Lecture 5

- Last time:
  - Bode plots for first-order transfer functions (low-pass and high-pass filters)
- Today :
  - Rapid sketching techniques for more complicated transfer functions

Bode Plots

Technique for plotting complicated transfer functions (e.g., several $(1+j\omega\tau_i)$ factors in numerator and denominator)

$$H(j\omega) = \frac{Aj\omega(1+j\omega\tau_2)(1+j\omega\tau_{4.})...(1+j\omega\tau_n)}{(1+j\omega\tau_1)(1+j\omega\tau_3)...(1+j\omega\tau_{n-1})}$$

$\omega_i = (1/\tau_i)$ are the break frequencies

Denominator factors: *poles*

Numerator factors: *zeroes*

Summary of Individual Factors

Poles:

Zeroes:

$j\omega\tau$ factors:

Example

$$H(j\omega) = \frac{10^{-5}\, j\omega(1 + j\omega\tau_2)}{(1 + j\omega\tau_1)(1 + j\omega\tau_3)}$$

$\tau_1 = 200$ ns

$\tau_2 = 20$ ns

$\tau_3 = 200$ ps

Break frequencies: invert time constants

$\omega_1 = 5$ Mrad/s $\omega_2 = 50$ Mrad/s $\omega_3 = 5$ Grad/s

$$H(j\omega) = \frac{(j\omega/10^5)(1 + j\omega/\omega_2)}{(1 + j\omega/\omega_1)(1 + j\omega/\omega_3)}$$

Breaking Down the Magnitude

$$\left|H(j\omega)\right|_{dB} = 20\log\left|\frac{10^{-5}\, j\omega(1+j\omega\tau_2)}{(1+j\omega\tau_1)(1+j\omega\tau_3)}\right|$$

$$= 20\log\left|j\omega/10^5\right| + 20\log\left|1+j\omega\tau_2\right|$$

$$- 20\log\left|1+j\omega\tau_1\right| - 20\log\left|1+j\omega\tau_3\right|$$

→ Plot the terms separately and add them graphically!

Breaking Down the Phase

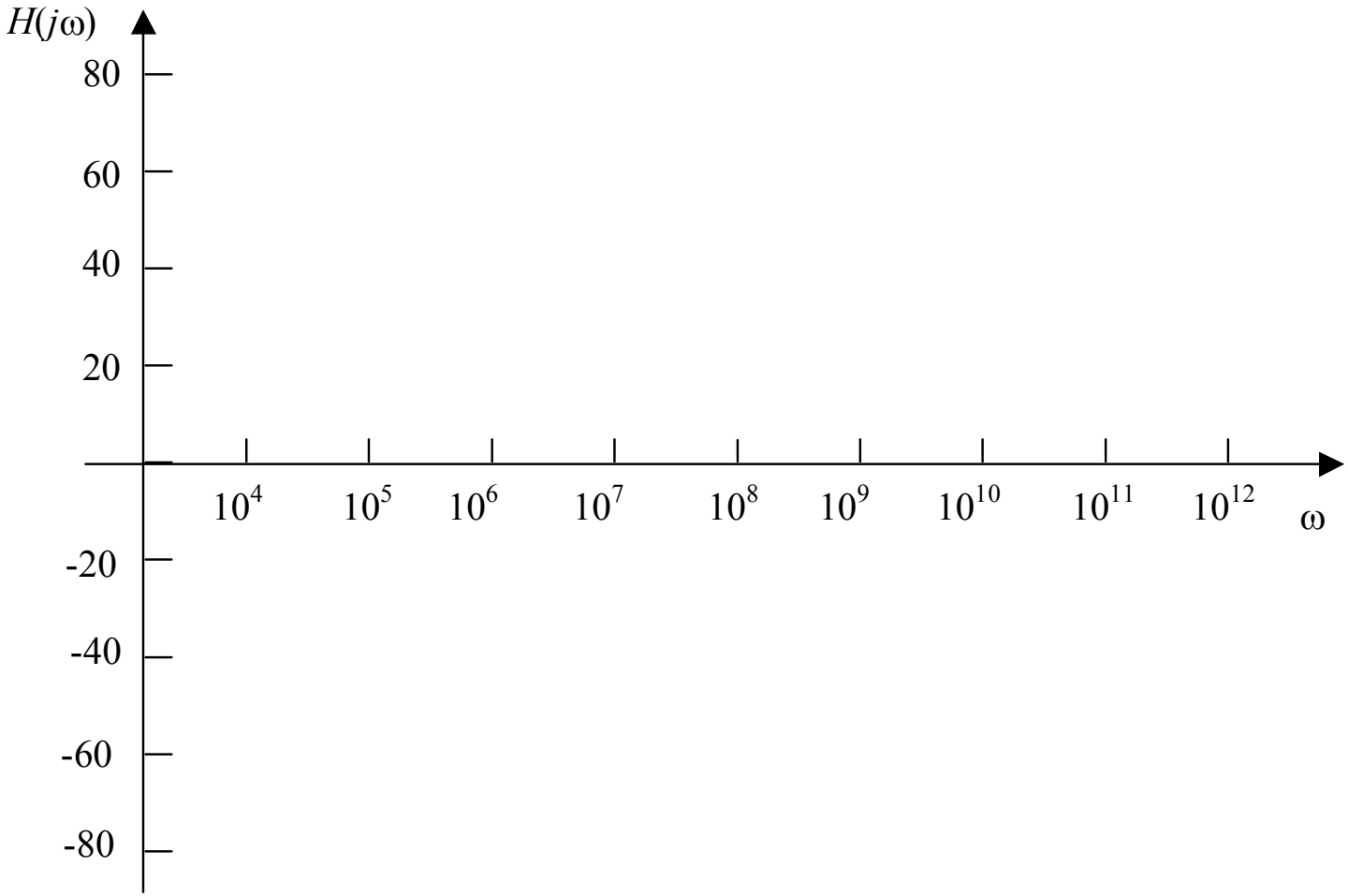
$$\angle H(j\omega) = \angle \frac{10^{-5}\, j\omega(1+j\omega\tau_2)}{(1+j\omega\tau_1)(1+j\omega\tau_3)}$$

But we know $\angle z_1 z_2 = \angle z_1 + \angle z_2$

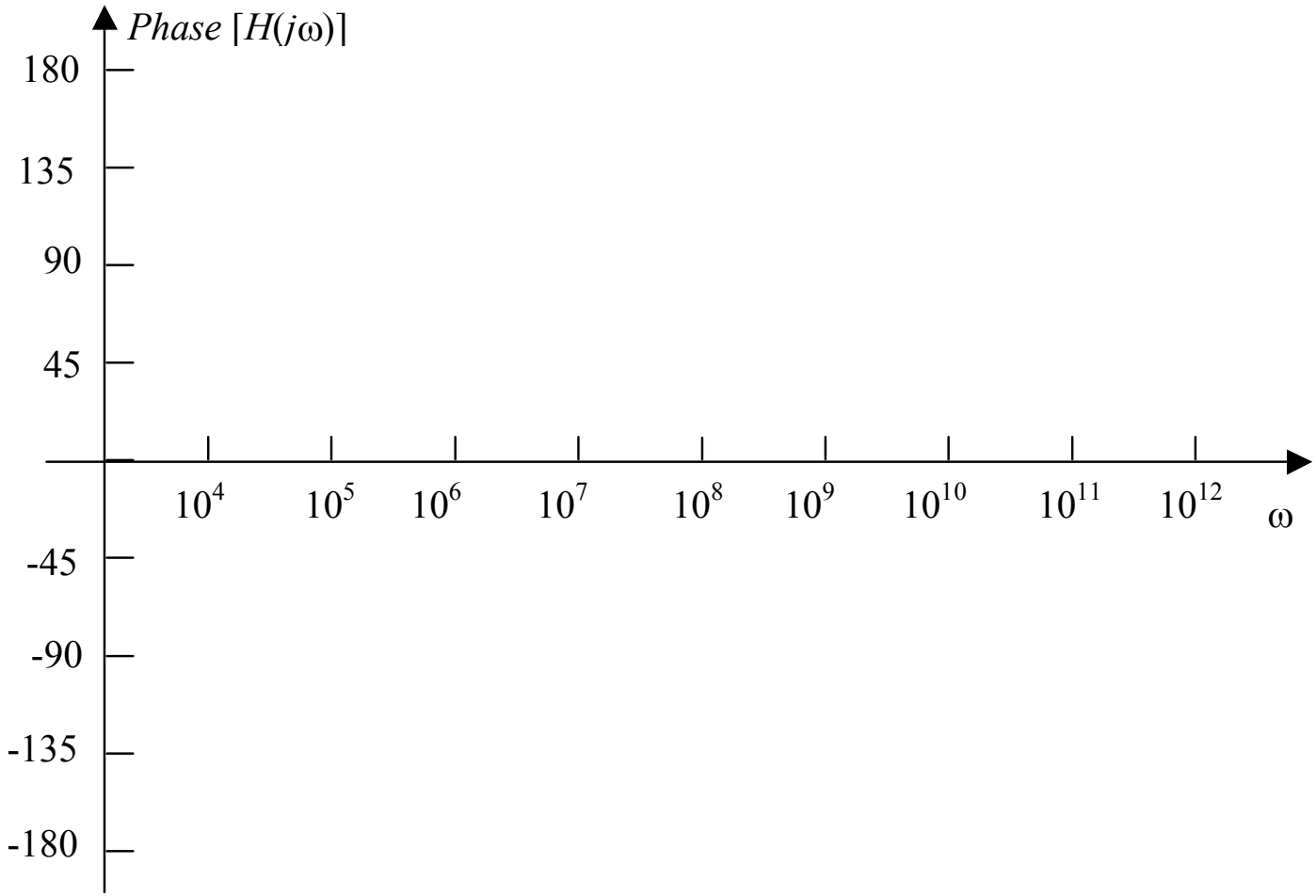
$$\angle H(j\omega) = \angle j\omega/10^5 + \angle(1+j\omega\tau_2) - \angle(1+j\omega\tau_1) - \angle(1+j\omega\tau_3)$$

→ Plot each term separately and add them graphically!

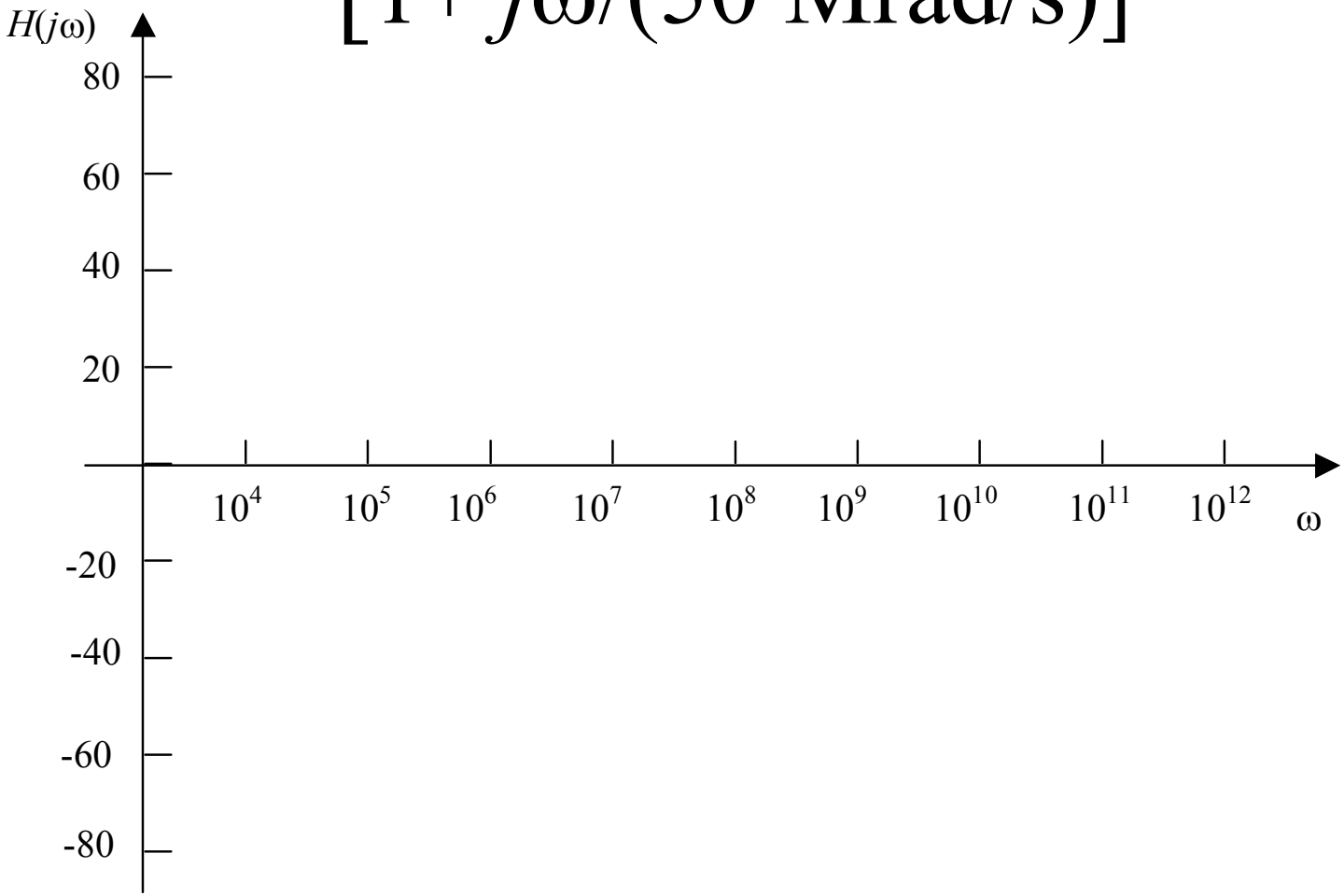
Magnitude Bode Plot: $j\omega/10^5$



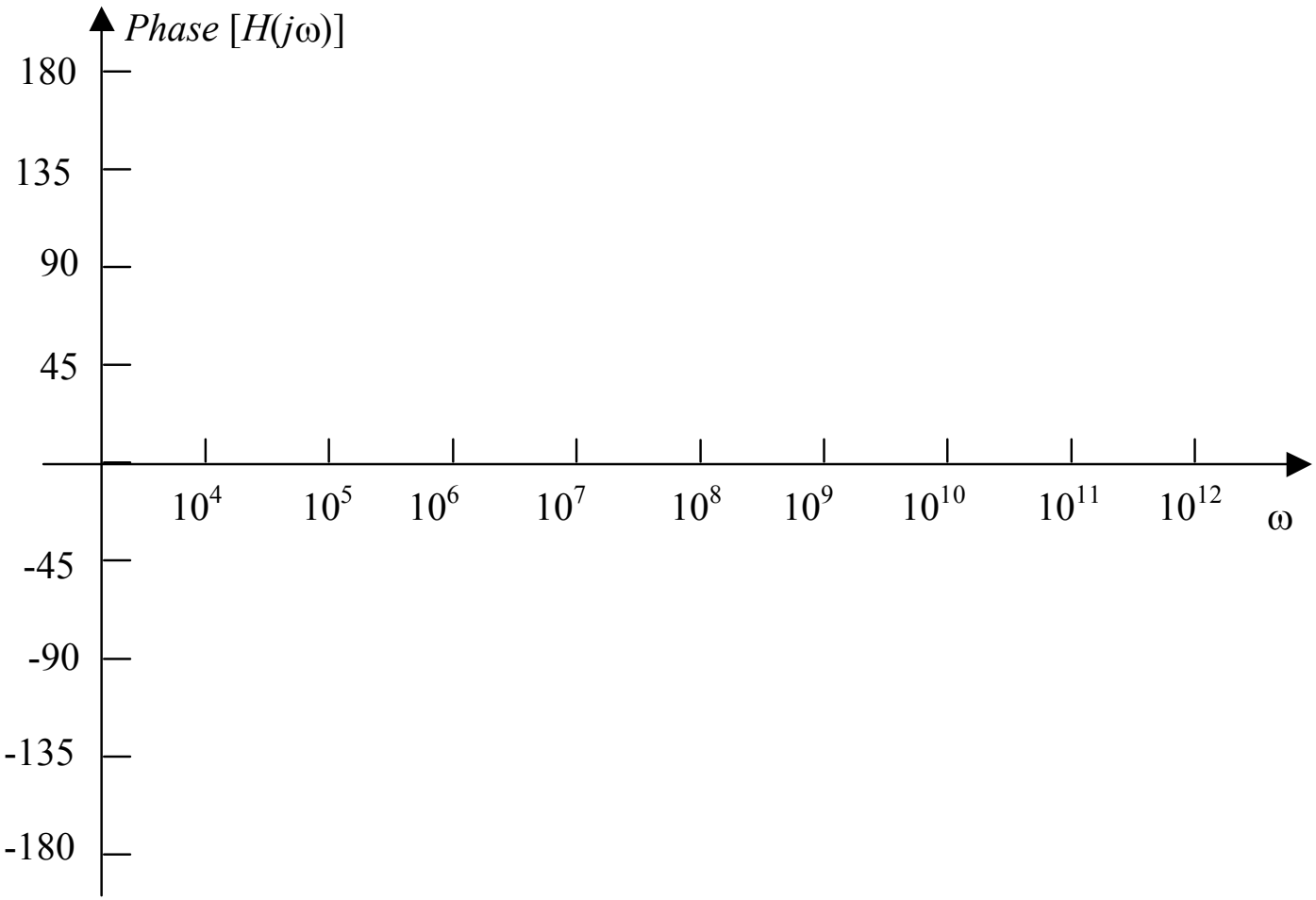
Phase Bode Plot: $j\omega/10^5$



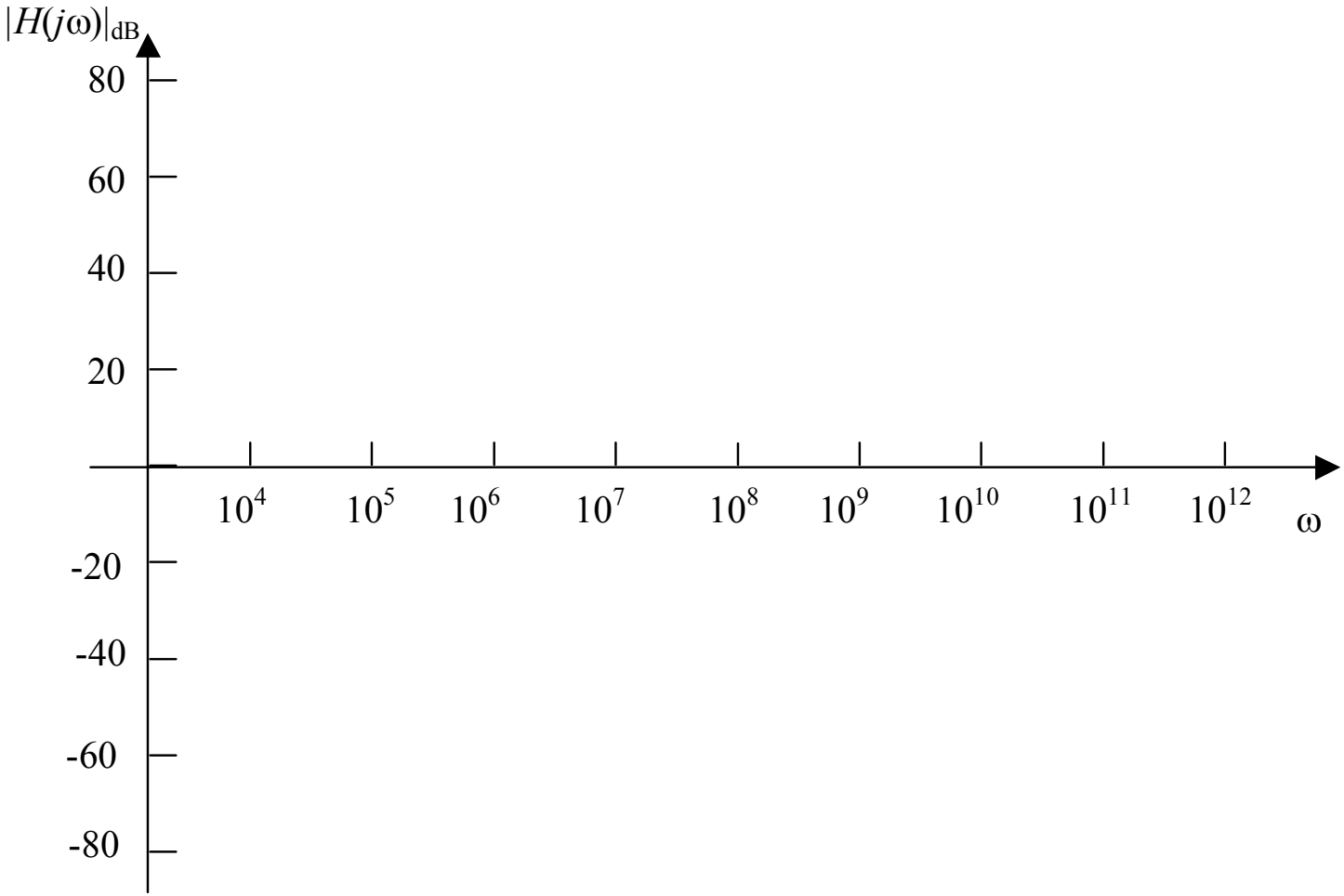
Magnitude Bode Plot: [1+ *j*ω/(50 Mrad/s)]



Phase Bode Plot: [1+ $j\omega/(50$ Mrad/s)]



Magnitude Bode Plot: $|H(j\omega)|_{dB}$



Phase Bode Plot: $\angle H(j\omega)$