Lecture 4

- Last time:
  - Circuit analysis with phasors: impedances
  - Bode plot of low-pass filter (start)
- Today :
  - Bode plot sketching for first-order transfer functions
  - Low-pass and high-pass filters

Bode Plots for Low-Pass Filter

1. Plot magnitude $|H|$ in dB vs. ω (log scale)
2. Plot phase $\angle H$ in degrees vs. ω (log scale)

$$|H|_{dB} = \left| \frac{1}{1 + j\omega\tau} \right|_{dB} = \left[\frac{|1|}{|1 + j\omega\tau|} \right]_{dB}$$

Why?

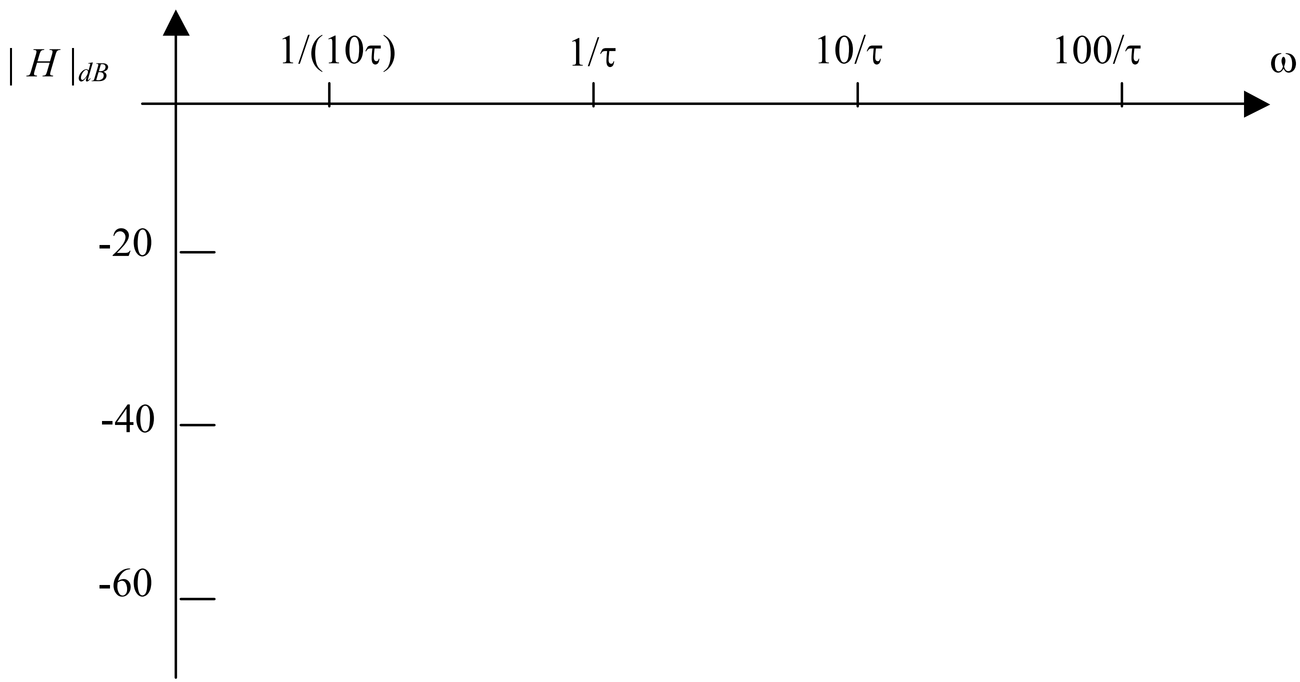
Sketching the Magnitude Plot

$$|H|_{dB} = \left[\frac{|1|}{|1 + j\omega\tau|}\right]_{dB} = 20\log\left[\frac{1}{\sqrt{1+(\omega\tau)^2}}\right]$$

Low-frequency (ω τ <<1) asymtote

High-frequency (ω τ >>1) asymtote

The Break Frequency $\omega_{-3dB} = (1/\tau)$



Finding the Phase Plot

$$\angle(H) = \angle\left[\frac{1}{1+j\omega\tau}\right] = 0 - \arctan(\omega\tau)$$

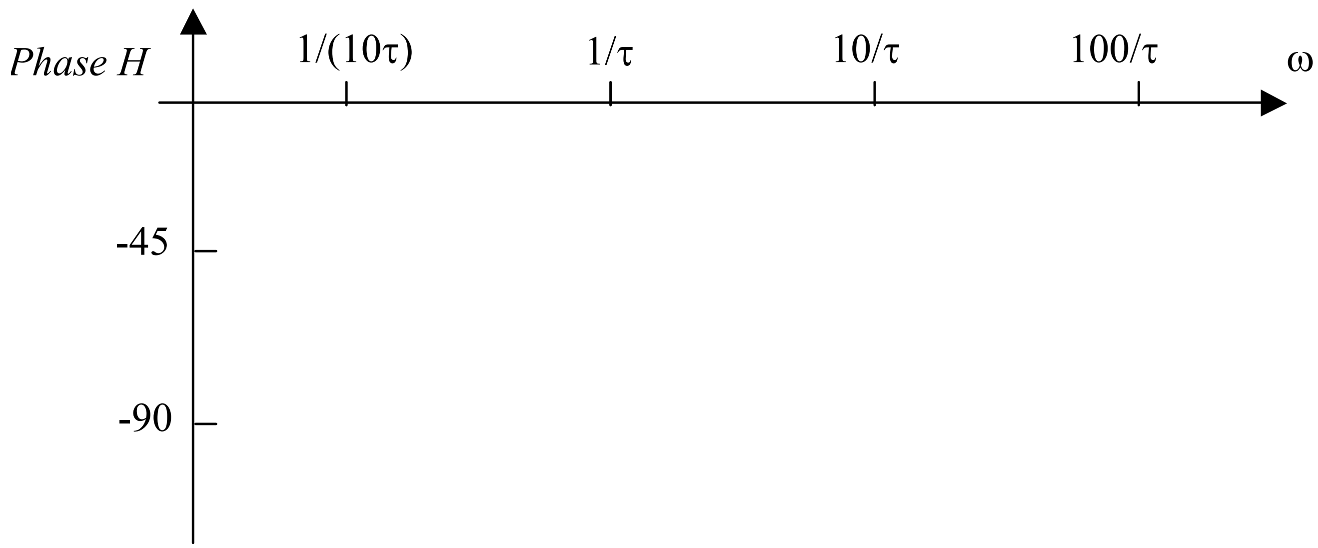
Why?

Low-frequency asymtote

High-frequency asymtote

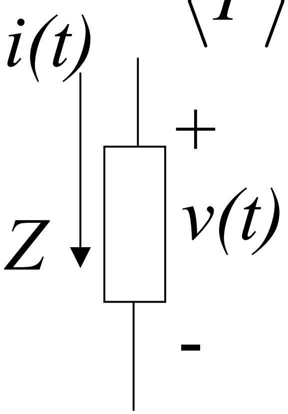
Approx. linear with ω for $1/(10\tau) < \omega < 10/\tau$

Rapidly Sketching the Phase Plot



Average Power and Phasors

Integrate $P(t)$ over one period:



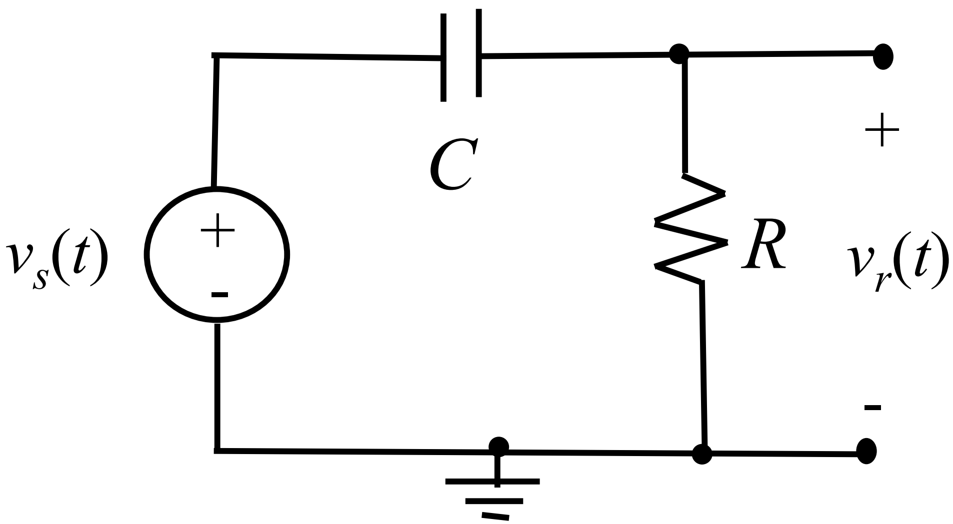
$$\langle P \rangle = \int_0^T i(t)v(t)dt = \int_0^T |I|\cos(\omega t + \angle I)|V|\cos(\omega t + \angle V)dt$$

$$= |I||V|\left\langle \sum \left\{\cos(\omega t), \cos(2\omega t), \sin(\omega t), \sin(2\omega t)\right\}\right\rangle +$$

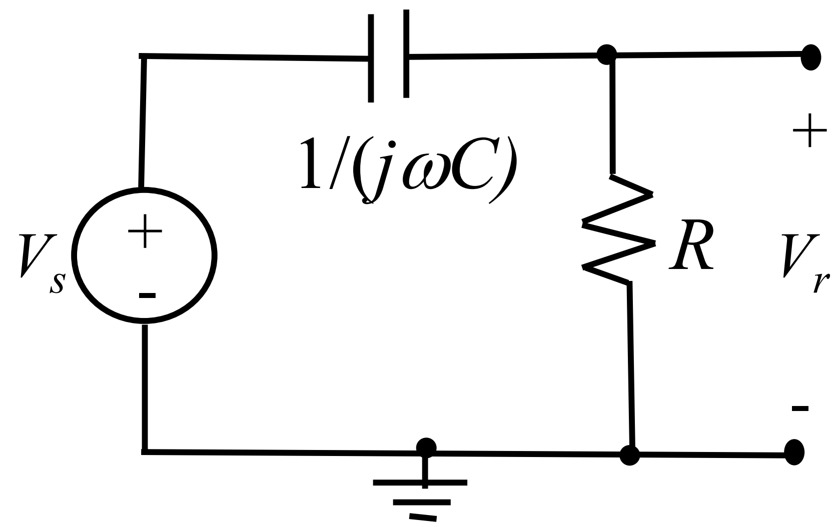
$$\frac{|I||V|}{2}\left(\cos\angle I \cos\angle V + \sin\angle I \sin\angle V\right)$$

Result: $\langle P \rangle = \dfrac{|I||V|}{2}\cos(\angle I - \angle V) = \mathrm{Re}\{I \cdot V^*\}$

The High-Pass Filter



Impedance Divider



Insight:

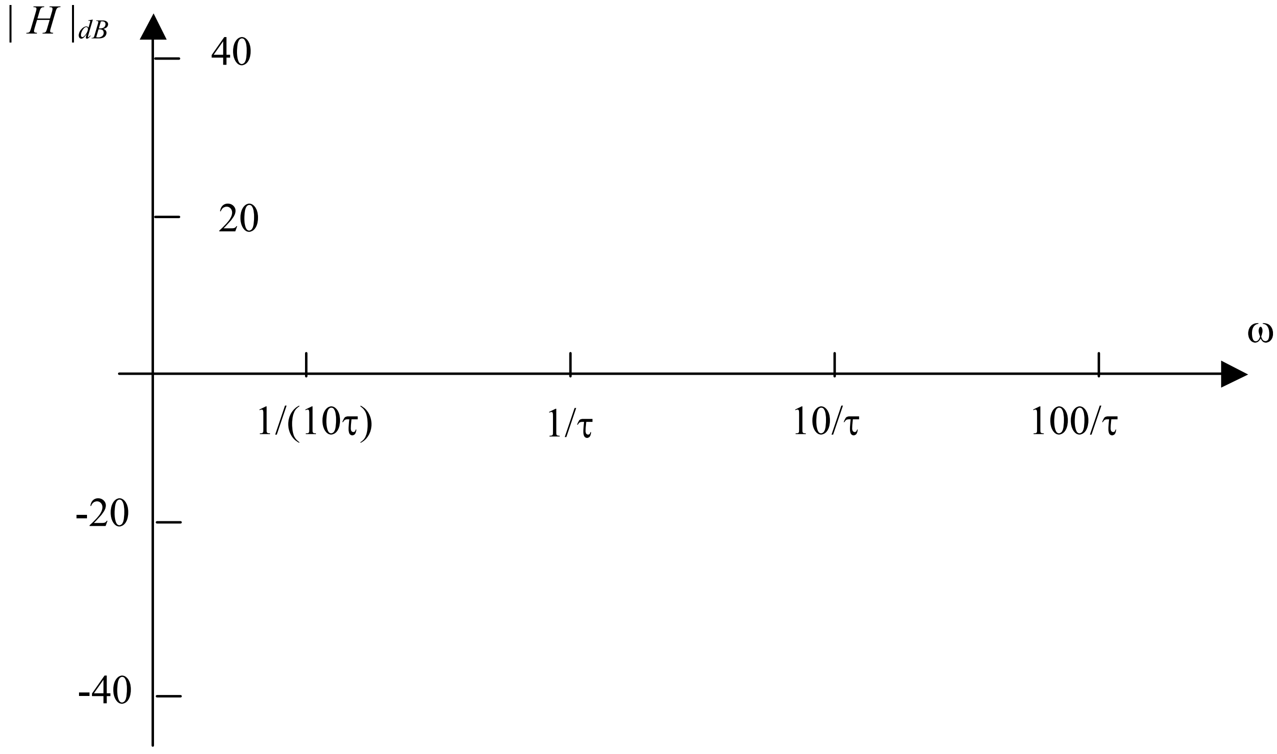
$$H = \frac{V_r}{V_s} =$$

Magnitude Bode Plot

$$|H|_{dB} = \left|\frac{j\omega\tau}{1+j\omega\tau}\right|_{dB} = |j\omega\tau|_{dB} + \left|\frac{1}{1+j\omega\tau}\right|_{dB}$$

First term (numerator):

Graphical Addition of Magnitudes



Phase Bode Plot for HPF