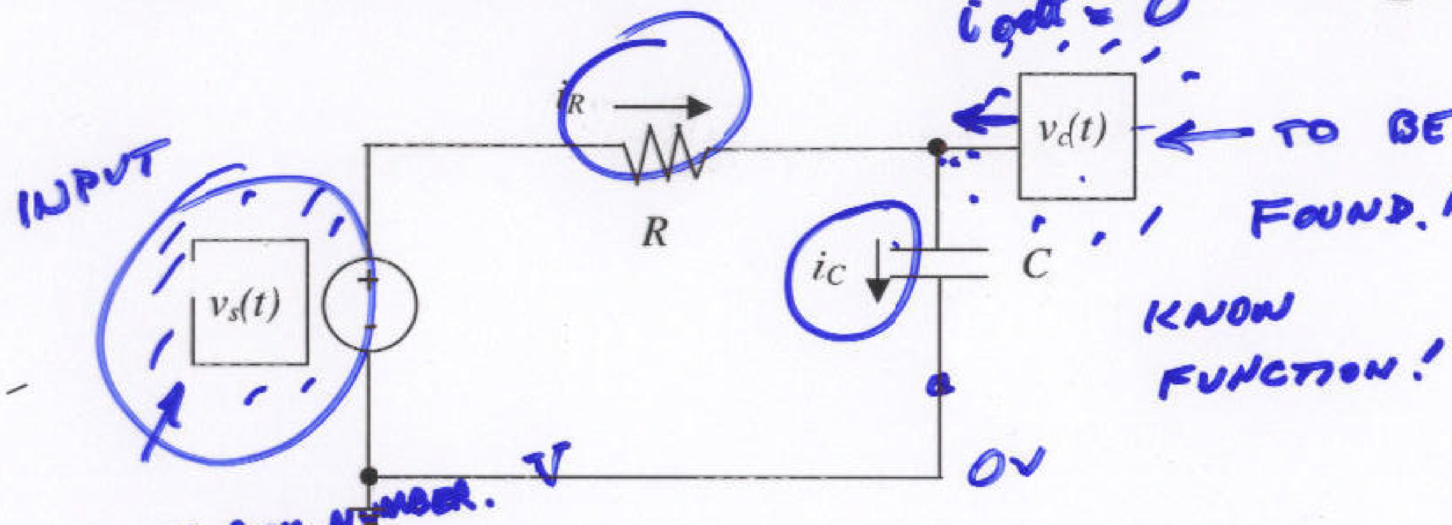
- WAITLISTS → CHECK THAT YOU'RE ENROLLED.

Lecture 2

- LABS: START MONDAY → SHOULD CHECK ON WEB PAGE. STOP BY 353 CORY AT 1 PM.

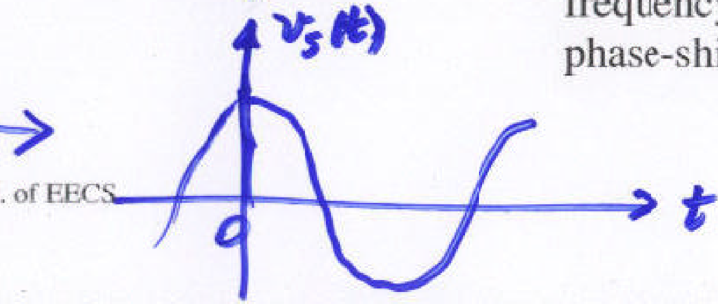
- Last time:
  - Course overview
  - Sinusoidal signals !
  - *RC* circuit with sinusoidal input voltage
- Today :
  - Finish *RC* circuit: introduce deciBel (dB)
  - Introduce phasor representation of sinusoids

RC Circuit with Sinusoidal Input



$v_s(t) = V_s \cos(\omega t)$: set phase of source to zero (use as the reference)

$v_c(t) = V_c \cos(\omega t + \phi)$: solution is a sinusoidal signal with the same frequency, but with a different amplitude and phase-shifted with respect to the source



Circuit Analysis

$$i_R = \frac{v_S - v_C}{R} \; ; \quad i_C = C\frac{dv_C}{dt}$$

$$i_R = i_C \Rightarrow \frac{v_S - v_C}{R} = C\frac{dv_C}{dt} \quad \boxed{C = \text{const.}}$$

$$v_S - v_C = RC\frac{dv_C}{dt} = \tau\frac{dv_C}{dt} \quad \checkmark$$

$$\tau = s.$$

$$\boxed{v_S(t) = \tau\frac{dv_C}{dt} + v_C.}$$

we want $(\phi, V_C/V_S)$ variable

$$v_C(t) = V_C\cos(\omega t + \phi)$$

$$\frac{dv_C}{dt} = -V_C\,\omega\sin(\omega t + \phi)$$

$$V_S\cos\omega t = \tau\left[-\omega V_C\boxed{\sin(\omega t+\phi)}\right] + V_C\cos(\omega t + \phi)$$

$$\sin(x+y) = \sin x\cos y + \cos x\sin y.$$

$\cos(x+y) = \cos x \cos y - \sin x \sin y$

Circuit Analysis (Continued)

$\sin(\omega t + \phi) = \sin\omega t \cos\phi + \cos\omega t \sin\phi$

$$V_s \cos\omega t = -\omega\tau V_c \{\overbrace{\sin\omega t \cos\phi + \cos\omega t \sin\phi}\}$$

$$+ V_c \underbrace{\cos(\omega t + \phi)} \rightarrow \cos\omega t \cos\phi - \sin\omega t \sin\phi$$

$$\boxed{V_s \cos\omega t} = \{-\omega\tau V_c \cos\phi - V_c \sin\phi\} \sin\omega t - \omega\tau V_c \sin\phi \cos\omega t + V_c \cos\phi \cos\omega t$$

$$= \sin\omega t \{\overbrace{-\omega\tau V_c \cos\phi - V_c \sin\phi}^{\phi}\} + \cos\omega t \{\underbrace{-\omega\tau V_c \sin\phi + \cdot}_{V_s}\}$$

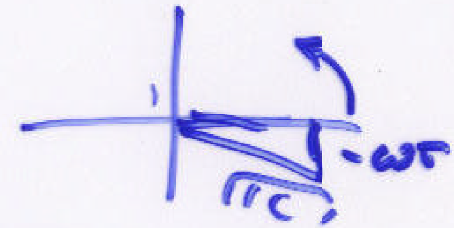
\#2

$0 = \cancel{V_c}[-\omega\tau \cos\phi - \sin\phi]$

$\sin\phi / \cos\phi = -\omega\tau \Rightarrow \boxed{\phi = \tan^{-1}(-\omega\tau)}$

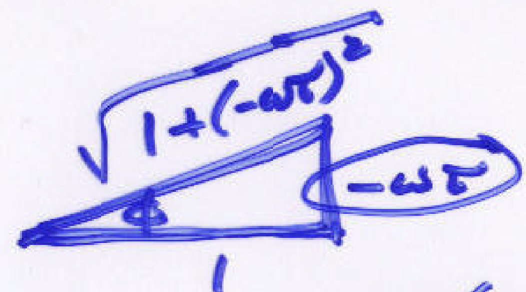
- KNOW ϕ ... FIND V_c/V_s

$$V_s = -\omega\tau V_c \sin\phi + V_c \cos\phi$$



$$V_s/V_c = -\omega\tau \sin\phi + \cos\phi.$$

$$\sin\left[\tan^{-1}(-\omega\tau)\right]$$



$$\sin\phi = \frac{-\omega\tau}{\sqrt{1+(\omega\tau)^2}}$$

$$\cos\phi = \frac{1}{\sqrt{}}$$

$$\frac{V_s}{V_c} = \frac{-\omega\tau(-\omega\tau)}{\sqrt{1+(\omega\tau)^2}} + \frac{1}{\sqrt{1+(\omega\tau)^2}}$$

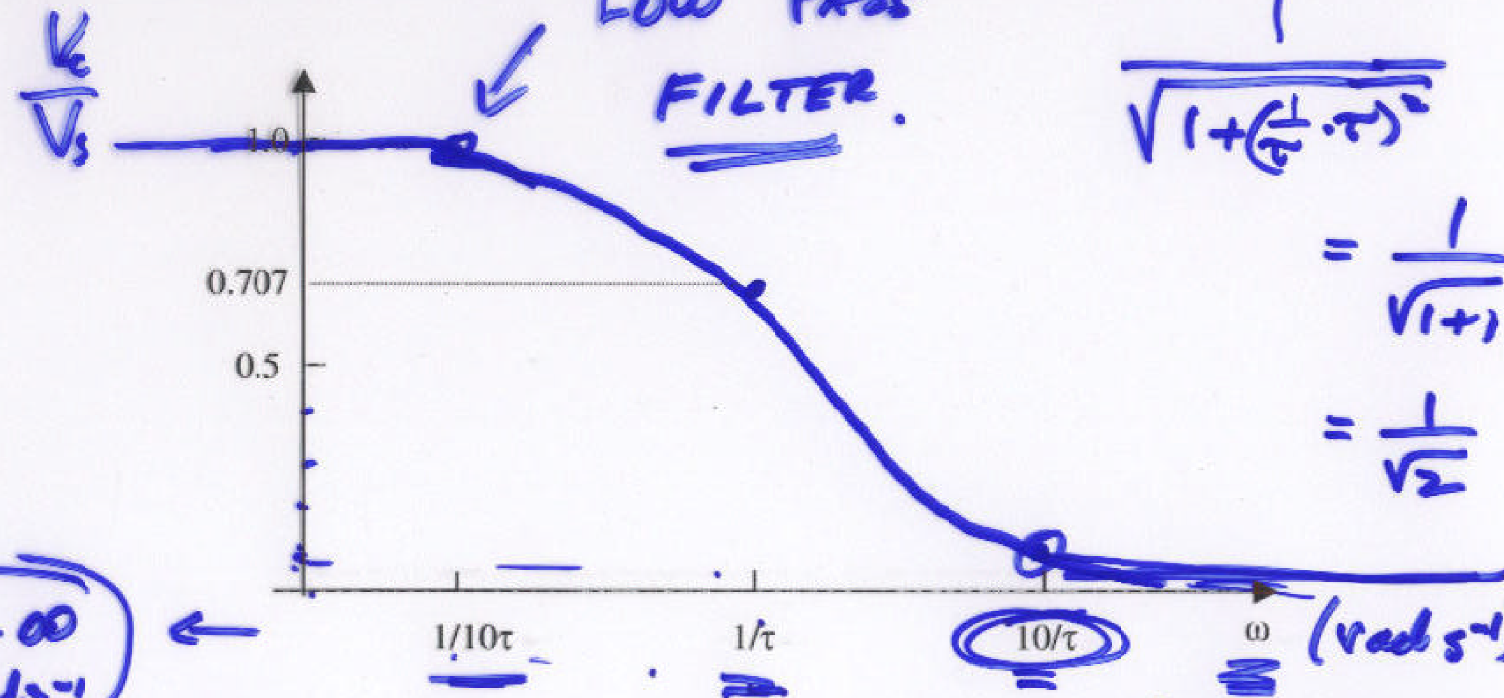
- $$\frac{V_s}{V_c} = \frac{1+(\omega\tau)^2}{\sqrt{1+(\omega\tau)^2}} = \sqrt{1+(\omega\tau)^2}$$

- $$V_c/V_s = \frac{1}{\sqrt{1+(\omega\tau)^2}}.$$

$$\frac{V_c}{V_s} = \frac{1}{\sqrt{1+(\omega\tau)^2}}.$$

Graphical Result for Amplitude Ratio.

LOW PASS FILTER.



$$\frac{1}{\sqrt{1+\left(\frac{1}{\tau}\cdot\tau\right)^2}} = \frac{1}{\sqrt{1+1}} = \frac{1}{\sqrt{2}}$$

(LOG SCALE)

$\leftarrow -\infty$

0 rad s^{-1}

$f(\omega\tau)$

$\tau = 1\ \mu s$

$1/\tau = 1$ Mrad s^{-1}

0 — 10^{10}

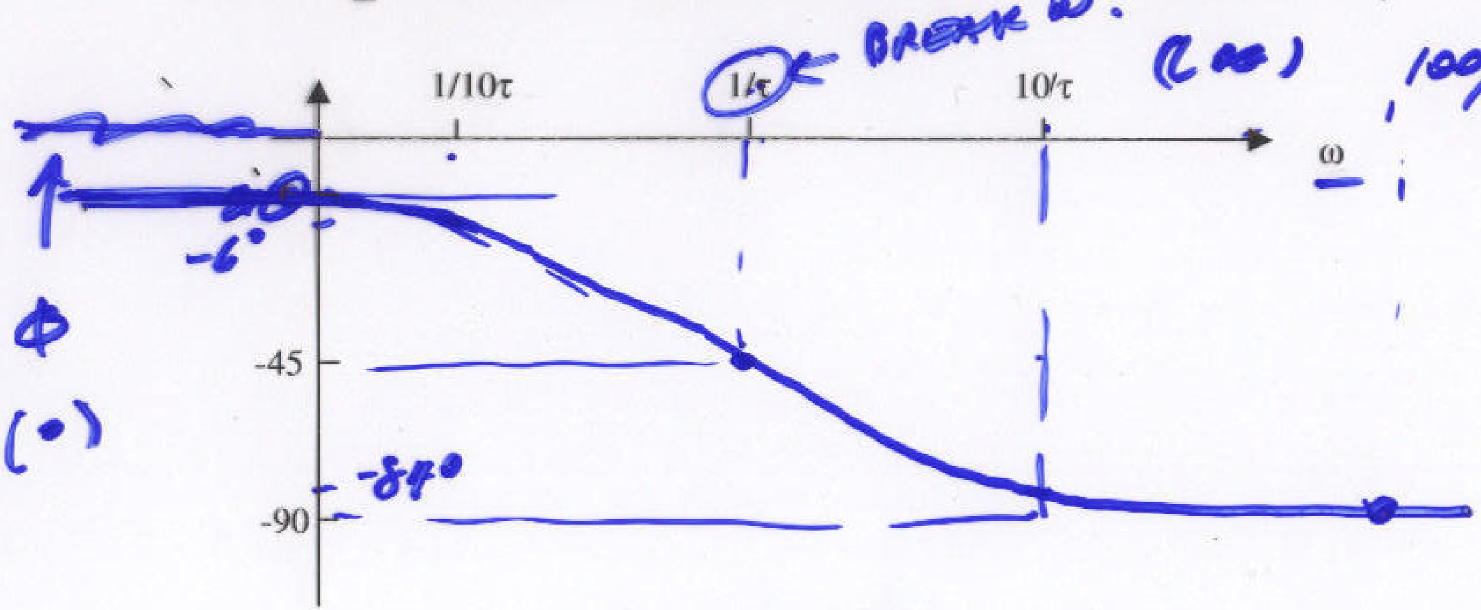
$$v_c(t) = V_c \cos(\omega t + \phi)$$

$$V_c = \check{V}_s\left(\frac{1}{\sqrt{1+(\omega\tau)^2}}\right)$$

$$\phi = \tan^{-1}(-\omega\tau)$$

$$\boxed{\phi = \tan^{-1}(-\omega\tau)}$$

Graphical Result for Phase ϕ



$0,\ 1/10$

$\phi = \tan^{-1}(-\omega\tau) \approx 0^\circ$ $\quad \tan^{-1}(-\infty)$

$\tan^{-1}(-1) = -45^\circ$ $\quad = -90^\circ$

Amplitude: a new representation

- We are interested in *very* small ratios (e.g., $V_c/V_s = 0.0001$)

- Therefore, we use a log plot … but we also define a new function called the deciBel (after Alex. Graham Bell)

$$(V_c/V_s)_{dB} = 20 \log_{10} (V_c/V_s)$$

10^{-4}

$\log x^2 = 2 \log x$

- Examples: $V_c/V_s = 0.0001 \rightarrow (V_c/V_s)_{dB} = -80 \text{ dB}$

  $V_c/V_s = 0.707 \rightarrow (V_c/V_s)_{dB} = -3 \text{ dB}$

$\omega = \frac{1}{\tau}$

$\omega\tau = 1$

$\cancel{\cos\omega t}$ $\cancel{\sin\omega t}$ $e^{(i\omega t)}$

A Better Technique

- It is much more efficient to work with *imaginary exponentials* as "representing" sinusoids, since these functions are direct solutions of linear differential equations:

$$\frac{d}{dt}(e^{j\omega t}) = j\omega(e^{j\omega t})$$

- Note that EEs use $\boxed{j = (-1)^{1/2}}$ rather than i, since the symbol i is already taken for current

Using Imaginary Exponentials

From the example in Lecture 1:

$$\overset{\tau}{RC}\frac{dv_c}{dt} + v_c = v_s$$

Substitute:

$$\begin{cases} v_s(t) = v_s e^{j\omega t} \longleftarrow \\ v_c(t) = v_c e^{j(\omega t + \phi)} \longleftarrow \end{cases} \quad Re\{ \quad \}$$

Result:

$$\tau(j\omega)v_c e^{j(\omega t+\phi)} + v_c e^{j(\omega t+\phi)} = v_s e^{j\omega t}$$

Finding the Amplitude Ratio

$$[\tau(j\omega)v_c e^{j\phi} + v_c e^{j\phi}]\cancel{e^{j\omega t}} = v_s \cancel{e^{j\omega t}}$$

$$\boxed{[j\omega\tau e^{j\phi} + e^{j\phi}]v_c = v_s}$$ ... use to find amplitude and phase

Amplitude Ratio: $$\frac{v_c}{v_s} = \frac{1}{j\omega\tau e^{j\phi} + e^{j\phi}} = \left|\frac{e^{-j\phi}}{(1 + j\omega\tau)}\right|$$

- Answer is a real number, so take magnitude

$$\frac{v_c}{v_s} = \frac{1}{\sqrt{1+(\omega\tau)^2}}$$

$z = x + jy$

$|z| = \sqrt{x^2+y^2}$

$|e^{-j\phi}| = |\cos(-\phi) + j\sin(-\phi)|$

Finding the Phase

$$[j\omega\tau e^{j\phi} + e^{j\phi}] = \underline{v_s / v_c} \quad \text{(a real number)}$$

Use Euler's formula to convert to rectangular form:

$$j\omega\tau\underbrace{(\cos\phi + j\sin\phi)} + (\cos\phi + j\sin\phi) = v_s / v_c$$

Collect real and imaginary parts; latter must be zero:

$$\mathrm{Im}(\cdot) = \omega\tau\cos\phi + \sin\phi = 0$$

$$\boxed{\tan\phi = -\omega\tau}$$