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LAB THIS WEEK

→ SEE TAs ... TO GET PSPICE!

Lecture 3

[PS 1 DUE @ 4 PM TUESDAY ...]

- Last time:
  - Imaginary exponentials: simplify the math
  - $\boxed{\text{Phasor}}$: complex "prefactor" for $e^{j\omega t}$
- Today :
  - Complex number review
  - Circuit analysis with phasors

ACTUAL / MEASURED

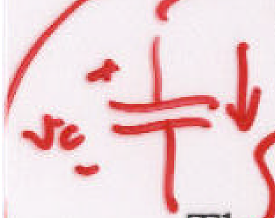
FUNCTION $v(t)$

Finding the "Real" Waveform

- How to connect the imaginary exponential solution to the measured waveform *v(t)*? Conventionally, *v(t)* is the *real part* of the of the imaginary exponential

$$\mathrm{Re}(\underbrace{ve^{j(\omega t+\phi)}}_{\text{ASSUMED}}) = \underbrace{v\cos(\omega t+\phi)}_{\text{MEASURED.}}$$

$$\mathrm{Re}\{x + jy\} = x$$



$i_C = \left(\frac{dv_C}{dt}\right) C \Longrightarrow V$ small letter

Pushing This Idea Further …

There are two parameters needed to define a sinusoidal signal:

* magnitude ✓
* phase ✓

Why not work with a *complex number* as the signal and eliminate the imaginary exponential from the analysis (it cancelled out)?

Define the complex number consisting of the amplitude and phase a sinusoidal signal as a **phasor** V. LECTURE 2

DECEPTIVELY HARD CONCEPT

(in V)

$$\underbrace{v(t) = \hat{v}\cos(\omega t + \phi)} \Leftrightarrow \underbrace{v(t) = Ve^{j\omega t}}.$$

$$\boxed{V = ve^{j\phi}}$$

$$v\cos(\omega t+\phi) \rightarrow ve^{j(\omega t+\phi)} = \left[ve^{j\phi}\right]e^{j\omega t}$$

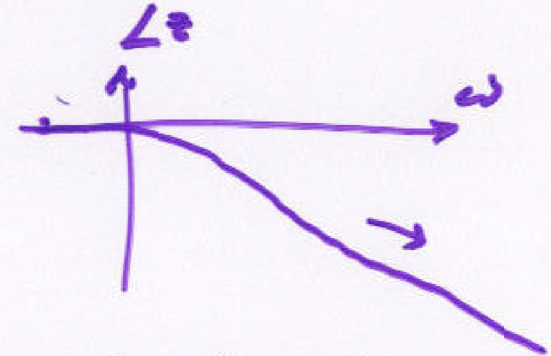
Complex Number Summary

i

- Rectangular form: $z = x + jy$
  - Magnitude $|z| = \sqrt{x^2+y^2}$
  - Phase $\angle z = \tan^{-1}(y/x)$

USEFUL

- Polar form: $z = |z| e^{j\angle z}$



- Useful results (easily shown in polar form):
  - $|z_1 z_2| = |z_1| \cdot |z_2|$ $\quad \angle(z_1 z_2) = \phi_1 + \phi_2$.
  - $|z_1/z_2| = |z_1| / |z_2|$. $\quad \angle z_1/z_2 = \phi_1 - \phi_2$

Question: $\sqrt{j} = \sqrt{\sqrt{-1}}$

$|z_1 z_2| = \left| |z_1| e^{j\phi_1} \cdot |z_1| e^{j\phi_2} \right| = \left| |z_1||z_2| e^{j(\phi_1+\phi_2)} \right|$

$$|z_1 z_2| = \left| \underbrace{Z}_{|z_1||z_2|} e^{j\overbrace{\phi}^{\phi_1+\phi_2}} \right|$$

$$e^{j(\phi_1+\phi_2)} = \cos(\phi_1+\phi_2) + j\sin(\phi_1+\phi_2)$$

$$\left| e^{j(\phi_1+\phi_2)} \right| = 1.$$

GO POLAR!

$$j = 0 + j1 \quad \ldots \quad \text{POLAR FORM} \quad e^{j\cdot\pi/2} = e^{j\cdot 90^\circ}$$

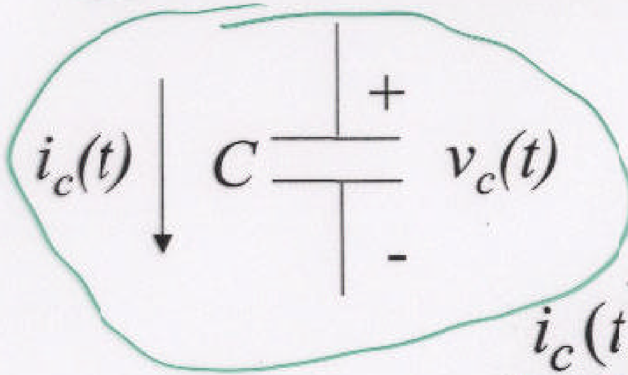
CHECK THIS... KNOW IT

$$j = e^{j\cdot\pi/2}.$$

$$\sqrt{j} = \sqrt{e^{j\pi/2}}$$

$$= \left\{ e^{j\cdot\pi/2} \right\}^{1/2}$$

Using Phasors: Capacitor Current



$$i_c(t) = C\frac{dv_c}{dt}$$

PHASOR

$$i_c(t) = I_c e^{j\omega t} \leftarrow \text{TIME FUNCTION}$$

PHASOR.

$$v_c(t) = V_c e^{j\omega t}$$

$$V_c = |V_c| e^{j\phi_c}$$

$$I_c e^{j\omega t} = C\left\{\frac{d}{dt}\cdot\left[V_c e^{j\omega t}\right]\right\}$$

$$I_c \cancel{e^{j\omega t}} = C \cdot V_c \cdot j\omega \cancel{e^{j\omega t}}$$

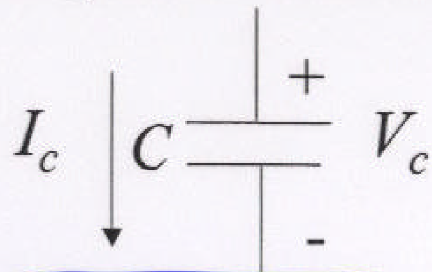
Result:

$$\boxed{I_c = (j\omega C)\cdot V_c}$$

i_R R v_R $\Rightarrow$ $i_R(t) = \left(\frac{1}{R}\right) \cdot v_R(t).$ $q = C v$ $\left(\frac{C}{V}\right) = F$

Impedance of a Capacitor

Definition: the impedance Z of a two-terminal circuit element is the ratio of the phasor voltage to the phasor current (positive reference convention)



$I_c = (j\omega C) \cdot V_c$

$Z_c = \frac{V_c}{I_c}$

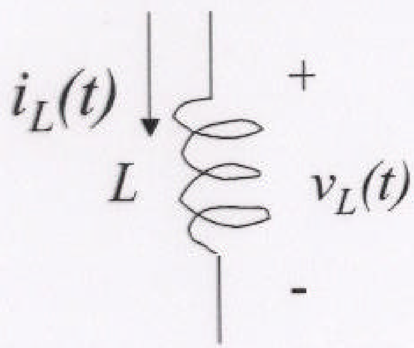
$Z_c = \frac{V_c}{j\omega C \cdot V_c} = \boxed{\frac{1}{j\omega C}}.$

$\boxed{V_c = Z_c \cdot I_c}$

Admittance: $\boxed{Y_c = 1 / Z_c = j\omega C}$

generalized conductance G

Using Phasors: Inductor Voltage



$$\boxed{v_L(t) = L\frac{di_L}{dt}}$$

$$v_L(t) = \underline{\underline{V_L}}e^{j\omega t} \qquad i_L(t) = \underline{\underline{I_L}}e^{j\omega t}$$

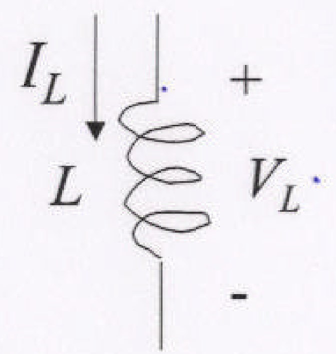
$$v_L = V_L \cancel{e^{j\omega t}} = L \cdot \frac{d}{dt}\{ I_L e^{j\omega t} \}$$

$$= L I_L \cdot j\omega \cancel{e^{j\omega t}}$$

Result: $\boxed{V_L = (j\omega L) \cdot I_L}$

$Z_L = j\omega L.$

Inductor Impedance



$\boxed{\Omega = s^{-1} \cdot H}$

$Z_L = j\omega L$

$\omega \to 0 \text{ rad s}^{-1} \ldots Z_L = j \cdot 0 = 0\,\Omega$ SHORT.

$\omega \to \infty \quad Z_L = j \cdot \infty$ BIG!

Admittance: $Y_L = 1 / Z_L = \dfrac{1}{j\omega L}$

$\omega^* \Rightarrow$

$Z_L(\omega^*) + Z_C(\omega^*) = 0\,\Omega.$

$j\omega L = Z_L$

$\dfrac{1}{j\omega C} = Z_C. = \dfrac{-j}{\omega C}$

$i_R(t)$ R $v_R(t) = R \cdot i_R(t)$

$$\begin{cases} i_R(t) = I_R e^{j\omega t} \\ v_R(t) = V_R e^{j\omega t} \end{cases}$$

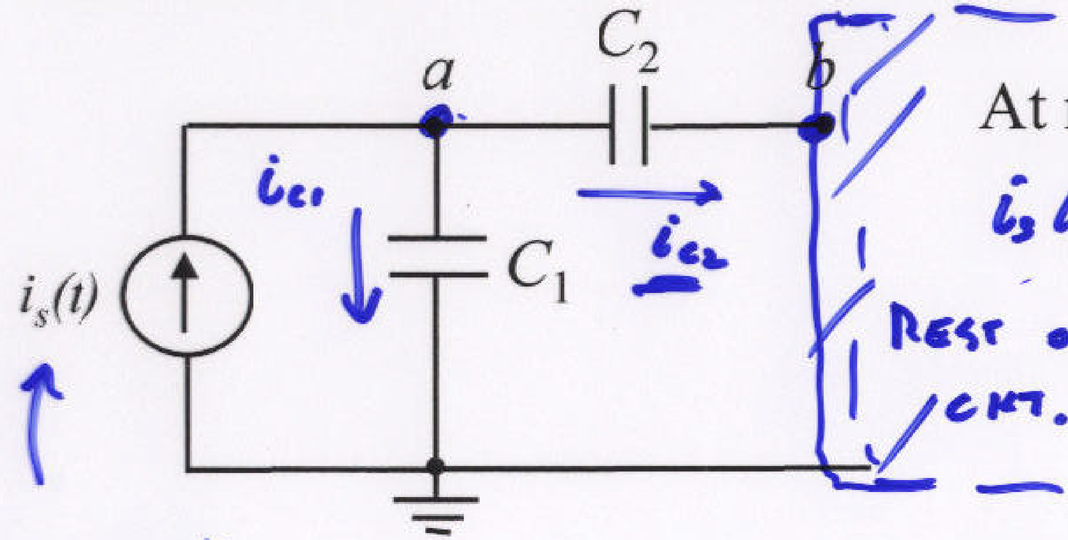
$V_R \cancel{e^{j\omega t}} = R \cdot I_R \cancel{e^{j\omega t}}$

$V_R = R \cdot I_R$

$Z_R = \dfrac{V_R}{I_R} = R.$

KCL

Kirchhoff's Current Law Example



At node a:

$i_s(t) = i_{c1}(t) + i_{c2}(t)$

$I_s e^{j\omega t}$

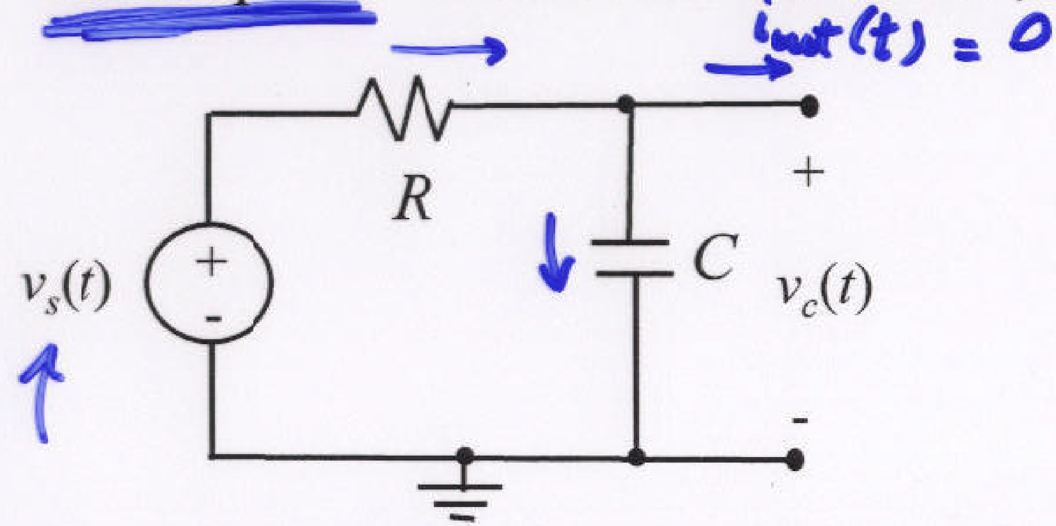
$$i_{c2} = I_{c2} e^{j\omega t} = C_2 \frac{d(v_{c2})}{dt} = C_2 \frac{d[v_a - v_b)}{dt}.$$

$$i_{c1} = I_{c1} e^{j\omega t} = C_1 \frac{dv_{c1}}{dt} = C_1 \frac{dv_a}{dt}.$$

$$v_a(t) = V_a e^{j\omega t} ; \quad v_b(t) = V_b e^{j\omega t}.$$

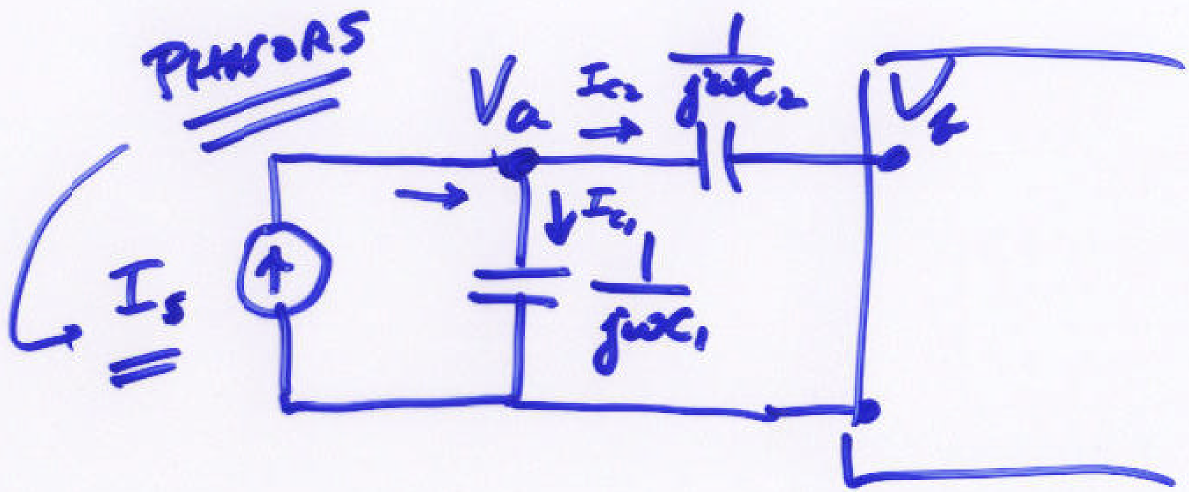
Circuit Analysis with Phasors

Assumption: sources are sinusoidal, steady-state!



$v_s(t) = V_s \cos(\omega t + 0^\circ)$

PICK 0°.



$$I_s = I_{C_1} + I_{C_2}$$

PHASORS!

KCL

$$\boxed{I_s = \underbrace{(j\omega C_1)\cdot V_a}_{} + (j\omega C_2)(V_a - V_o)}$$

$$\frac{V_a}{Z_{C_1}} = \frac{V_a}{\left(\frac{1}{j\omega C_1}\right)}$$