R. T. Howe

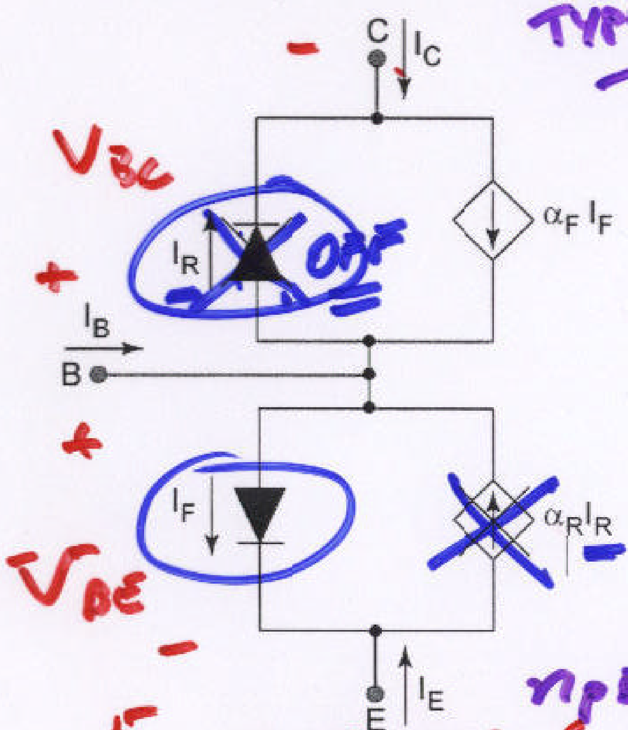
Lecture 21

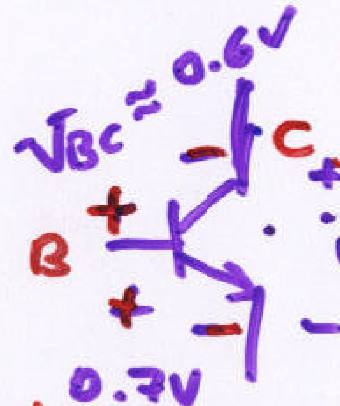
- Last time:
  - Large-signal model under forward bias
  - Ebers-Moll model, start small signal model
- Today :
  - Small-signal model of the npn bipolar transistor

Forward-Active Model

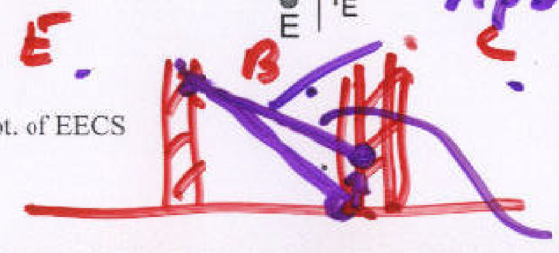
- B-C junction is not forward-biased → I_R is very small

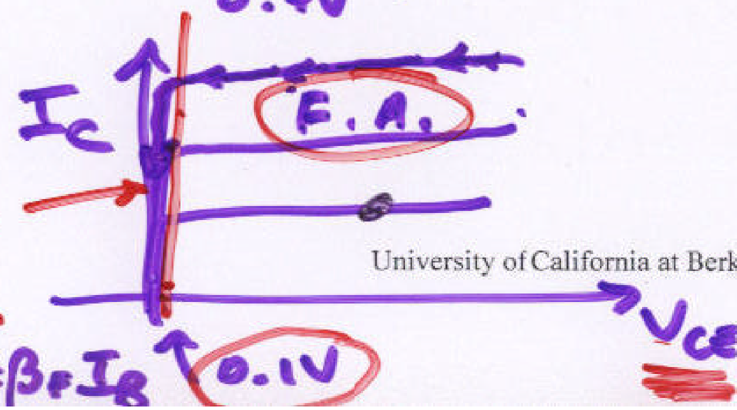
Typical: $V_{BE} = 0.7\,V$, $V_{CE} = 2\,V$ … $V_{BC} = -1.3\,V$.





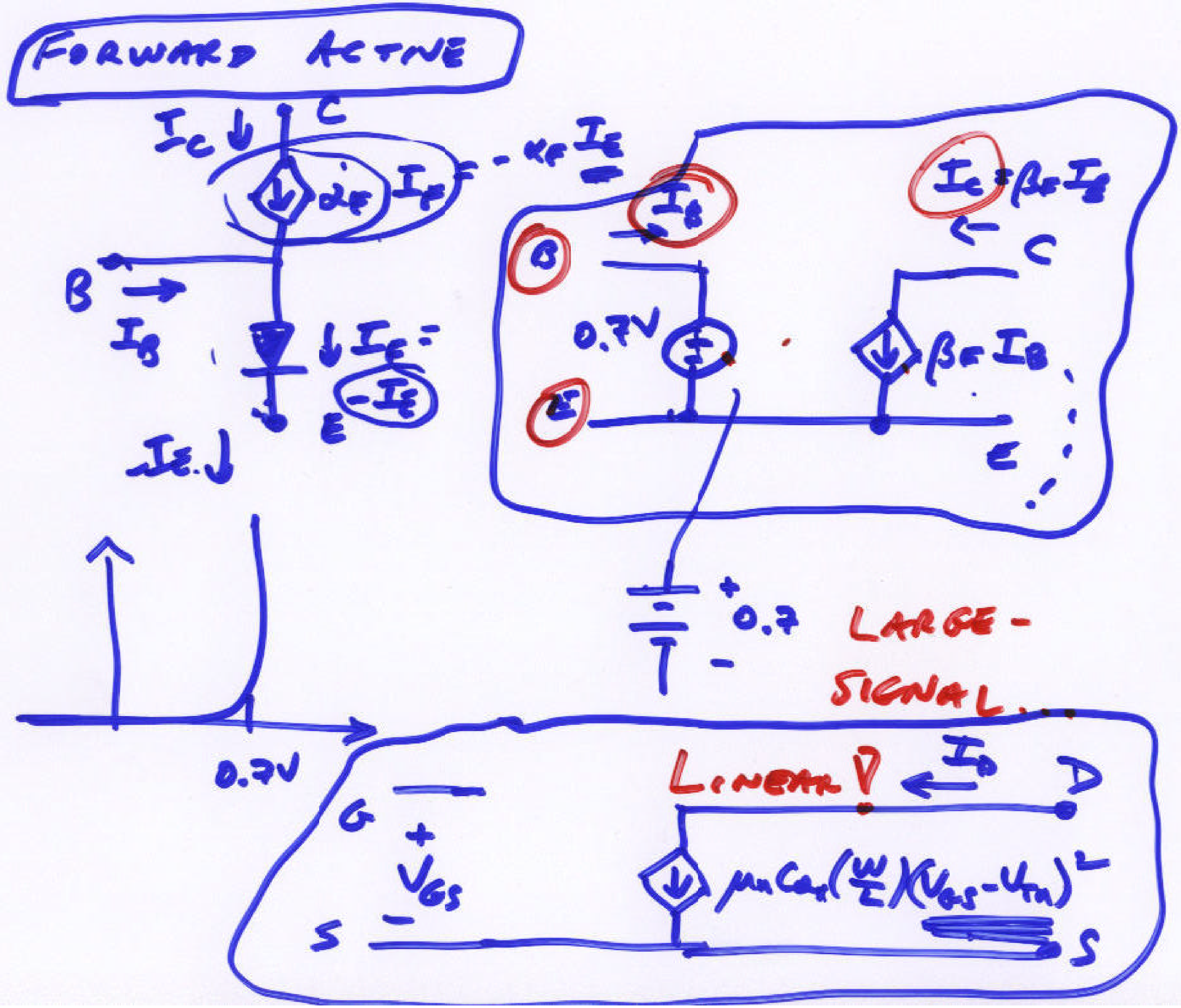
$V_{CE_{sat}} \approx 0.1\,V$.





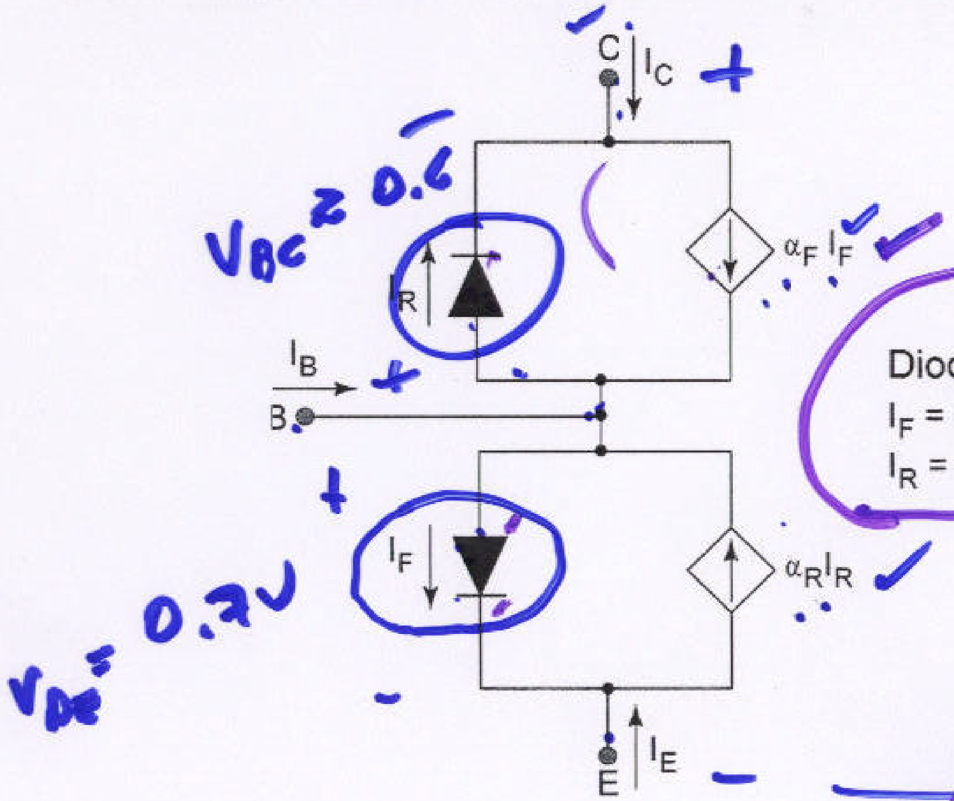
BJT SAT.

$I_C \neq \beta_F I_B$


FORWARD ACTIVE
I_C
C
$\alpha_F I_F = -\alpha_F I_E$
B
I_B
$I_F = -I_E$
E
I_E
0.7V
I_B
$I_C = \beta_F I_B$
0.7V
$\beta_F I_B$
E
+ 0.7 −
LARGE-SIGNAL...
LINEAR?
I_D
D
G
+ V_{GS} −
S
$\mu_n C_{ox}(\frac{W}{L})(V_{GS}-V_{TH})^2$
S

Ebers-Moll Equivalent Circuit

Building blocks: diodes and *I*-controlled *I* sources

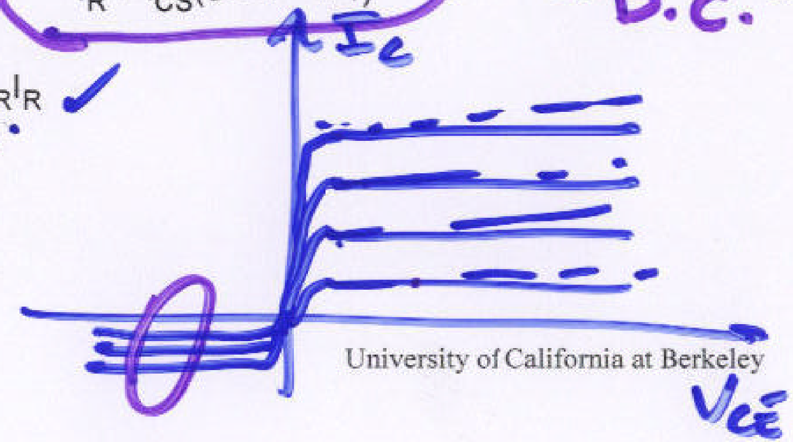


Diode Currents:

$I_F = I_{ES}(e^{V_{BE}/V_{th}} - 1)$

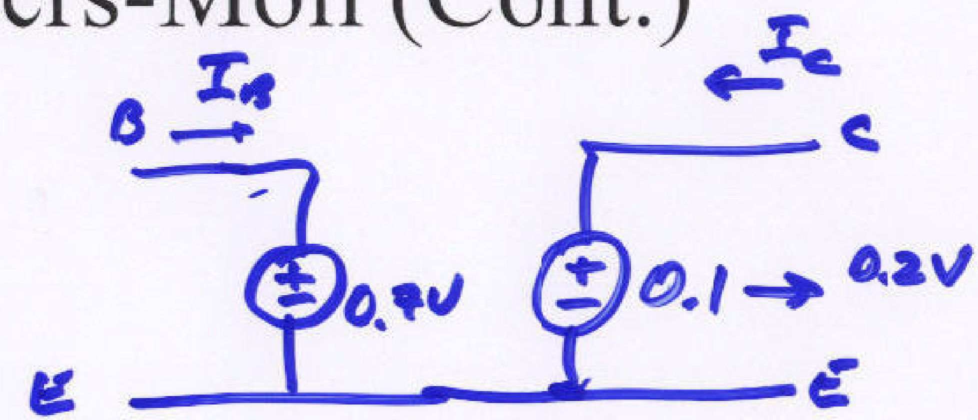
$I_R = I_{CS}(e^{V_{BC}/V_{th}} - 1)$

LINEAR ELEMENTS... EVEN FOR "D.C."



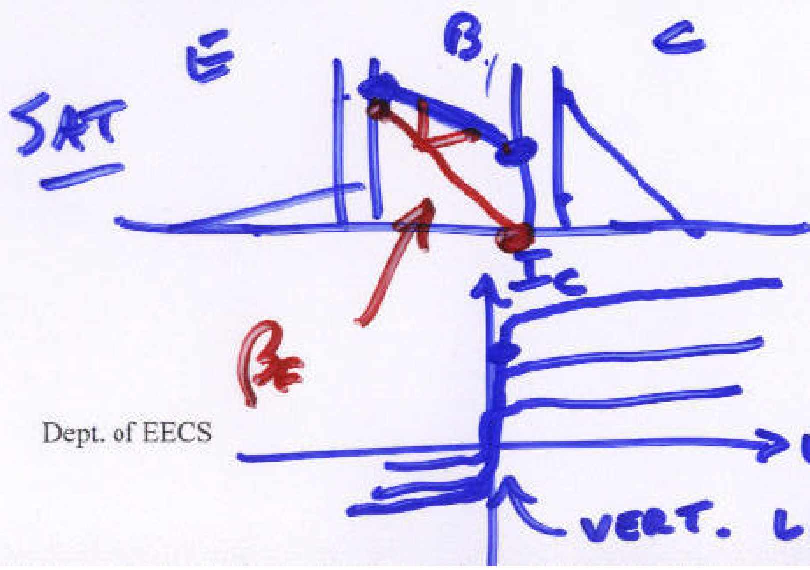
Simplified Ebers-Moll (Cont.)

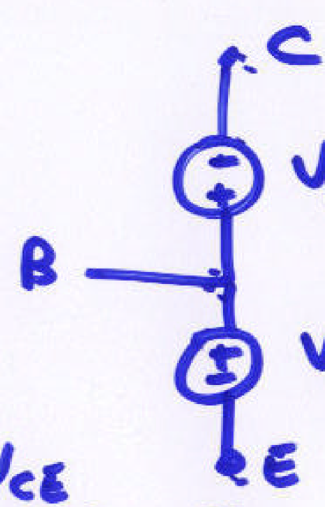
~~Forward-Active Case~~ ✓



Saturation: both diodes are forward-biases → batteries

$$I_C < \beta_F I_B$$



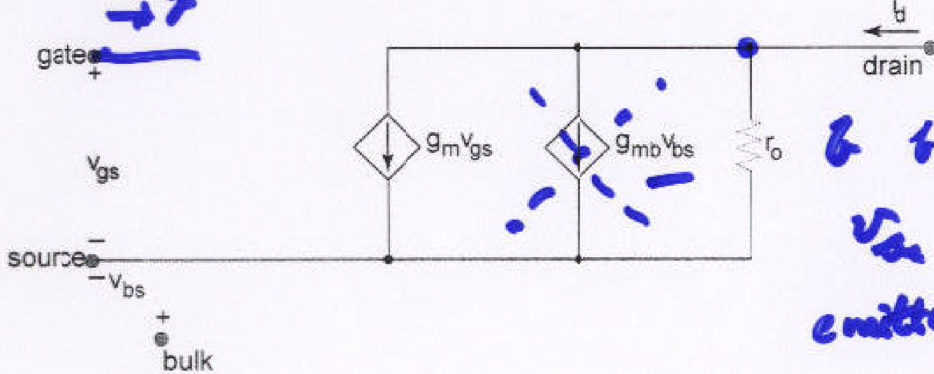


Small-Signal Model

Analogy from MOSFET s.s. model:

BACKGATE

$$i_D = f(v_{GS}, v_{DS}, v_{BS})$$



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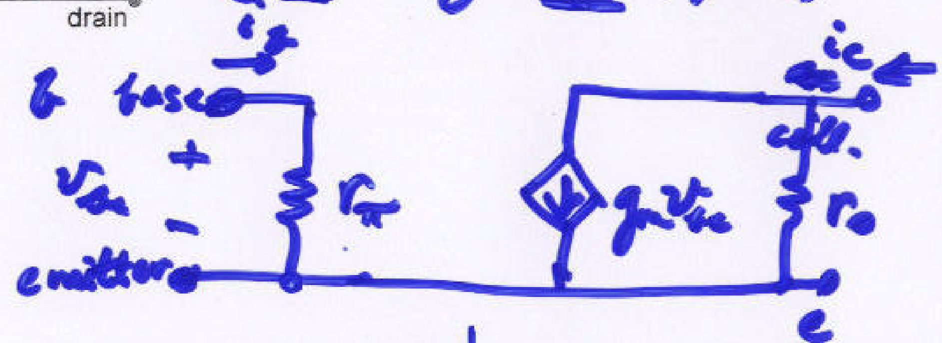
$$i_C = f(v_{BE}, v_{CE})$$

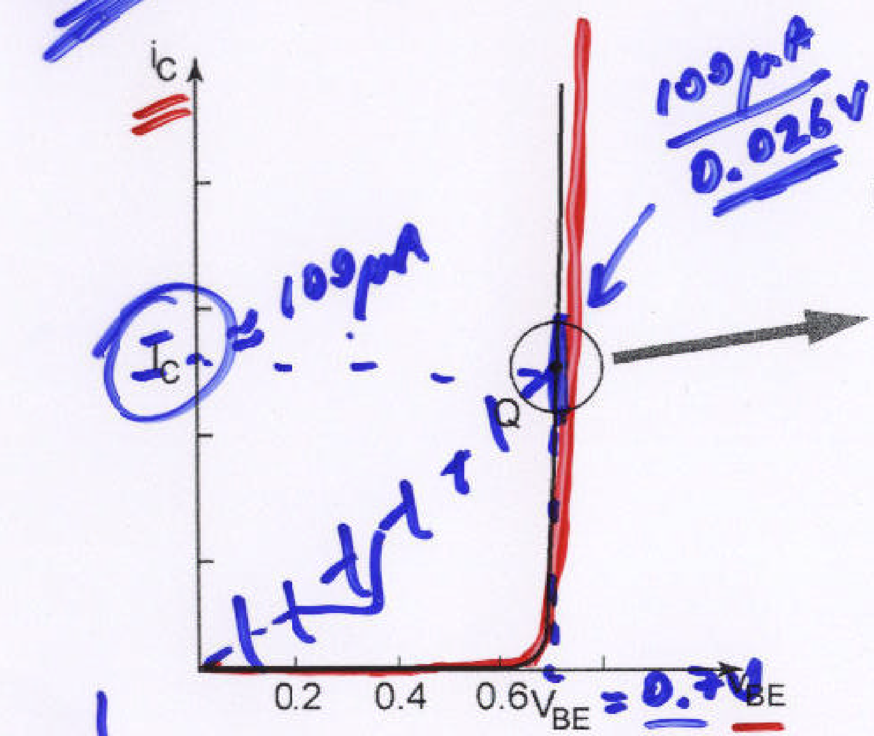
$$i_B = g(v_{BE}, \cancel{v_{CE}})$$

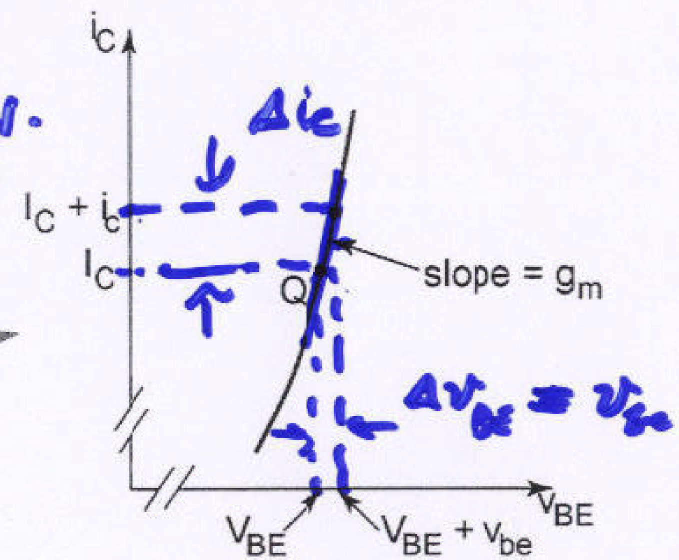


$$r_\pi = \frac{1}{\left\{ \left. \frac{\partial i_B}{\partial v_{BE}} \right|_Q \right\}}$$

Transconductance g_m

$V_{BE} \approx 0.7V$





$$i_C = I_S e^{v_{BE}/V_{th}}$$

constants.

$$g_m \equiv \frac{\Delta i_C}{\Delta v_{BE}}\Big|_Q$$

$$g_m = \frac{i_c}{v_{be}}$$

Transconductance (cont.)

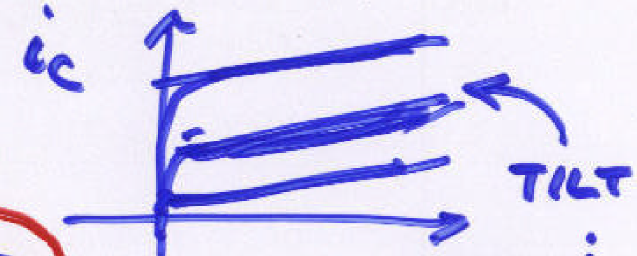
- Forward-active large-signal current:

  - $i_C = I_S e^{v_{BE}/V_{th}} (1 + v_{CE}/V_A)$

    FUDGE FACTOR. $\frac{1}{V_A}$ ↔ $(1 + \lambda_n V_{DS})$

- Differentiating and evaluating at $Q = (V_{BE}, V_{CE})$

V_A = Early voltage



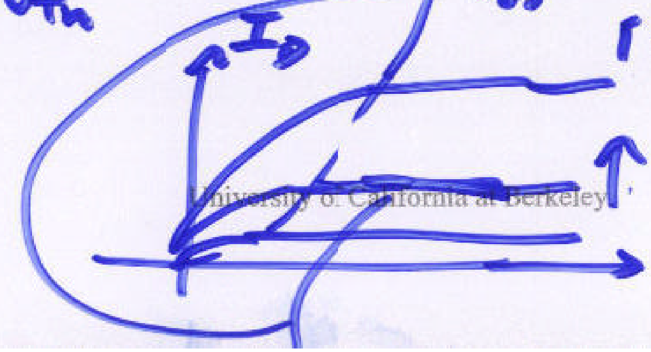
$$g_m = \left.\frac{\partial i_C}{\partial v_{BE}}\right|_Q = \left[\frac{I_S}{V_{th}} e^{V_{BE}/V_{th}}\right]\left[1 + \frac{V_{CE}}{V_A}\right] \leftarrow I_C$$

few %.

$$g_m = I_C / V_{th}$$

Comparison with MOSFET g_m

- Bipolar transistor:
- MOSFET:
- Typical bias point: drain/coll. current = 100 μA; Select $(W/L) = 8/1$, $\mu_n C_{ox} = 100\ \mu A/V^2$



$i_B = I_B + i_b$ $\quad I_C/\beta_F \approx \frac{99\mu A}{100} = 1\mu A.$

What about the Base Current?

Unlike MOSFET, there is a DC current into the base terminal of a bipolar transistor:

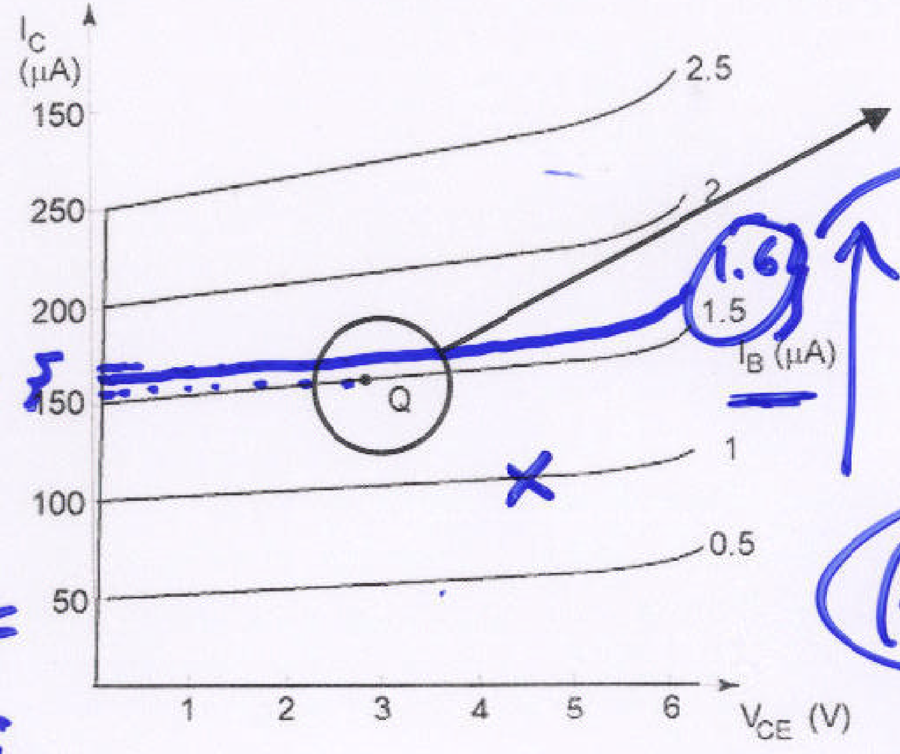
$$I_B = I_C / \beta_F = (I_S / \beta_F) e^{V_{BE}/V_{th}} (1 + V_{CE}/V_{th})$$

To find the change in base current due to change in base-emitter voltage:

$$\left.\frac{\partial i_B}{\partial v_{BE}}\right|_Q = \underbrace{\left.\frac{\partial i_B}{\partial i_C}\right|_Q}_{\beta_o^{-1}} \underbrace{\left.\frac{\partial i_C}{\partial v_{BE}}\right|_Q}_{g_m} = \left(\frac{1}{\beta_o}\right) g_m.$$

$\beta_F = \frac{\alpha_F}{1-\alpha_F} = \ldots \quad \alpha_F \approx 0.99. \quad \beta_F \pm 50\%$

Small-Signal Current Gain β_o



$\Delta i_B = 100\,nA = 1.6 - 1.5\ \mu A$

$\Delta i_C = i_c$

$\beta_o = \frac{\Delta i_c}{\Delta i_B} \approx \beta_F \pm 50\%$

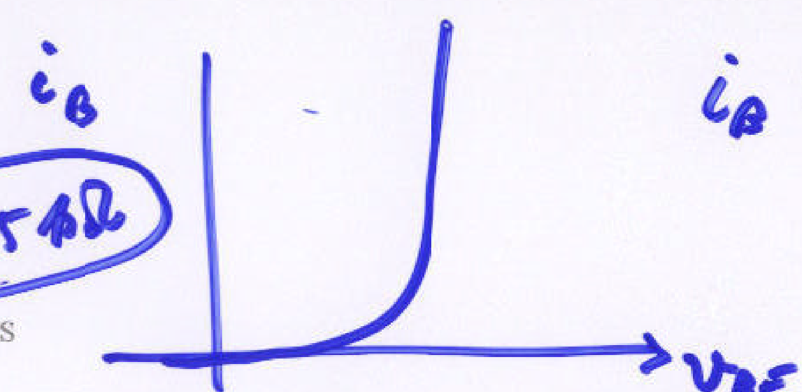
NOT $\left(\frac{I_C}{I_B}\right)$

Input Resistance r_π

$$(r_\pi)^{-1} = \frac{\partial i_B}{\partial v_{BE}}\bigg|_Q = \left(\frac{1}{\beta}\right)\frac{\partial i_C}{\partial v_{BE}}\bigg|_Q = \frac{g_m}{\beta_o} \leftarrow \text{s.s.}$$

In practice, the DC current gain β_F and the small-signal current gain β_o are both highly variable (+/- ~~25~~ 50%)

Typical bias point: DC collector current = 100 μA



$i_B \approx \frac{i_C}{\beta_F}$

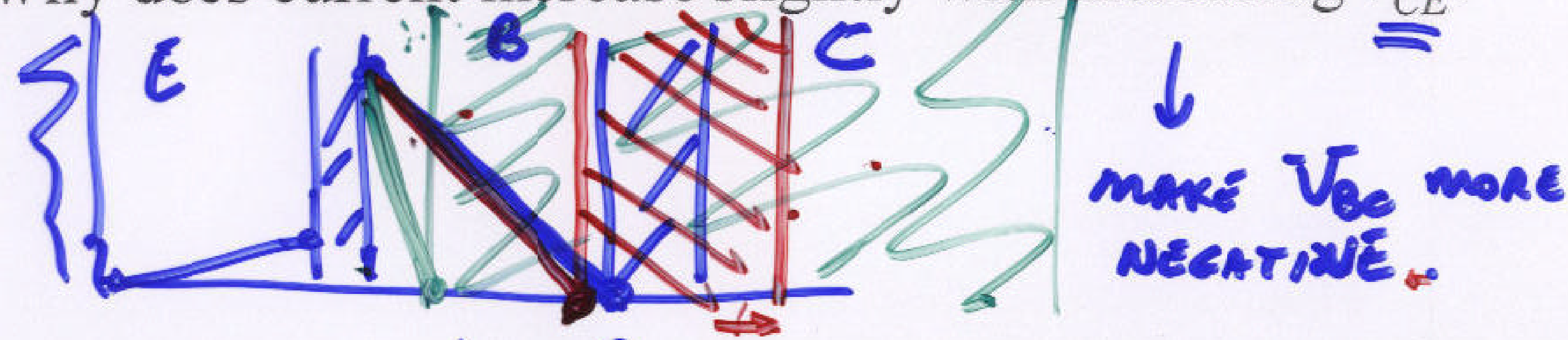
$g_m = 4\text{ mS} = \frac{100\,\mu A}{\left(\frac{1}{40}\text{ V}\right)}$

$r_\pi = \frac{\beta_o}{g_m} = \frac{100}{4\text{ mS}}$

$r_\pi = 25\text{ k}\Omega$

Output Resistance r_o

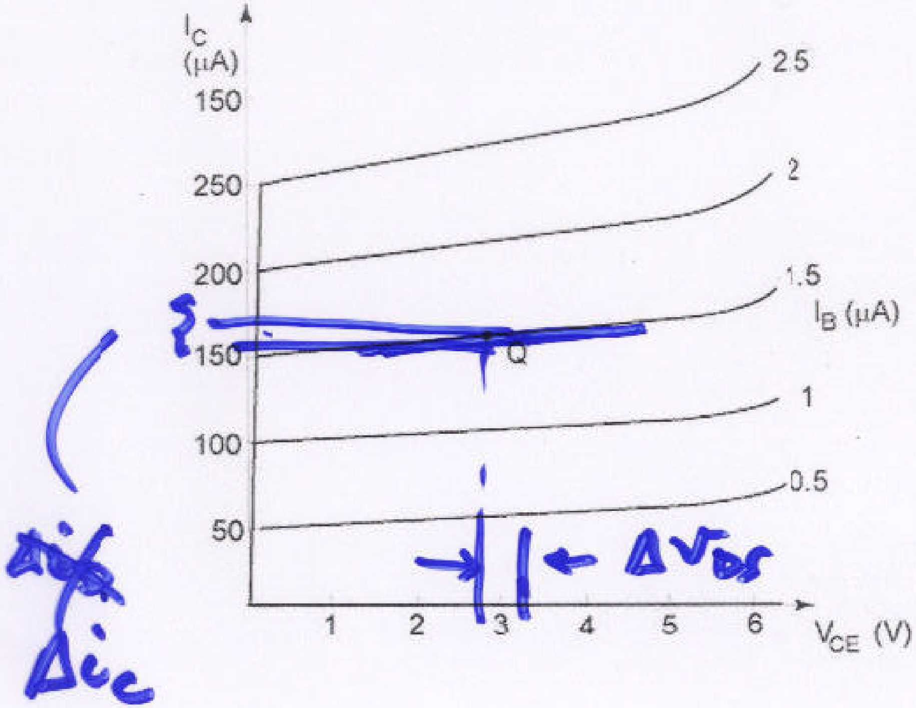
- Why does current increase slightly with increasing v_{CE}?



Model: math is a mess, so introduce the Early voltage

$$i_C = I_S e^{v_{BE}/V_{th}} (1 + v_{CE}/V_A)$$

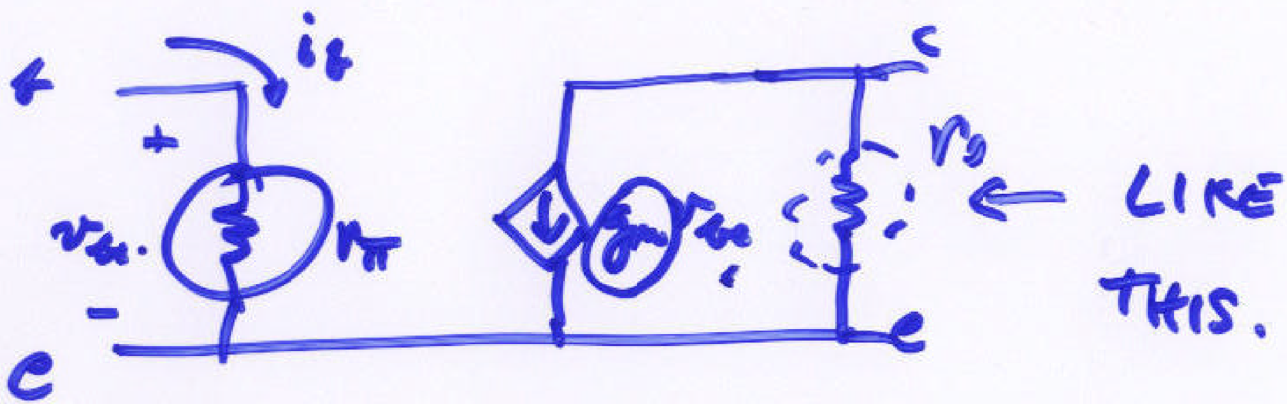
Graphical Interpretation of r_o

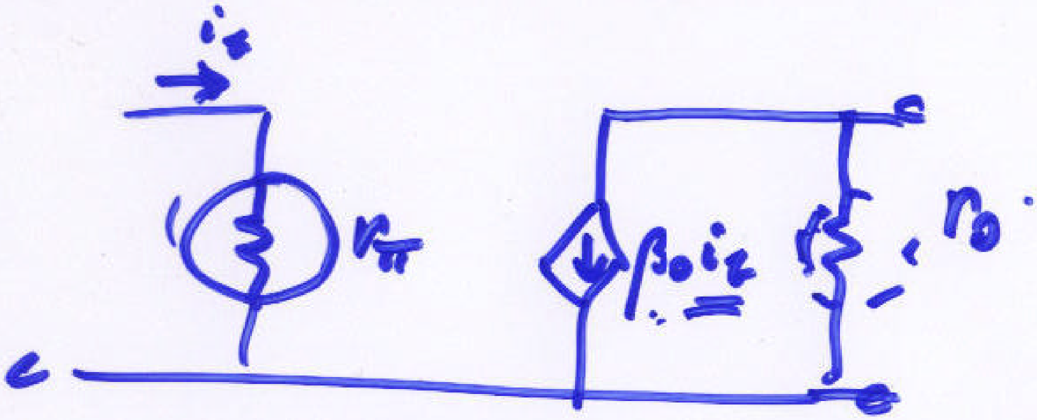


$$r_o \approx \frac{V_A}{I_C} = \frac{35V}{100\mu A} = 350 k\Omega.$$

Typical value:

$V_A = 35V.$





$$v_{be} = r_\pi i_b$$

$$i_c = g_m v_{be} = (g_m r_\pi) i_b = \beta_o i_b$$