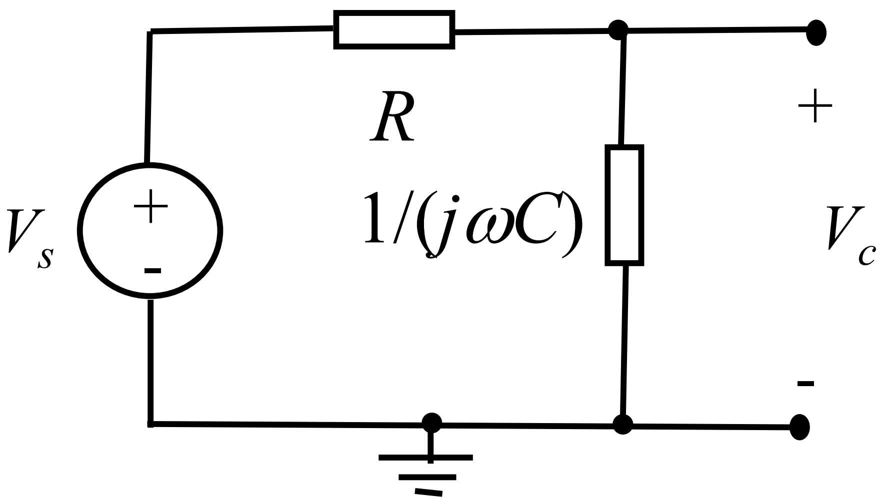
At node *a*:

Circuit Analysis with Phasors

Assumption: sources are sinusoidal, steady-state!



Redrawing the Circuit with Impedances



Note: this is not a “real” circuit that could be built and tested!

Transfer Function

Ratio of output to input phasor is called the transfer function of the circuit:

$$H = \frac{V_c}{V_s} =$$

Bode Plots

1. Plot magnitude $|H|$ in dB vs. ω (log scale)
2. Plot phase $\angle H$ in degrees vs. ω (log scale)