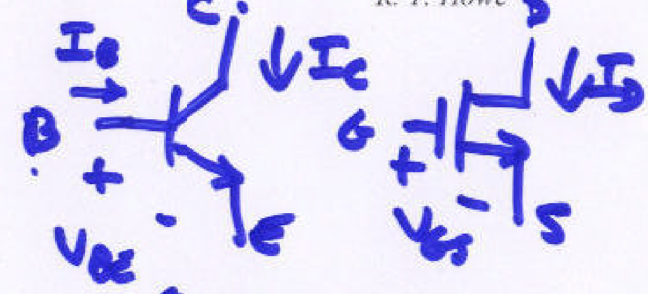
R. T. Howe

$\boxed{\text{LAB}}$: BJTs

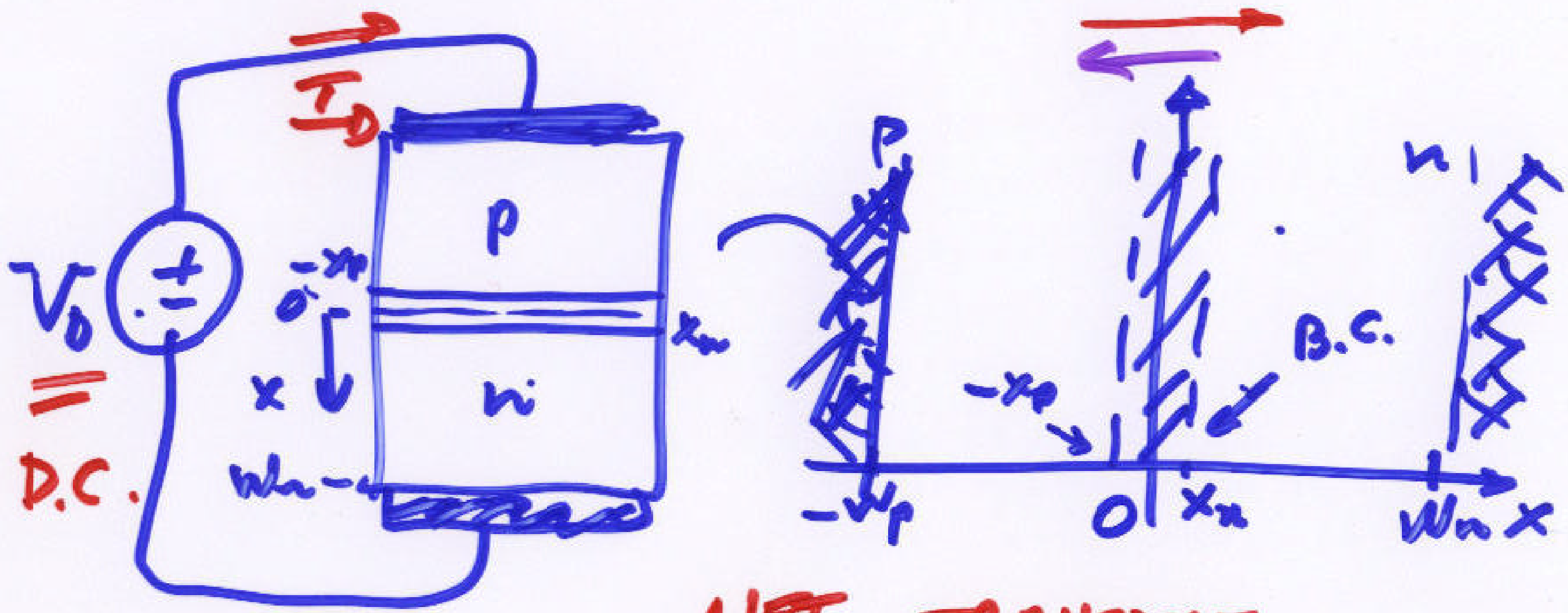
CHAPTER 7.
OLD F '01 LECTURE NOTES

Lecture 18



... "EBERS-MOLL."

- Last time:
  - pn junctions under *forward* bias ($V_D = 0.7$ V)
    - IMBALANCE OF DRIFT & DIFFUSION
      $\Rightarrow$ LARGE NET CURRENT.
- Today :
  - TODAY : BUILD THE MODEL
  - DC and small-signal model of the forward-biased diode



NET TRANSPORT

[HOLES → N-SIDE
ELECTRONS → P-SIDE.]

- **BOUNDARY CONDITIONS**

- $p_n(x = x_n)$; $p_n(x = W_n)$
- $n_p(x = -x_p)$; $n_p(x = -W_p)$

CHAPTER 6

Minority Carriers at Junction Edges

Minority carrier concentration at boundaries of depletion region increase as barrier lowers … the function is

$$\frac{p_n(x = x_n)}{p_p(x = -x_p)} = \frac{\text{(minority) hole conc. on n-side of barrier}}{\text{(majority) hole conc. on p-side of barrier}}$$

$$= \boxed{e^{-(\underline{Barrier\ Energy})/kT}} \therefore$$

← NOT GOING TO PROVE!

B.

T.E. "10^{-14}"

$$\boxed{\frac{p_n(x = x_n)}{N_A} = e^{-\underbrace{q(\phi_B - V_D)}_{\sim 1\,\mathrm{eV}}/kT}}$$

(Boltzmann's Law)

BARRIER: $q\phi_B$ $q(\phi_B - V_D)$

$kT = \underline{0.026\,\mathrm{eV}}.$

The Thermal Voltage

Define $V_{th} = (q / kT)^{-1}$ as the *thermal voltage* $\left(\frac{kT}{q}\right)$

Value: $q = 1.6 \times 10^{-19}$ C, $k = 1.38 \times 10^{-23}$ J/K
$T = 300$ K

$V_{th} = 26$ mV at room temperature

$= 0.026$ V.

"Law of the Junction"

Minority carrier concentrations at the edges of the depletion region are given by: (BOUNDARY COND.)

BOLTZMANN FACTOR ... $(10^{-(\text{big})})$

$$\boxed{\begin{aligned} p_n(x = x_n) &= N_A e^{-q(\phi_B - V_D)/kT} \\ n_p(x = -x_p) &= N_D e^{-q(\phi_B - V_D)/kT} \end{aligned}}$$

HOLES ON P-SIDE (T.E.) ...

$n_{si}(x = x_n)$

Note 1: N_A and N_D are the majority carrier concentrations on the *other* side of the junction

Note 2: we can reduce these equations further by substituting $V_D = 0$ V (thermal equilibrium)

Note 3: $\underbrace{\text{assumption that } p_n << N_D \text{ and } n_p << N_A}_{\text{LOW-LEVEL INJECTION.}}$

$V_D = 0\,V.$

Thermal Equilibrium Case

Define p_{no} as thermal equilibrium hole concentration on the n-side of the junction … ← T.E. MASS ACTION LAW.

$$p_{no} = \frac{n_i^2}{N_D} = \underbrace{N_A e^{-(\phi_B - 0)/V_{th}}}_{\text{L. of J.}}$$

Solve for the built-in barrier

$$\frac{n_i^2}{N_A N_D} = e^{-\phi_B/V_{th}} \quad \ldots \quad \ln\left[\frac{n_i^2}{N_A N_D}\right] = \frac{-\phi_B}{V_{th}}$$

$$\boxed{\phi_B = V_{th} \ln\left[\frac{N_A N_D}{n_i^2}\right]}$$

$V_{th} = 26\,mV$

Alternative form of junction law:

$$\boxed{\phi_B = (60\,mV)\log_{10}\left[\frac{N_A N_D}{n_i^2}\right]} \approx 1\,V$$

$= 60 \times 14\ mV.$
$= 0.84\ V.$

$\left.\begin{matrix} N_A \sim 10^{17} \\ N_D \sim 10^{17} \end{matrix}\right] \; 10^{34}\,cm^{-6}/10^{20} = 10^{14}$

PLUG BACK INTO LAW OF JUNCTION.

$$\rightarrow p_n(x = x_n) = N_A e^{-[\phi_B - V_D]/V_{th}}$$

$$= \boxed{N_A e^{-\phi_B/V_{th}}} e^{V_D/V_{th}}$$

$$\phi_B = V_{th} \ln\left[\frac{N_A N_D}{n_i^2}\right]$$

$$N_A e^{-[\cancel{V_{th}} \ln[\frac{N_A N_D}{n_i^2}]]/\cancel{V_{th}}}$$

$$N_A e^{\ln[\frac{n_i^2}{N_A N_D}]} = \cancel{N_A} \frac{n_i^2}{\cancel{N_A} N_D}$$

$$= \frac{n_i^2}{N_D} \equiv p_{no}$$

equal.

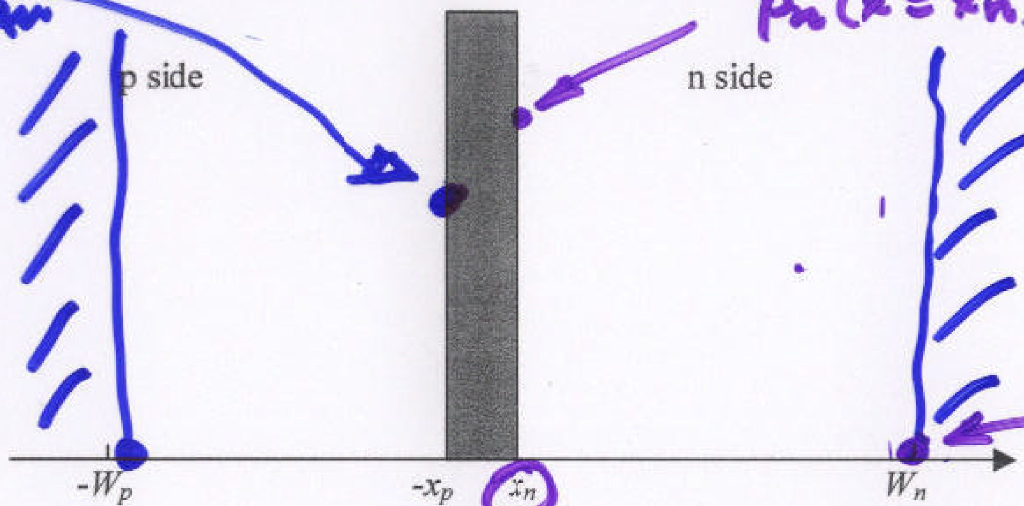
$$\boxed{p_n(x = x_n) = p_{no} e^{+V_D/V_{th}}}$$

Boundary Conditions

$n_p(x=-x_p) = n_{po}e^{V_D/V_{th}}$

$\left(n_{po} = \frac{n_i^2}{N_A}\right)$

$p_n(x=x_n) = p_{no}e^{V_D/V_{th}}$



Depletion region edges: ✓ (LAW OF JUNCTION)

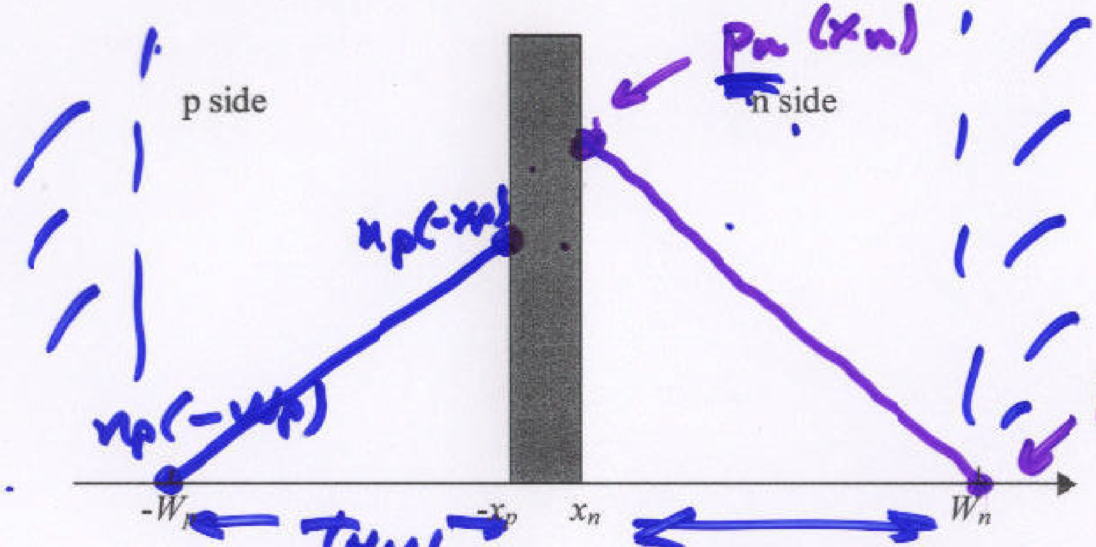
Ohmic contacts: KEPT AT THERMAL EQUILIBRIUM.

CHAPTER 6

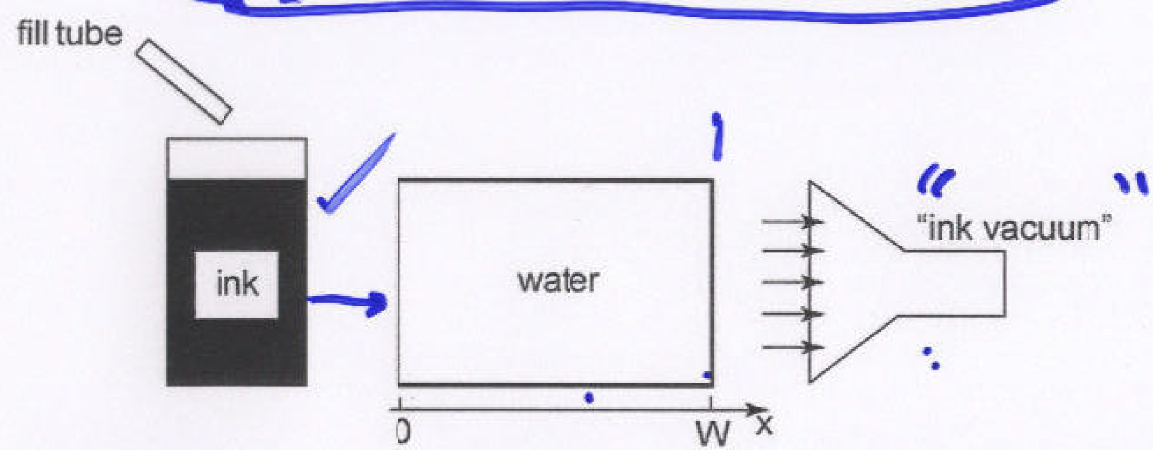
$I_D \leftarrow$ D.C. $\underline{\underline{V_D}}$ ~~f(t)~~

Steady-State Concentrations

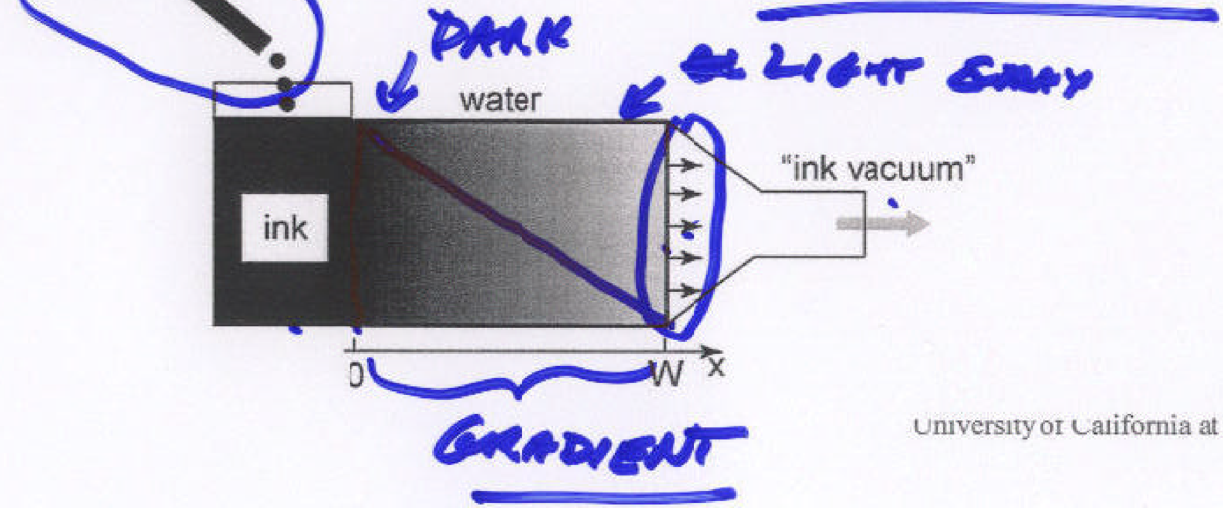
Assume that none of the diffusing holes and electrons recombine → get straight lines … ALL MAKE IT!



Diffusion Analogy



WAIT ... STEADY - STATE.



$n_{po} e^{V_D/V_{th}}$

Diode Current Densities

$$J_n^{diff} = qD_n \frac{dn_p}{dx}\Bigg|_{x=-x_p} = qD_n \left[\frac{n_p(x=-x_p) - n_{po}}{-x_p - (-W_p)} \right].$$

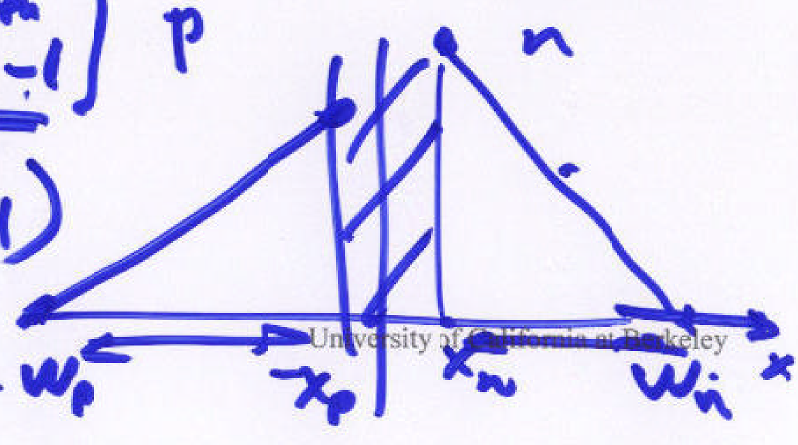
$$J_p^{diff} = -qD_p \frac{dp_n}{dx}\Bigg|_{x=x_n} = -qD_p \left[\frac{p_n(W_n) - p_n(x_n)}{W_n - x_n} \right]$$

Total current: $J = J_n^{diff} + J_p^{diff} \leftarrow$ @ JUNCTION.

$$J \cong \left[\frac{qD_n}{W_p}\right] n_{po}\left[e^{V_D/V_{th}} - 1\right] + \left(\frac{qD_p}{W_n}\right) p_{no}\left(e^{V_D/V_{th}} - 1\right)$$

$\ll W_n, W_p$

$x_n, x_p \to 0.$



$$I_D = \boxed{A \cdot \left[\frac{q D_n}{W_p} + \frac{q D_p}{W_n}\right]}\left(e^{V_D/V_{th}} - 1\right)$$

I_0

pn Junctions in ICs